Tutorial On Indoor Localization Using Magnetic-Fields

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Tutorial structure and presenters

Tutorial structure:

Lecture #1: Introduction to Magnetic-Field Localization, by I. Skog

Lecture #2: Magnetic-Field Based Odometry, by G. Hendeby

Demo: Magnetic Source Localization

Lecture #3: Magnetic-Field SLAM, by M. Kok

Presenters

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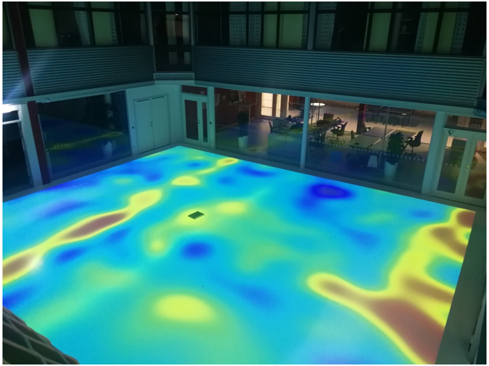
Lecture #1 outline

- Magnetic-fields
  - Dipole fields
  - Geomagnetic field
  - Hard and soft iron effects
  - Magnetic-field properties and modeling
- Localization using dipole models
  - Magnetic-object tracking using multiple magnetometers
  - Self-localization using a single magnetometer and multiple active dipoles
- Localization using magnetic field anomalies
  - Mapping and finger-printing
  - Simultaneous localization and mapping (SLAM)
  - Odometry using magnetic field “images”



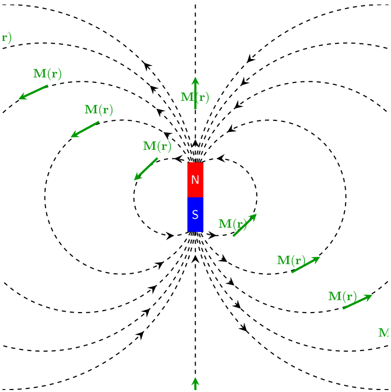
Magnetic fields

Dipole fields

Dipole field:

$$\mathbf{M}(\mathbf{r}) = \frac{\mu_o}{4\pi\|\mathbf{r}\|^3}(3\mathbf{u}_r\mathbf{u}_r^\top - \mathbf{I})\mathbf{m}$$

- μ_o Permeability of free space.
- $\mathbf{r}$ Location w.r.t "center" of dipole.
- $\mathbf{u}_r$ Unit vector pointing towards location r, i.e., $u_r = r/\|r\|$.
- $\mathbf{m}$ Magnetic dipole moment.



Noteworthy:

- Decays cubically
- Rotation invariant around one axis
- SI unit: Tesla [T]

Geomagnetic field 1 (2)

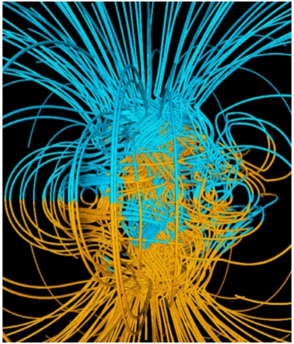
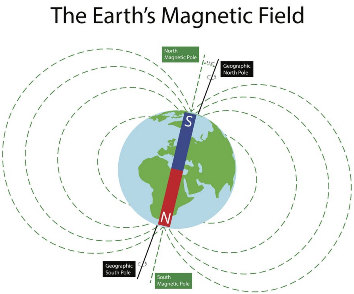


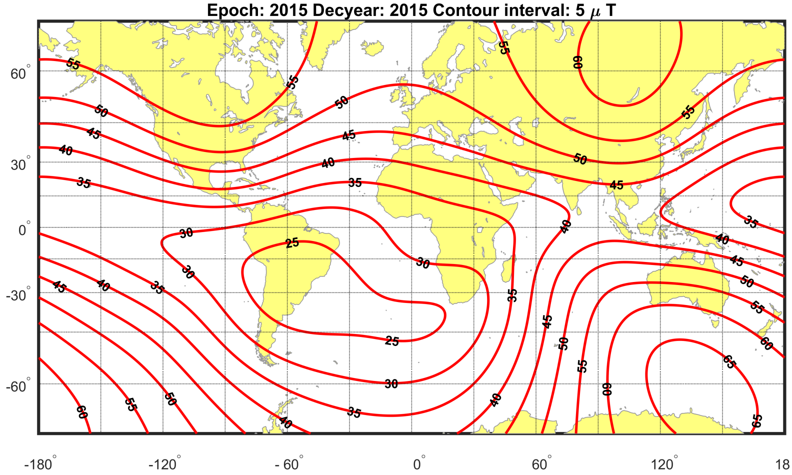
Figure from [4]



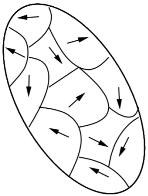
- The field can to first order be approximated as a dipole field (accounts for 80–90% of the true field).
- Magnitude: 25–65 [μT] (0.25-0.65 [G]).
- Tilt angle approx. 11°.

Geomagnetic field 2 (2)

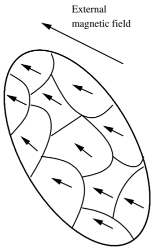
Isodynamic chart of the geomagnetic field intensity calculated World Magnetic Model 2015 (WMM2015)



Magnetized ferromagnetic materials



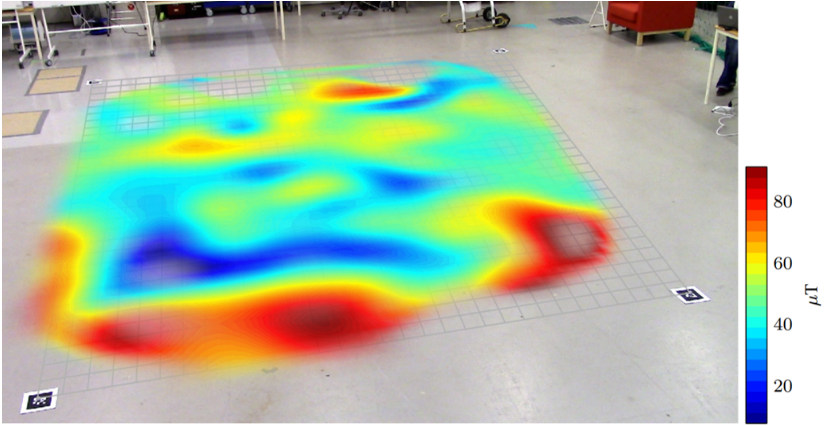
Non-magnetized ferromagnetic material. All dipoles moments are independent of each other and in average we have zero magnetization [12].



Magnetized ferromagnetic material. All dipoles moments are aligned in the direction of the external and gives a large multiplication of the applied field [12].

- Hard-iron magnetization:
  - The material **stay magnetized** after the external field has been removed.
  - Magnetization is aligned with the reference frame of the magnetized object.
- Soft-iron magnetization:
  - The material **do not stay magnetized** after the external field has been removed.
  - Magnetization is aligned with the applied magnetic field.

Local distortion of the geomagnetic field



Complex field created by ferromagnetic materials in the floor and walls interacting with geomagnetic-field. Theoretically it is possible to model the field as a (infinite) sum of dipoles, but in practise it hard to do identify the parameters in such a model.

Magnetic field properties and modeling

Maxwell's equations (in vacuum, no charges or currents):

$$\nabla \cdot E = 0 \qquad \nabla \times E = -\frac{\partial M}{\partial t}$$

$$\nabla \cdot M = 0 \qquad \nabla \times M = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

E electric field

M magnetic field

μ_0 permeability of free space

ε_0 permittivity of free space

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ speed of light

Observations:

- The magnetic field is divergence free: $\nabla \cdot M = 0$
- In a static electric field, the magnetic field is curl free: $\nabla \times M = 0$

$\implies$ A model $\mathcal{M}(\mathbf{r}; \boldsymbol{\theta})$ ($\boldsymbol{\theta} =$ model parameters) of the magnetic-field should (preferably) fulfill this property. Examples of models that can be designed to have these properties are:

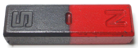
- Sum of dipoles
- Polynomial [9]
- Gaussian process [13]
- Neural networks [5]

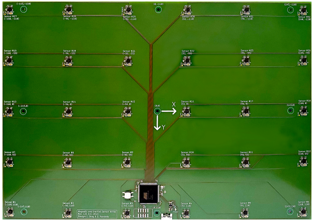
Localization using dipole models

Dipole localization using multiple magnetometers 1 (3)

Measurements from N magnetometers at locations $\{\mathbf{p}^{(i)}\}_{i=1}^{N}$:

$$\underbrace{\begin{bmatrix} \mathbf{y}^{(1)} \\ \vdots \\ \mathbf{y}^{(N)} \end{bmatrix}}_{\triangleq \mathbf{y}} = \underbrace{\begin{bmatrix} \mathcal{M}(\mathbf{p}^{(1)} - \mathbf{r}; \boldsymbol{\theta}) \\ \vdots \\ \mathcal{M}(\mathbf{p}^{(N)} - \mathbf{r}; \boldsymbol{\theta}) \end{bmatrix}}_{\triangleq \mathbf{h}(\mathbf{r}, \boldsymbol{\theta})} + \underbrace{\begin{bmatrix} \mathbf{e}^{(1)} \\ \vdots \\ \mathbf{e}^{(N)} \end{bmatrix}}_{\triangleq \mathbf{e}}$$





With the geomagnetic field subtracted, the measurements are well described by a dipole model. The dipole model is a separable function in the location $\mathbf{r}$ and the dipole moment $\boldsymbol{\theta} \equiv \mathbf{m}$, i.e.,

$$\mathbf{y} = \mathbf{h}(\mathbf{r}, \mathbf{m}) + \mathbf{e} = \mathbf{H}(\mathbf{r})\mathbf{m} + \mathbf{e}$$

where

$$\mathbf{H}(r) = \begin{bmatrix} \mathbf{A}(\mathbf{p}^{(1)} - \mathbf{r}) \\ \vdots \\ \mathbf{A}(\mathbf{p}^{(N)} - \mathbf{r}) \end{bmatrix} \qquad \mathbf{A}(\mathbf{z}) = \frac{\mu_o}{4\pi \|\mathbf{z}\|^3}(3\mathbf{u}_z\mathbf{u}_z^\top - \mathbf{I})$$

Dipole localization using multiple magnetometers 2 (3)

Non-linear least squares problem

$$\{\hat{\mathbf{r}}, \hat{\mathbf{m}}\} = \arg\min_{\mathbf{r},\mathbf{m}} \|\mathbf{y} - \mathbf{H}(\mathbf{r})\mathbf{m}\|^2$$

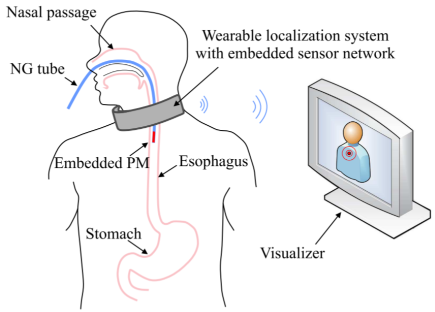
For a fix $\mathbf{r}$ then $\hat{\mathbf{m}}(\mathbf{r}) = \mathbf{H}(\mathbf{r})^\dagger \mathbf{y}$, so the minimization problem can be rewritten as

$$\hat{\mathbf{r}} = \arg\max_{\mathbf{r}} \|\mathbf{H}(\mathbf{r})\mathbf{H}(\mathbf{r})^\dagger \mathbf{y}\|^2$$

Hence, only numerical minimization in 3-dim needed. See [10] for details.

Noteworthy:

- The orientation of the dipole can be estimated from $\hat{\mathbf{m}}$.
- A demonstration of a dipole tracking system will be shown during the coffee break.



Example of a non-invasive magnetic localization system for nasogastric intubation [11].

Dipole localization using multiple magnetometers 3 (3)

By introducing a model for the motion of the magnetic object (dipole) the localization performance can be improved.

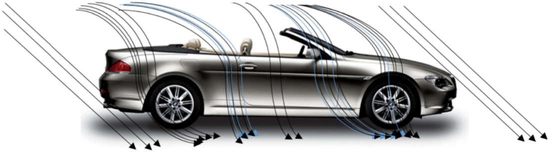
$$\mathbf{x}_k \triangleq \begin{bmatrix} \mathbf{r}_k^\top & \mathbf{v}_k^\top & \mathbf{m}_k^\top \end{bmatrix}^\top$$

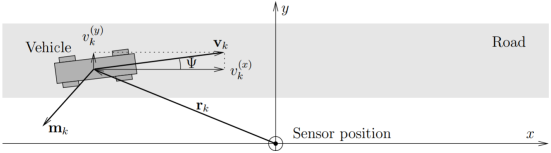
$$\begin{aligned} \mathbf{x}_{k+1} &= f(\mathbf{x}_k) + \mathbf{w}_k \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}_k) + \mathbf{e}_k \end{aligned}$$

where $\mathbf{w}_k \sim \text{AWGN}(\mathbf{Q}_k)$ and $\mathbf{v}_k \sim \text{AWGN}(\mathbf{R}_k)$.

Challenges:

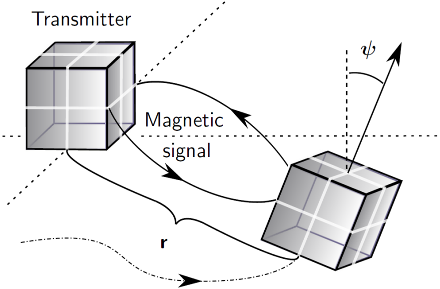
- "Constant" dipole moment may be a poor approximation.
- Extended/large objects $\rightarrow$ Multi-dipole model needed.

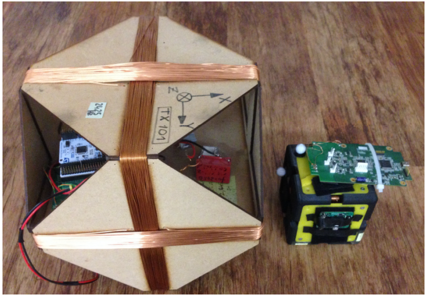




Example of magnetic field based vehicle tracking system. In the far-field, the dipole field is a good first-order model of the magnetic field induced by a vehicle [2, 12].

Self-localization using a single magnetometer and multiple active dipoles

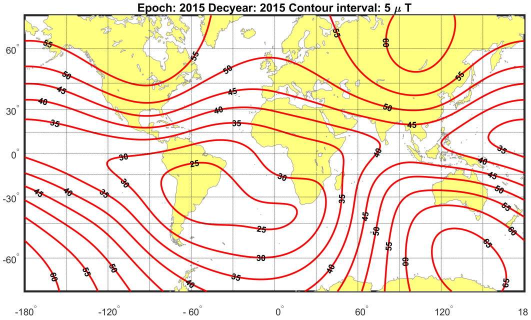


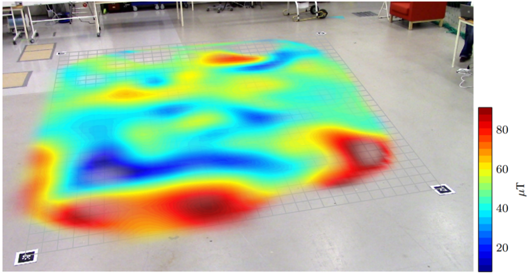


- One dipole + multiple magnetometers $\Longleftrightarrow$ Multiple dipoles + one magnetometer.
- The generated fields must be modulated to separate different dipole sources. See [3, 14] for details.

Localization using geomagnetic field anomalies

Mapping and finger-printing 1 (2)





Basic idea: Correlate measurements with a map of the magnetic field anomalies.

Application examples:

- Macro scale: Satellite localization [7]
- Meso scale: Validation of GNSS data [6]
- Micro scale: Indoor localization [1]

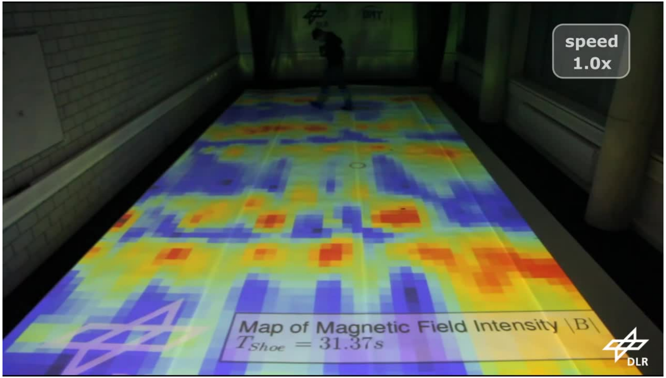
Challenges:

- Non-unique features in the map
- Scalable mathematical represent of the map
- Construction and updating of the map

Mapping and finger-printing 2 (2)

- To avoid the use of scale invariant mapping method, need displacement estimate between meas.
- Fuse with a dead-reckoning system. Short-term resolution + long-term stability
- $\boldsymbol{\eta}$ – magnetic field map.

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \tilde{\mathbf{u}}_k) && \text{(Dead-reckoning)} \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \boldsymbol{\eta}) + \mathbf{e}_k && \text{(Map-matching)} \end{aligned}$$



https://youtu.be/eavk1akiLV4

Mapping and finger-printing 2 (2)

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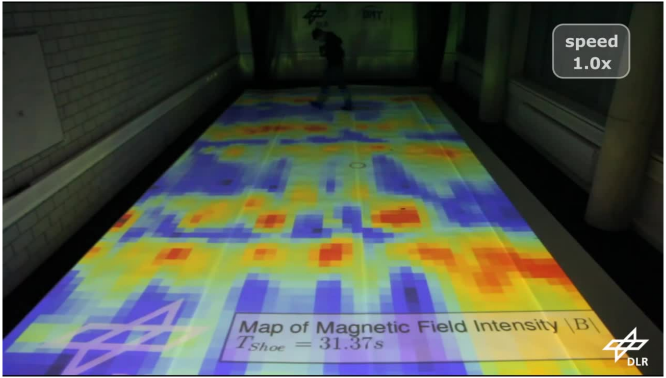
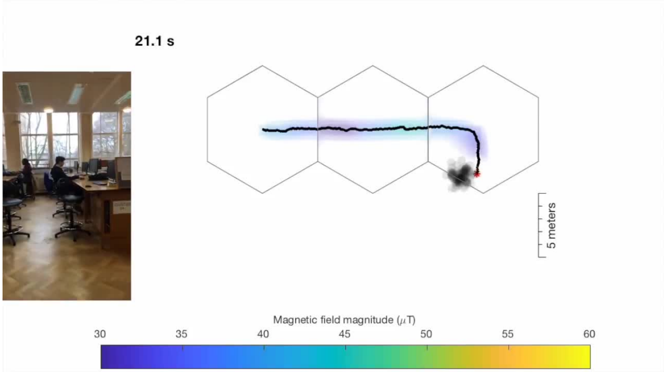


https://youtu.be/eavk1akiLV4

Simultaneous localization and mapping (SLAM)

Idea: Build a map $\boldsymbol{\eta}_k$ of the field on the fly by adding the map as a state in the state-space model.

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \tilde{\mathbf{u}}_k)\\ \boldsymbol{\eta}_{k+1} &= \boldsymbol{\eta}_k\\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \boldsymbol{\eta}_k) + \mathbf{e}_k\end{aligned}$$



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Details in lecture #3, where the magnetic field $\boldsymbol{\eta}_k$ is modeled using a curl-free Gaussian process.

Simultaneous localization and mapping (SLAM)

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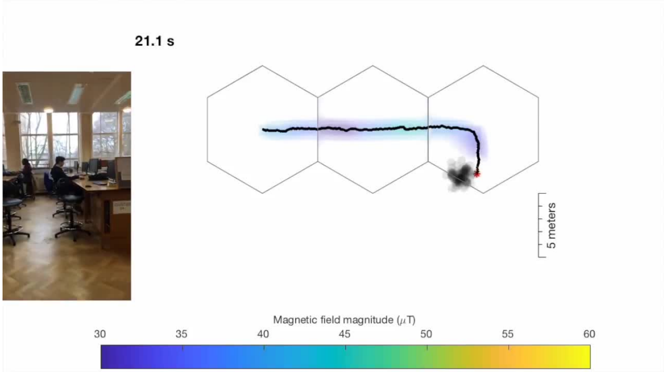


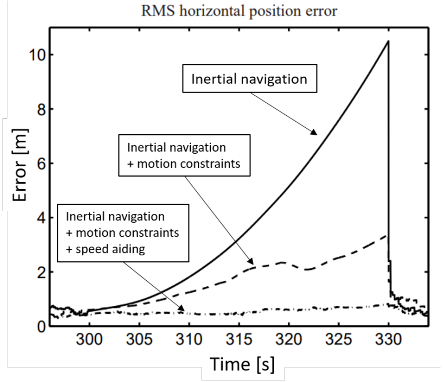
https://youtu.be/pbwWLoh6mvI

Details in lecture #3, where the magnetic field $\boldsymbol{\eta}_k$ is modeled using a curl-free Gaussian process.

Exploration phase challenges in SLAM

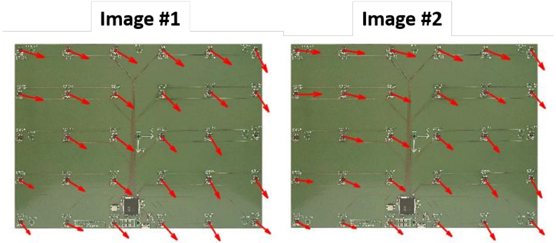
- The fast navigation error growth in low-cost inertial navigation systems together with the non-unique "features" in the observed field limits the allowable length of the exploration phase, i.e., the time between "loop-closures".

- Techniques to reduce the error growth rate:
  - Motion constraints
  - Aiding sensors
- Techniques to get more unique "features":
  - Go from point estimate to "image" of the magnetic-field.



Example of the position error growth in an inertial navigation system using low-cost sensor [8].

Magnetic field odometry 1 (2)



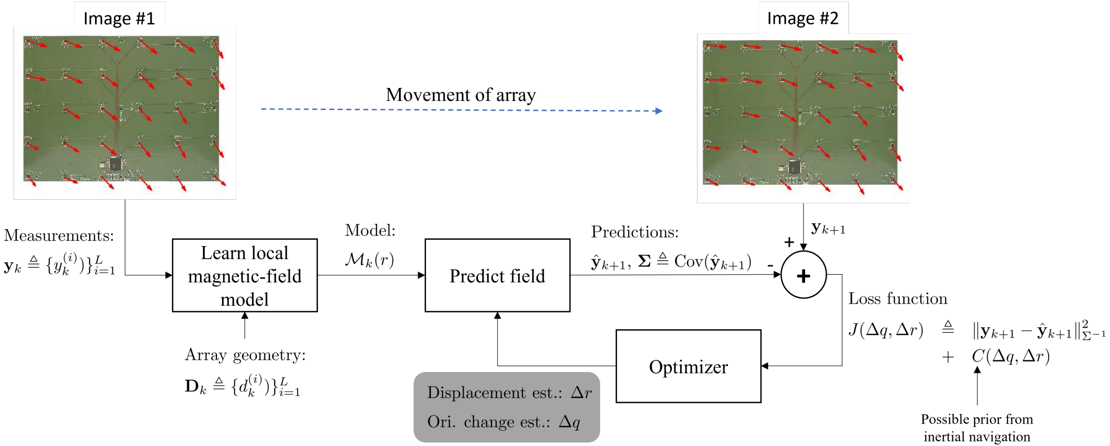
Visual image:

- High-resolution, i.e., many feature points.
- Unstructured environment.

Magnetic-field image:

- Low-resolution, i.e., few feature points.
- Structured environment.

Magnetic field odometry 2 (2)



An optical flow inspired approach to magnetic-field odometry. Details during lecture #2.

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