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Concerning an Heuristic Point of View Toward the Emission and Transformation of Light

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A profound formal distinction exists between the theoretical concepts which physicists have formed regarding gases and other ponderable bodies and the Maxwellian theory of electromagnetic processes in so–called empty space. While we consider the state of a body to be completely determined by the positions and velocities of a very large, yet finite, number of atoms and electrons, we make use of continuous spatial functions to describe the electromagnetic state of a given volume, and a finite number of parameters cannot be regarded as sufficient for the complete determination of such a state. According to the Maxwellian theory, energy is to be considered a continuous spatial function in the case of all purely electromagnetic phenomena including light, while the energy of a ponderable object should, according to the present conceptions of physicists, be represented as a sum carried over the atoms and electrons. The energy of a ponderable body cannot be subdivided into arbitrarily many or arbitrarily small parts, while the energy of a beam of light from a point source (according to the Maxwellian theory of light or, more generally, according to any wave theory) is continuously spread an ever increasing volume.

The wave theory of light, which operates with continuous spatial functions, has worked well in the representation of purely optical phenomena

and will probably never be replaced by another theory. It should be kept in mind, however, that the optical observations refer to time averages rather than instantaneous values. In spite of the complete experimental confirmation of the theory as applied to diffraction, reflection, refraction, dispersion, etc., it is still conceivable that the theory of light which operates with continuous spatial functions may lead to contradictions with experience when it is applied to the phenomena of emission and transformation of light.

It seems to me that the observations associated with blackbody radiation, fluorescence, the production of cathode rays by ultraviolet light, and other related phenomena connected with the emission or transformation of light are more readily understood if one assumes that the energy of light is discontinuously distributed in space. In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.

In the following I wish to present the line of thought and the facts which have led me to this point of view, hoping that this approach may be useful to some investigators in their research.

1. Concerning a Difficulty with Regard to the Theory of Blackbody Radiation

We start first with the point of view taken in the Maxwellian and the electron theories and consider the following case. In a space enclosed by completely reflecting walls, let there be a number of gas molecules and electrons which are free to move and which exert conservative forces on each other on close approach: i.e. they can collide with each other like molecules in the kinetic theory of gases.[1] Furthermore, let there be a number of electrons which are bound to widely separated points by forces proportional to their distances from these points. The bound electrons are also to participate in conservative interactions with the free molecules and electrons when the latter come

[1] This assumption is equivalent to the supposition that the average kinetic energies of gas molecules and electrons are equal to each other at thermal equilibrium. It is well known that, with the help of this assumption, Herr Drude derived a theoretical expression for the ratio of thermal and electrical conductivities of metals.

very close. We call the bound electrons "oscillators": they emit and absorb electromagnetic waves of definite periods.

According to the present view regarding the origin of light, the radiation in the space we are considering (radiation which is found for the case of dynamic equilibrium in accordance with the Maxwellian theory) must be identical with the blackbody radiation — at least if oscillators of all the relevant frequencies are considered to be present.

For the time being, we disregard the radiation emitted and absorbed by the oscillators and inquire into the condition of dynamical equilibrium associated with the interaction (or collision) of molecules and electrons. The kinetic theory of gases asserts that the average kinetic energy of an oscillator electron must be equal to the average kinetic energy of a translating gas molecule. If we separate the motion of an oscillator electron into three components at angles to each other, we find for the average energy $\overline{E}$ of one of these linear components the expression

$$\overline{E} = (R/N)\ T,$$

where R denotes the universal gas constant. N denotes the number of "real molecules" in a gram equivalent, and T the absolute temperature. The energy $\overline{E}$ is equal to two-thirds the kinetic energy of a free monatomic gas particle because of the equality the time average values of the kinetic and potential energies of the oscillator. If through any cause—in our case through radiation processes—it should occur that the energy of an oscillator takes on a time-average value greater or less than $\overline{E}$, then the collisions with the free electrons and molecules would lead to a gain or loss of energy by the gas, different on the average from zero. Therefore, in the case we are considering, dynamic equilibrium is possible only when each oscillator has the average energy $\overline{E}$.

We shall now proceed to present a similar argument regarding the interaction between the oscillators and the radiation present in the cavity. Herr Planck has derived[2] the condition for the dynamics equilibrium in this case under the supposition that the radiation can be considered a completely random process.[3] He found

$$(\overline{E}_\nu) = (L^3/8\pi\nu^2)\rho_\nu,$$

[2] M. Planck, Ann. Phys. 1, 99 (1900).

[3] This problem can be formulated in the following manner. We expand the Z component of the electrical force (Z) at an arbitrary point during the time interval between $t = 0$ and $t = T$ in a Fourier series in which $A_\nu \geq 0$ and $0 \leq \alpha_\nu \leq 2\pi$: the time T is taken to

where $(\overline{E}_\nu)$ is the average energy (per degree of freedom) of an oscillator with eigenfrequency ν, L the velocity of light, ν the frequency, and $\rho_\nu d\nu$ the energy per unit volume of that portion of the radiation with frequency between ν and $\nu + d\nu$.

If the radiation energy of frequency ν is not continually increasing or decreasing, the following relations must obtain:

$$(R/N)\ T = \overline{E} = \overline{E}\nu = (L^3/8\pi\nu^2)\rho_\nu,$$

$$\rho_\nu = (R/N)(8\pi\nu^2/L^3)\ T.$$

These relations, found to be the conditions of dynamic equilibrium, not only fail to coincide with experiment, but also state that in our model there can be not talk of a definite energy distribution between ether and matter. The wider the range of wave numbers of the oscillators, the greater will be the radiation energy of the space, and in the limit we obtain

$$\int_0^\infty \rho_\nu\ d\nu = \frac{R}{N} \cdot \frac{8\pi}{L^3} \cdot T \int_0^\infty \nu^2\ d\nu = \infty.$$

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be very large relative to all the periods of oscillation that are present:

$$Z = \sum_{\nu=1}^{\nu=\infty} A_\nu \sin\left(2\pi\nu\frac{t}{T} + \alpha_\nu\right),$$

If one imagines making this expansion arbitrary often at a given point in space at randomly chosen instants of time, one will obtain various sets of values of A_ν and α_ν. There then exist for the frequency of occurrence of different sets of values of A_ν and α_ν (statistical) probabilities dW of the form:

$$dW = f(a_1, A_2, \ldots, \alpha_1, \alpha_2, \ldots)dA_1 dA_2 \ldots d\alpha_1 d\alpha_2 \ldots,$$

The radiation is then as disordered as conceivable if

$$f(A_1, A_2, \ldots \alpha_1, \alpha_2, \ldots) = F_1(A_1)F_2(A_2)\ldots f_1(\alpha_1)f_2(\alpha_2)\ldots,$$

i.e., if the probability of a particular value of A or α is independent of other values of A or α. The more closely this condition is fulfilled (namely, that the individual pairs of values of A_ν and α_ν are dependent upon the emission and absorption processes of specific groups of oscillators) the more closely will radiation in the case being considered approximate a perfectly random state.

2. Concerning Planck's Determination of the Fundamental Constants

We wish to show in the following that Herr Planck's determination of the fundamental constants is, to a certain extent, independent of his theory of blackbody radiation.

Planck's formula,[4] which has proved adequate up to this point, gives for ρ_ν

$$\rho_\nu = \frac{\alpha\nu^3}{e^{\beta\nu/T} - 1},$$

$$\alpha = 6.10 \times 10^{-56},$$

$$\beta = 4.866 \times 10^{-11}.$$

For large values of T/ν; i.e. for large wavelengths and radiation densities, this equation takes the form

$$\rho_\nu = (\alpha/\beta)\ \nu^2 T.$$

It is evident that this equation is identical with the one obtained in Sec. 1 from the Maxwellian and electron theories. By equating the coefficients of both formulas one obtains

$$(R/N)(8\pi/L^3) = (\alpha/\beta)$$

or

$$N = (\beta/\alpha)(8\pi R/L^3) = 6.17 \times 10^{23}.$$

i.e., an atom of hydrogen weighs $1/N$ grams $= 1.62 \times 10^{-24}$ g. This is exactly the value found by Herr Planck, which in turn agrees with values found by other methods.

We therefore arrive at the conclusion: the greater the energy density and the wavelength of a radiation, the more useful do the theoretical principles we have employed turn out to be: for small wavelengths and small radiation densities, however, these principles fail us completely.

In the following we shall consider the experimental facts concerning blackbody radiation without invoking a model for the emission and propagation of the radiation itself.

[4] M. Planck, Ann. Phys. 4, 561 (1901).

3. Concerning the Entropy of Radiation

The following treatment is to be found in a famous work by Herr W. Wien and is introduced here only for the sake of completeness.

Suppose we have radiation occupying a volume v. We assume that the observable properties of the radiation are completely determined when the radiation density $\rho(\nu)$ is given for all frequencies.[5] Since radiation of different frequencies are to be considered independent of each other when there is no transfer of heat or work, the entropy of the radiation can be represented by

$$S = v \int_0^\infty \varphi(\rho, \nu)\ d\nu,$$

where φ is a function of the variables ρ and ν.

φ can be reduced to a function of a single variable through formulation of the condition that the entropy of the radiation is unaltered during adiabatic compression between reflecting walls. We shall not enter into this problem, however, but shall directly investigate the derivation of the function φ from the blackbody radiation law.

In the case of blackbody radiation, ρ is such a function of ν that the entropy is maximum for a fixed value of energy; i.e.,

$$\delta \int_0^\infty \varphi\ (\rho, \nu)\ d\nu = 0,$$

providing

$$\delta \int_0^\infty \rho d\nu = 0.$$

From this it follows that for every choice of $\delta\rho$ as a function of ν

$$\int_0^\infty \left(\frac{\partial \varphi}{\partial \rho} - \lambda \right) \delta\rho d\nu = 0,$$

where λ is independent of ν. In the case of blackbody radiation, therefore, $\partial\varphi/\partial\rho$ is independent of ν.

[5] This assumption is an arbitrary one. One will naturally cling to this simplest assumption as long as it is not controverted experiment.

The following equation applies when the temperature of a unit volume of blackbody radiation increases by dT

$$dS = \int_{\nu=0}^{\nu=\infty} \left(\frac{\partial\varphi}{\partial\rho}\right) \, d\rho d\nu,$$

or, since $\partial\varphi/\partial\rho$ is independent of ν.

$$dS = (\partial\varphi/\partial\rho) \, dE.$$

Since dE is equal to the heat added and since the process is reversible, the following statement also applies

$$dS = (1/T) \, dE.$$

By comparison one obtains

$$\partial\varphi/\partial\rho = 1/T.$$

This is the law of blackbody radiation. Therefore one can derive the law of blackbody radiation from the function φ, and, inversely, one can derive the function φ by integration, keeping in mind the fact that φ vanishes when $\rho = 0$.

4. Asymptotic from for the Entropy of Monochromatic Radiation at Low Radiation Density

From existing observations of the blackbody radiation, it is clear that the law originally postulated by Herr W. Wien,

$$\rho = \alpha\nu^3 e^{-\beta\nu/T},$$

is not exactly valid. It is, however, well confirmed experimentally for large values of ν/T. We shall base our analysis on this formula, keeping in mind that our results are only valid within certain limits.

This formula gives immediately

$$(1/T) = -(1/\beta\nu) \, \ln \, (\rho/\alpha\nu^3)$$

and then, by using the relation obtained in the preceeding section,

$$\varphi(\rho,\nu) = -\frac{\rho}{\beta\nu}\left[\ln\left(\frac{\rho}{\alpha\nu^3}\right) - 1\right].$$

Suppose that we have radiation of energy E, with frequency between ν and $\nu + d\nu$, enclosed in volume v. The entropy of this radiation is:

$$S = v\varphi(\rho,\nu)d\nu = -\frac{E}{\beta\nu}\left[\ln\left(\frac{E}{v\alpha\nu^3 d\nu}\right) - 1\right].$$

If we confine ourselves to investigating the dependence of the entropy on the volume occupied by the radiation, and if we denote by S_0 the entropy of the radiation at volume v_0, we obtain

$$S - S_0 = (E/\beta\nu)\ \ln\ (v/v_0).$$

This equation shows that the entropy of a monochromatic radiation of sufficiently low density varies with the volume in the same manner as the entropy of an ideal gas or a dilute solution. In the following, this equation will be interpreted in accordance with the principle introduced into physics by Herr Boltzmann, namely that the entropy of a system is a function of the probability its state.

5. Molecular–Theoretic Investigation of the Dependence of the Entropy of Gases and Dilute solutions on the volume

In the calculation of entropy by molecular–theoretic methods we frequently use the word "probability" in a sense differing from that employed in the calculus of probabilities. In particular "gases of equal probability" have frequently been hypothetically established when one theoretical models being utilized are definite enough to permit a deduction rather than a conjecture. I will show in a separate paper that the so-called "statistical probability" is fully adequate for the treatment of thermal phenomena, and I hope that by doing so I will eliminate a logical difficulty that obstructs the application of Boltzmann' s Principle. here, however, only a general formulation and application to very special cases will be given.

If it is reasonable to speak of the probability of the state of a system, and futhermore if every entropy increase can be understood as a transition to a state of higher probability, then the entropy S_1 of a system is a function of W_1, the probability of its instantaneous state. If we have two noninteracting systems S_1 and S_2, we can write

$$S_1 = \varphi_1(W_1),$$

$$S_2 = \varphi_2(W_2).$$

If one considers these two systems as a single system of entropy S and probability W, it follows that

$$S = S_1 + S_2 = \varphi(W)$$

and

$$W = W_1 \cdot W_2.$$

The last equation says that the states of the two systems are independent of each other.

From these equation it follows that

$$\varphi(W_1 \cdot W_2) = \varphi_1(W_1) + \varphi_2(W_2)$$

and finally

$$\varphi_1(W_1) = C\ \ln(W_1) + \text{const},$$

$$\varphi_2(W_2) = C\ \ln(W_2) + \text{const},$$

$$\varphi(W) = C\ \ln(W) + \text{const}.$$

The quantity C is therefore a universal constant; the kinetic theory of gases shows its value to be R/N, where the constants R and N have been defined above. If S_0 denotes the entropy of a system in some initial state and W denotes the relative probability of a state of entropy S, we obtain in general

$$S - S_0 = (R/N)\ \ln\ W.$$

First we treat the following special case. We consider a number (n) of movable points (e.g., molecules) confined in a volume v_0. Besides these points, there can be in the space any number of other movable points of any kind. We shall not assume anything concerning the law in accordance with which the points move in this space except that with regard to this motion, no part of the space (and no direction within it) can be distinguished from

any other. Further, we take the number of these movable points to be so small that we can disregard interactions between them.

This system, which, for example, can be an ideal gas or a dilute solution, possesses an entropy S_0. Let us imagine transferring all n movable points into a volume v (part of the volume v_0) without anything else being changed in the system. This state obviously possesses a different entropy (S), and now wish to evaluate the entropy difference with the help of the Boltzmann Principle.

We inquire: How large is the probability of the latter state relative to the original one? Or: How large is the probability that at a randomly chosen instant of time all n movable points in the given volume v_0 will be found by chance in the volume v?

For this probability, which is a "statistical probability", one obviously obtains:

$$W = (v/v_0)^n;$$

By applying the Boltzmann Principle, one then obtains

$$S - S_0 = R\ (n/N)\ \ln\ (v/v_0).$$

It is noteworthy that in the derivation of this equation, from which one can easily obtain the law of Boyle and Gay–Lussac as well as the analogous law of osmotic pressure thermodynamically,[6] no assumption had to be made as to a law of motion of the molecules.

6. Interpretation of the Expression for the volume Dependence of the entropy of Monochromatic Radiation in Accordance with Boltzmann's Principle

In Sec. 4, we found the following expression for the dependence of the entropy of monochromatic radiation on the volume

$$S - S_0 = (E/\beta\nu)\ \ln\ (v/v_0).$$

[6] If E is the energy of the system, one obtains:

$$-d \cdot (E - TS) = pdv = TdS = RT \cdot (n/N) \cdot (dv/v);$$

therefore

$$pv = R \cdot (n/N) \cdot T.$$

If one writes this in the from

$$S - S_0 = (R/N)\ \ln\left[(v/v_0)^{(N/R)(E/\beta\nu)}\right].$$

and if one compares this with the general formula for the Boltzmann principle

$$S - S_0 = (R/N)\ \ln W,$$

one arrives at the following conclusion:

If monochromatic radiation of frequency ν and energy E is enclosed by reflecting walls in a volume v_0, the probability that the total radiation energy will be found in a volume v (part of the volume v_0) at any randomly chosen instant is

$$W = (v/v_0)^{(N/R)(E/\beta\nu)}.$$

From this we further conclude that: Monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as though it consisted of a number of independent energy quanta of magnitude $R\beta\nu/N$.

We still wish to compare the average magnitude of the energy quanta of the blackbody radiation with the average translational kinetic energy of a molecule at the same temperature. The latter is ${}^3/_2(R/N)T$, while, according to the Wien formula, one obtains for the average magnitude of an energy quantum

$$\int_0^\infty \alpha\nu^3 e^{-\beta\nu/T} d\nu \Big/ \int_0^\infty \frac{N}{R\beta\nu}\ \alpha\nu^3 e^{-\beta\nu/T} d\nu = 3(RT/N).$$

If the entropy of monochromatic radiation depends on volume as though the radiation were a discontinuous medium consisting of energy quanta of magnitude $R\beta\nu/N$, the next obvious step is to investigate whether the laws of emission and transformation of light are also of such a nature that they can be interpreted or explained by considering light to consist of such energy quanta. We shall examine this question in the following.

7. Concerning Stokes's Rule

According to the result just obtained, let us assume that, when monochromatic light is transformed through photoluminescence into light of a different

frequency, both the incident and emitted light consist of energy quanta of magnitude $R\beta\nu/N$, where ν denotes the relevant frequency. The transformation process is to be interpreted in the following manner. Each incident energy quantum of frequency ν_1 is absorbed and generates by itself–at least at sufficiently low densities of incident energy quanta – a light quantum of frequency ν_2; it is possible that the absorption of the incident light quanta can give rise to the simultaneous emission of light quanta of frequencies ν_3, ν_4 etc., as well as to energy of other kinds, e.g., heat. It does not matter what intermediate processes give rise to this final result. If the fluorescent substance is not a perpetual source of energy, the principle of conservation of energy requires that the energy of an emitted energy quantum cannot be greater than that of the incident light quantum; it follows that

$$R\ \beta\nu_2/N \leq R\ \beta\nu_1/N$$

or

$$\nu_2 \leq \nu_1.$$

This is the well–known Stokes's Rule.

It should be strongly emphasized that according to our conception the quantity of light emitted under conditions of low illumination (other conditions remaining constant) must be proportional to the strength of the incident light, since each incident energy quantum will cause an elementary process of the postulated kind, independently of the action of other incident energy quanta. In particular, there will be no lower limit for the intensity of incident light necessary to excite the fluorescent effect.

According to the conception set forth above, deviations from Stokes's Rule are conceivable in the following cases:

1. when the number of simultaneously interacting energy quanta per unit volume is so large that an energy quantum of emitted light can receive its energy from several incident energy quanta;

2. when the incident (or emitted) light is not of such a composition that it corresponds to blackbody radiation within the range of validity of Wien's Law, that is to say, for example, when the incident light is produced by a body of such high temperature that for the wavelengths under consideration Wien's Law is no longer valid.

The last-mentioned possibility commands especial interest. According to the conception we have outlined, the possibility is not excluded that a "non-Wien radiation" of very low density can exhibit an energy behavior different from that of a blackbody radiation within the range of validity of Wien's Law.

8. Concerning the Emission of Cathode Rays Through Illumination of Solid Bodies

The usual conception that the energy of light is continuously distributed over the space through which it propagates, encounters very serious difficulties when one attempts to explain the photoelectric phenomena, as has been pointed out in Herr Lenard's pioneering paper.[7]

According to the concept that the incident light consists of energy quanta of magnitude $R\beta\nu/N$, however, one can conceive of the ejection of electrons by light in the following way. Energy quanta penetrate into the surface layer of the body, and their energy is transformed, at least in part, into kinetic energy of electrons. The simplest way to imagine this is that a light quantum delivers its entire energy to a single electron: we shall assume that this is what happens. The possibility should not be excluded, however, that electrons might receive their energy only in part from the light quantum.

An electron to which kinetic energy has been imparted in the interior of the body will have lost some of this energy by the time it reaches the surface. Furthermore, we shall assume that in leaving the body each electron must perform an amount of work P characteristic of the substance. The ejected electrons leaving the body with the largest normal velocity will be those that were directly at the surface. The kinetic energy of such electrons is given by

$$R\ \beta\nu/N - P.$$

In the body is charged to a positive potential Π and is surrounded by conductors at zero potential, and if Π is just large enough to prevent loss of electricity by the body, if follows that:

$$\Pi\epsilon = R\beta\nu/N - P$$

where ϵ denotes the electronic charge, or

$$\Pi E = R\beta\nu - P'$$

where E is the charge of a gram equivalent of a monovalent ion and P' is the potential of this quantity of negative electricity relative to the body.[8]

[7] P. Lenard, Ann. Phys., 8, 169, 170 (1902).

[8] If one assumes that the individual electron is detached from a neutral molecule by light with the performance of a certain amount of work, nothing in the relation derived above need be changed; one can simply consider P' as the sum of two terms.

If one takes $E = 9.6 \times 10^3$, then $\Pi \cdot 10^{-8}$ is the potential in volts which the body assumes when irradiated in a vacuum.

In order to see whether the derived relation yields an order of magnitude consistent with experience, we take $P' = 0$, $\nu = 1.03 \times 10^{15}$ (corresponding to the limit of the solar spectrum toward the ultraviolet) and $\beta = 4.866 \times 10^{-11}$. We obtain $\Pi \cdot 10^7 = 4.3$ volts, a result agreeing in order magnitude with those of Herr Lenard.[9]

If the derived formula is correct, then Π, when represented in Cartesian coordinates as a function of the frequency of the incident light, must be a straight line whose slope is independent of the nature of the emitting substance.

As far as I can see, there is no contradiction between these conceptions and the properties of the photoelectric observed by Herr Lenard. If each energy quantum of the incident light, independently of everything else, delivers its energy of electrons, then the velocity distribution of the ejected electrons will be independent of the intensity of the incident light; on the other hand the number of electrons leaving the body will, if other conditions are kept constant, be proportional to the intensity of the incident light.[10]

Remarks similar to those made concerning hypothetical deviations from Stokes's Rule can be made with regard to hypothetical boundaries of validity of the law set forth above.

In the foregoing it has been assumed that the energy of at least some of the quanta of the incident light is delivered completely to individual electrons. If one does not make this obvious assumption, one obtains, in place of the last equation:

$$\Pi E + P' \leq R\beta\nu.$$

For fluorescence induced by cathode rays, which is the inverse process to the one discussed above, one obtains by analogous considerations:

$$\Pi E + P' \geq R\beta\nu.$$

In the case, of the substances investigated by Herr Lenard, PE [11]is always significantly greater than $R\beta\nu$, since the potential difference, which the cathode rays must traverse in order to produce visible light, amounts in some cases to hundreds and in others to thousands of volts.[12] It is therefore to

[9] P.Lenard, Ann. Phys. 8, pp. 163, 185, and Table I, Fig. 2 (1902).
[10] P. Lenard, Ref. 9, p. 150 and p. 166–168.
[11] Should be ΠE (translator's note).
[12] P. Lenard, Ann. Phys., 12, 469 (1903).

be assumed that the kinetic energy of an electron goes into the production of many light energy quanta.

9. Concerning the Ionization of Gases by Ultraviolet Light Solid Bodies

We shall have to assume that, the ionization of a gas by ultraviolet light, an individual light energy quantum is used for the ionization of an individual gas molecule. From this is follows immediately that the work of ionization (i.e., the work theoretically needed for ionization) of a molecule cannot be greater than the energy of an absorbed light quantum capable of producing this effect. If one denotes by J the (theoretical) work of ionization per gram equivalent, then it follows that:

$$R\,\beta\nu \geq J.$$

According to Lenard's measurements, however, the largest effective wavelength for air is approximately 1.9×10^{-5} cm: therefore:

$$R\,\beta\nu = 6.4 \cdot 10^{12}\ \text{erg} \geq J.$$

An upper limit for the work of ionization can also be obtained from the ionization potentials of rarefied gases. according to J. Stark[13] the smallest observed ionization potentials for air (at platinum anodes) is about 10 V.[14] One therefore obtains 9.6×10^{12} as an upper limit for J, which is nearly equal to the value found above.

There is another consequence the experimental testing of which seems to me to be of great importance. If every absorbed light energy quantum ionizes a molecule, the following relation must obtain between the quantity of absorbed light L and the number of gram molecules of ionized gas j:

$$j = L/R\beta\nu.$$

If our conception is correct, this relationship must be valid for all gases which (at the relevant frequency) show no appreciable absorption without ionization.

[13] J. Stark, Die Electrizitët in Gasen (Leipzig, 1902, p. 57)

[14] In the interior of gases the ionization potential for negative ions is, however, five times greater.

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