Direct prediction-error identification of unstable nonlinear systems applied to flight test data

Roger Larsson *,** **Zoran Sjanic** *,** **Martin Enqvist** * **Lennart Ljung** *

* *Linköping University, Linköping, 581 83 Sweden*
e-mail: {`roglar, zoran, maren, ljung`}`@isy.liu.se`
** *Saab Aerosystems, Linköping, 581 88 Sweden*

Abstract: Control system design for advanced, highly agile fighter aircraft, with unstable nonlinear aerodynamic characteristics, rely heavily on flight mechanical simulations. This makes the accuracy of the aerodynamic model in the simulators very important. Here, two methods for estimating parameters of nonlinear unstable systems where the control system is unknown are presented. Both approaches are direct prediction-error methods, either with a directly parametrized observer or with an Extended Kalman Filter as a predictor. These methods have been validated on simulated data, as well as on real flight test data and all approaches show promising results.

Keywords: System identification; Prediction-error method; Nonlinear system; Unstable; Kalman filter; Aerodynamics; Flight test.

1. INTRODUCTION

In the early year of motorized flight, the Wright brothers flew their Flyer I manually. Later, analysis [Culick, 2001] shows that this aircraft had both unstable and nonlinear aerodynamic pitch characteristics. Fortunately, the time constants were small enough for the brothers to handle. As design of aircraft led to higher speeds it was necessary to turn to inherently pitch stable solutions. Today's highly maneuverable fighter aircraft are designed to be unstable in the pitch plane in order to gain performance. The speed has increased so much that supersonic flight is not uncommon. Near transonic speed, i.e. close to the speed of sound, nonlinearities can occur due to aerodynamic shocks moving over the aircraft. The instability and nonlinearity have made the modern fighter aircraft dependent on control systems that aid the pilot in flying the aircraft. In order to design the control laws, high quality simulation models are needed. A modern fighter aircraft such as JAS 39 Gripen has a very complex flight control system which is gain scheduled and handles many different flight modes. It is therefore desirable to be able to use a direct identification method [Ljung, 1999] on flight test data, i.e. to be able to identify the system without any knowledge of the control system.

Several books on the subject of aircraft identification have been published recently [Jategaonkar, 2006, Klein and Morelli, 2006, Tischler and Remple, 2006]. However, most work in this field has focused either on unstable and linear [Ghaffari et al., 2007] or stable and nonlinear systems [Bruce and Kellett, 1998, Paris and Alaverdi, 2005]. There are some papers that mention both nonlinear and unstable system, like Horton [1997] and Jategaonkar and Thielecke [2000], but where the methods are different from the ones presented in this paper.

In this paper, methods for estimating parameters for a nonlinear unstable system are presented. The theory is based on the direct prediction-error method. Here, two different predictors have been used, a parametrized observer and an Extended Kalman Filter. These methods have been applied to both simulated data and real flight test data.

The methods are described in Section 2. In Section 3, results based on simulated data are presented. An estimation on real flight test data is given in Section 4 and then conclusions are made in Section 5.

2. METHOD

In this paper, the following discrete-time state-space representation of a nonlinear output-error model is used:

$$\begin{aligned} x(t+1) &= f(x(t), u(t); \theta), \\ y(t) &= h(x(t), u(t); \theta) + e(t), \end{aligned} \tag{1}$$

where θ is the unknown parameters to be identified, $e(t)$ is white noise with zero mean and covariance matrix R, $x(t)$ is a $n \times 1$ state vector, $u(t)$ is a $m \times 1$ input vector and $y(t)$ is a $p \times 1$ output vector. This is a nonlinear generalization of the methods in [Forssell and Ljung, 2000] which showed that for a linear unstable system with an output-error structure, the parameters could be estimated consistently.

A predictor of the output for the nonlinear model (1) can be written as

$$\begin{aligned} \hat{x}(t+1, \theta) &= f(\hat{x}(t, \theta), u(t); \theta) + K(t, \theta)\varepsilon(t, \theta), \\ \hat{y}(t, \theta) &= h(\hat{x}(t, \theta), u(t); \theta), \\ \varepsilon(t, \theta) &= y(t) - h(\hat{x}(t, \theta), u(t); \theta). \end{aligned} \tag{2}$$

The prediction error $\varepsilon(t,\theta)$ should ideallly represent the part of $y(t)$ that cannot be predicted from past measurements. The prediction error can be used to define a scalar loss function

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N}\frac{1}{2}\varepsilon(t,\theta)^T\varepsilon(t,\theta), \tag{3}$$

where Z^N represents the N input-output measurements. This loss function can be minimized to obtain an estimate

$$\hat{\theta} = \arg\min_{\theta} V_N(\theta, Z^N) \tag{4}$$

of θ.

This is called the prediction-error method (PEM) [Ljung, 1999]. In order to use PEM, a stable predictor is required. The choice of the predictor is not obvious if the system is unstable and nonlinear. Here, two approaches for calculating the observer gain $K(t,\theta)$ in (2) that gives a stable predictor have been studied. The first is based on a parametrized observer (PO) approach with direct estimation of K and the second are based on an Extended Kalman Filter (EKF) approach where the observer gain is time-varying. In this case the 25 first prediction errors have been excluded from the loss function (3) in order to reduce the effects of the transient from the EKF.

2.1 Parametrized Observer (PO) Approach

A simple approach is to let the PEM estimate the observer gain, i.e. to include K as a free time-invariant parameter in the θ vector as shown in (5).

$$\begin{aligned}\hat{x}(t+1,\theta) &= f(x(t,\theta),u(t);\theta) + K(\theta)\varepsilon(t,\theta)\\ vec(K) &= \theta_K\\ \theta &= [\theta_f^T\theta_K^T]^T.\end{aligned} \tag{5}$$

Here, θ_f are the parameters that appear in f. This approach is common in the linear case, i.e. when $f(\hat{x}(t,\theta),u(t);\theta) = A(t,\theta)\hat{x}(t,\theta) + B(t,\theta)u(t)$ [Ljung, 1999].

2.2 Extended Kalman Filter (EKF) Approach

The EKF is an extension of the Kalman filter [Kalman, 1960] to nonlinear systems. The main idea is to compute $K(t,\theta)$ at each time step using a linearized model. This linearization is performed by computing the partial derivatives of f with respect to x and u evaluated in $\hat{x}(t)$ and $u(t)$, giving the matrices $A(t,\theta)$ and $B(t,\theta)$, respectively

$$\begin{aligned}A(t,\theta) &= \frac{\partial}{\partial x}f(x,u;\theta)\Big|_{x=\hat{x}(t),\ u=u(t)}\\ B(t,\theta) &= \frac{\partial}{\partial u}f(x,u;\theta)\Big|_{x=\hat{x}(t),\ u=u(t)}\end{aligned} \tag{6}$$

Similarly, matrices $C(t,\theta)$ and $D(t,\theta)$ can be obtained by linearizing h. This gives the linearized model

$$\begin{aligned}x_{lin}(t+1) &= A(t,\theta)x_{lin}(t) + B(t,\theta)u(t)\\ y_{lin}(t) &= C(t,\theta)x_{lin}(t) + D(t,\theta)u(t) + e(t)\end{aligned} \tag{7}$$

Using (7) and (1), which both have no system noise, i.e. $Q = 0$, gives the following EKF recursion [Kailath et al., 2000]:

$$\begin{aligned}\hat{x}(t+1,\theta) =& f(\hat{x}(t,\theta),u(t);\theta) + K(t,\theta)\varepsilon(t,\theta)\\ K(t,\theta) =& [A(t,\theta)P(t,\theta)C^T(t,\theta)]\\ & [C(t,\theta)P(t,\theta)C^T(t,\theta)+R]^{-1}\\ P(t+1,\theta) =& A(t,\theta)P(t,\theta)A^T(t,\theta) -\\ & K(t,\theta)[C(t,\theta)P(t,\theta)C^T(t,\theta)+R]K^T(t,\theta)\\ \hat{y}(t,\theta) =& h(\hat{x}(t,\theta),u(t);\theta)\end{aligned} \tag{8}$$

with the initial conditions $\hat{x}(0,\theta) = x_0$ and $P(0,\theta) = P_0$. This method is called the EKF output-error approach (EKF OE).

2.3 Iterative search method

A general iterative search method, for the parameter vector θ to be estimated, is given by [Ljung, 1999] for the i:th iteration

$$\hat{\theta}_N^{(i+1)} = \hat{\theta}_N^{(i)} - \mu_N^{(i)}[R_N^{(i)}]^{-1}V_N'(\hat{\theta}_N^{(i)}, Z^N) \tag{9}$$

Here $\mu_N^{(i)}$ is the step size, $V_N'(\hat{\theta}_N^{(i)}, Z^N)$ is the gradient of the loss function (3) with respect to θ,

$$V_N'(\theta, Z^N) = -\frac{1}{N}\sum_{t=1}^{N}\psi(t,\theta)\varepsilon(t,\theta), \tag{10}$$

and $R_N^{(i)}$ is an approximation of the Hessian of V_N, that modifies the search direction. In this case, $R_N^{(i)}$ is chosen as in the Levenberg-Marquardt procedure (Gauss-Newton if $\lambda = 0$):

$$R_N^{(i)} = \frac{1}{N}\sum_{t=1}^{N}\psi(t,\hat{\theta}_N^{(i)})\psi^T(t,\hat{\theta}_N^{(i)}) + \lambda I \tag{11}$$

Here, $\psi(t,\theta)$ is the gradient of the prediction $\hat{y}(t,\theta)$, i.e.

$$\psi(t,\theta) = \frac{d}{d\theta}\hat{y}(t,\theta) = -\frac{d}{d\theta}\varepsilon(t,\theta) \tag{12}$$

The gradient has been calculated numerically.

3. ESTIMATION ON SIMULATED DATA

In order to validate the parameter estimators, simulated data has been generated from the following two-dimensional, nonlinear, unstable system on state-space form:

$$\begin{aligned}x(t+1) &= \begin{bmatrix}2.3 & 1.2\\ 0.0 & 1.7\end{bmatrix}\begin{bmatrix}\arctan(x_1(t))\\ \arctan(x_2(t))\end{bmatrix} + I_2u(t)\\ y(t) &= I_2x(t) + e(t)\end{aligned} \tag{13}$$

In the simulations, the system has been stabilized with the following feedback law:

$$u(t) = -\begin{bmatrix}2.3 & 1.2\\ 0.0 & 1.7\end{bmatrix}\begin{bmatrix}\arctan(x_1(t))\\ \arctan(x_2(t))\end{bmatrix} - kI_2y(t) + I_2r(t), \tag{14}$$

where I_n is an unit matrix of dimension n, k is a proportional feedback factor here chosen as 0.1, $r(t)$ is reference signal here chosen as white random binary signal with amplitude 1 and $e(t)$ is white Gaussian noise with zero mean and variance chosen in a way so that different signal-to-noise ratios (SNRs) are obtained.

The model structure

$$x(t+1) = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_3 & \theta_4 \end{bmatrix} \begin{bmatrix} \arctan(x_1(t)) \\ \arctan(x_2(t)) \end{bmatrix} + I_2 u(t) \\ y(t) = I_2 x(t) + e(t) \tag{15}$$

has been used in the identification experiment. Using this model, the true system is in the model set, i.e. with $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$ the true system is obtained for $\theta^0 = [2.3, 1.2, 0.0, 1.7]^T$. Both methods have been tested on data sets with 750 samples with 5 different SNRs (100, 133, 200, 388 and 10000) and 50 different noise realizations for each SNR. The initial setting was $\theta = [2.0, 1.5, 0.2, 1.5]^T$ for the EKF OE method and $\theta = [2.0, 1.5, 0.2, 1.5, 0.1, 0.1, 0.1, 0.1]^T$ for the PO method.

The obtained results for $\hat{\theta}_i$, $i = 1, \ldots, 4$ are shown in Tables 1 - 4 below.

Table 1. $|E[\hat{\theta}_1] - \theta_1^0|$ and standard deviation (σ) of $\hat{\theta}_1$ for each method and each SNR (values should be multiplied with 10^{-2}).

	PO	EKF OE
100	0.15, 0.45	15, 6.6
133	0.17, 0.41	13, 7.2
200	0.0016, 0.26	14, 9.0
388	0.048, 0.16	15, 8.8
10000	0.0054, 0.04	14, 7.8

Table 2. $|E[\hat{\theta}_2] - \theta_2^0|$ and standard deviation (σ) of $\hat{\theta}_2$ for each method and each SNR (values should be multiplied with 10^{-2}).

	PO	EKF OE
100	0.024, 0.38	0.60, 7.4
133	0.031, 0.31	0.80, 6.3
200	0.057, 0.29	1.05, 5.8
388	0.021, 0.23	1.05, 5.9
10000	0.0045, 0.037	0.84, 6.0

Table 3. $|E[\hat{\theta}_3] - \theta_3^0|$ and standard deviation (σ) of $\hat{\theta}_3$ for each method and each SNR (values should be multiplied with 10^{-2}).

	PO	EKF OE
100	0.056, 0.15	2.7, 2.2
133	0.036, 0.10	2.4, 2.0
200	0.0055, 0.11	2.3, 3.2
388	0.0098, 0.065	2.9, 2.1
10000	0.0009, 0.013	2.8, 1.9

As an illustration, the absolute value of $E[\hat{\theta}_1] - \theta_1^0$ and the variance of $\hat{\theta}_1$, as function of SNR are depicted in Fig. 1 and Fig. 2, respectively.

The conclusion that can be drawn from these simulations is that the PO approach gives a parameter estimator for which the bias and variance depend on the SNR for the signals in general, i.e. the higher SNR the lower bias and variance. The EKF OE approach gives a parameter

Table 4. $|E[\hat{\theta}_4] - \theta_4^0|$ and standard deviation (σ) of $\hat{\theta}_4$ for each method and each SNR (values should be multiplied with 10^{-2}).

	PO	EKF OE
100	0.0087, 0.10	0.28, 2.4
133	0.0016, 0.13	0.64, 2.2
200	0.019, 0.095	0.10, 2.7
388	0.0081, 0.06	0.88, 2.6
10000	0.0001, 0.01	0.93, 2.0

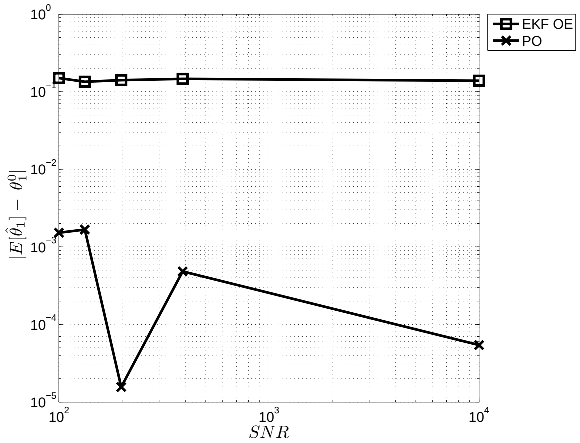

Fig. 1. Absolute value of $E[\hat{\theta}_1] - \theta_1^0$ as a function of SNR (log-scale on both axis).

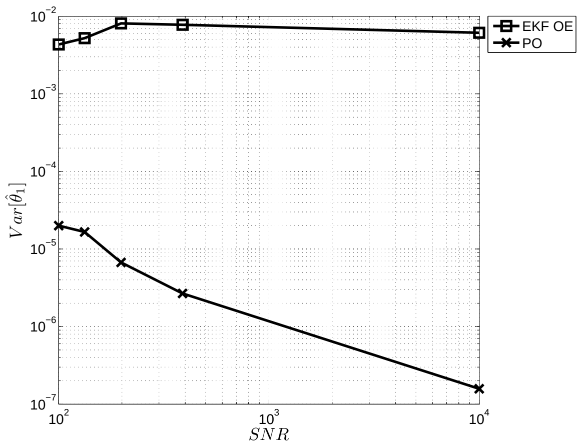

Fig. 2. Variance of $\hat{\theta}_1$ as a function of SNR (log-scale on both axis).

estimator for which the bias and variance do not depend on the SNR, but are fairly constant, and in some cases even decrease when the SNR is decreased which is not intuitive. One explanation can be that linearization effects and an absence of system noise to take care of these contribute to a large bias and variance.

In order to investigate the bias properties, simulations with increasing number of data points have been performed for SNR 200. The results in Fig. 3 show that, for both methods, $\hat{\theta}_1$ converges to values different from θ_1^0 i.e. the bias remains as the number of data points is increased.

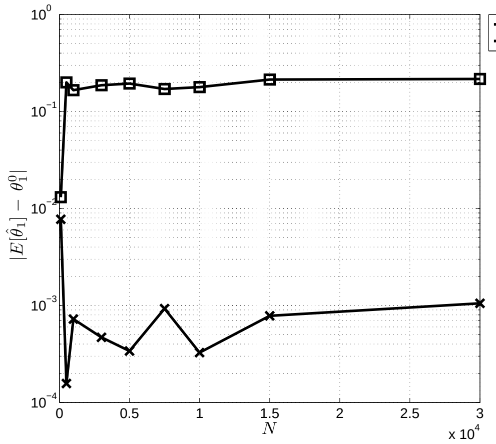

Fig. 3. Absolute value of $\hat{\theta}_1 - \theta_1^0$ as a function of N (log-scale on y-axis).

In any case, the PO approach gives promising results while the EKF OE approach gives a non-intuitive result which seems to be worse than the PO approach.

4. ESTIMATION ON REAL DATA

In this section, an example of estimation on real data is given. The data comes from a flight test with JAS 39 Gripen, which is a highly agile fighter aircraft.

4.1 Physical model

The physical model used for flight simulations is based on rigid body mechanics, originating from Newton's second law, treating all forces and moments acting on the aircraft. The main contribution in this model comes from aerodynamics, propulsion and inertia. Focusing on a pure pitch motion the simplified 2-DOF equations of motion are given as:

$$\begin{aligned} mV\dot{\alpha} &= N_{Aero} + N_{Thrust} \\ J_{yy}\dot{q} &= M_{Aero} + M_{Thrust} \end{aligned} \tag{16}$$

The left hand side of (16) is the total force and moment of inertia in the pitch plane, m, J_{yy} and V being the aircraft mass, moment of inertia and velocity, respectively. These are considered constant in the test case since their variations are limited during the performed maneuver. N_{Aero} and M_{Aero} are the aerodynamic force and moment to be estimated, N_{Thrust} and M_{Thrust} come from the engine thrust and are, in most cases, small in the pitch plane and can therefore be neglected. The aerodynamic components depend on variables such as the states, angle of attack (α) and pitch angle velocity (q), and the input in form of control surface deflections of the elevator (δ_e), canard (δ_c) and leading edge flaps (δ_{LE}). The definitions and signs are shown in Fig. 4.

For a highly maneuverable aircraft, such as the JAS 39 Gripen, the forces and moments can become nonlinear if the parameters get large. Typically, flight in the transonic region, i.e. near the speed of sound, leads to nonlinear effects that come from movements of aerodynamic shocks that appear near the speed of sound.

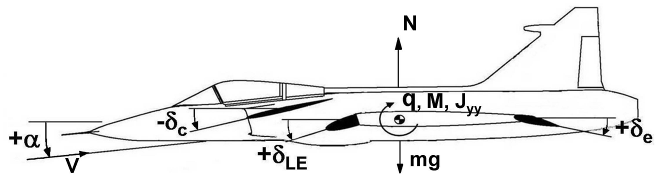

Fig. 4. Definition of the variables.

4.2 Estimation model

To get an estimation model, the equations of motion (16) are rewritten as

$$\begin{aligned} &\dot{\alpha}(t) = (1/mV) \cdot N_{Aero}(\alpha(t), q(t), \delta_e(t), \delta_c(t), \delta_{LE}(t)), \\ &\dot{q}(t) = (1/J_{yy}) \cdot M_{Aero}(\alpha(t), q(t), \delta_e(t), \delta_c(t), \delta_{LE}(t)), \\ &y(t) = [\alpha(t)\ q(t)]^T \end{aligned} \tag{17}$$

and then, turning into discrete time, the Euler's method for the derivatives are used

$$\begin{aligned} \dot{\alpha}(t) &= \frac{\alpha(t+T) - \alpha(t)}{T}, \\ \dot{q}(t) &= \frac{q(t+T) - q(t)}{T}. \end{aligned} \tag{18}$$

This results in the following nonlinear state-space description

$$\begin{aligned} x(t+T) &= f(x(t), u(t)), \\ y(t) &= Cx(t) + e(t) \end{aligned} \tag{19}$$

It is assumed that only the pitch stability, i.e pitching moment as a function of the angle of attack, is nonlinear and that all other relations are linear. This gives the following simplified state-space equation

$$\begin{aligned} x(t+T) &= a(x(t)) + Bu(t), \\ y(t) &= Cx(t) + e(t) \end{aligned} \tag{20}$$

where the state and input vectors are $x(t) = [\alpha(t)\ q(t)]^T$, $u(t) = [\delta_e(t)\ \delta_c(t)\ \delta_{LE}(t)]^T$ and the system matrices are given as

$$\begin{aligned} a &= \begin{bmatrix} N_\alpha \alpha(t) & N_q q(t) \\ f(\alpha(t)) & M_q q(t) \end{bmatrix}, \\ B &= \begin{bmatrix} N_{\delta_e} & N_{\delta_c} & N_{\delta_{LE}} \\ M_{\delta_e} & M_{\delta_c} & M_{\delta_{LE}} \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \tag{21}$$

The Ns and Ms are scaled partial derivatives of the aerodynamic force and moment with respect to the states and inputs, respectively. The scaling includes V, m, J_{yy}, T and dynamic pressure, q_a, reference wing area, S and reference wing chord, $\bar{c}$. $f(\alpha)$ is the nonlinear aerodynamic pitch stability model function which is built up as a piecewise affine function, similarly to the stucture for the present aerodynamic model for JAS 39 Gripen, with break-points positioned at $\alpha(i) = \alpha_{min}, \alpha_{min} + 1, ..., \alpha_{max}$ (see Fig. 5).

This gives the following for $\alpha(i) < \alpha < \alpha(i+1)$

$$f(\alpha) = \frac{f(\alpha(i+1)) - f(\alpha(i))}{\alpha(i+1) - \alpha(i)} \cdot (\alpha - \alpha(i)) + f(\alpha(i)) \tag{22}$$

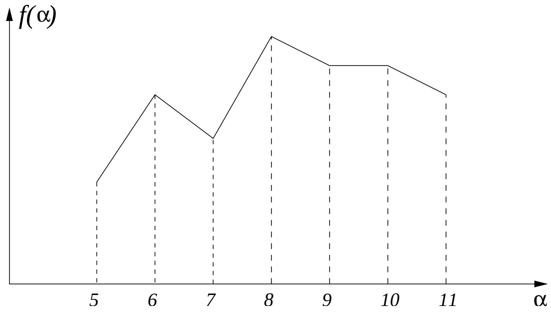

Fig. 5. Example of a piecewise affine function with $\alpha_{min} = 5$ and $\alpha_{max} = 11$.

All Ns, Ms and break-points $f(\alpha(i))$ are put into the parameter vector θ. Thus, the total parametrized model is given by:

$$x(t+T) = \begin{bmatrix} \theta_1\alpha(t) & \theta_2 q(t) \\ f(\alpha(t), \theta_{10}, \ldots, \theta_{21}) & \theta_3 q(t) \end{bmatrix} + \begin{bmatrix} \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & \theta_9 \end{bmatrix} \begin{bmatrix} \delta_e(t) \\ \delta_c(t) \\ \delta_{LE}(t) \end{bmatrix}, \tag{23}$$

$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + e(t).$$

4.3 Identification

The two methods have been evaluated on data from a flight test where a wind-up turn is performed. A wind-up turn is a flight maneuver where an initial roll of 90 degrees is performed followed by an almost pure, high angle of attack, pitching manoevre at almost constant speed. The identification is based on the data after the initial roll has been performed. Data have been collected at 60 Hz and the data set contained approximately 300 measurements. The estimation result, based on this data set, is shown in Fig. 6 and Table 5. All parameters were, in case of the PO approach, initialized to 1, and in case of the EKF OE approach, they were initialized to the result of the PO approach. The values from the present model were not used in the initialization.

As can be seen, both methods capture the nonlinearity around $\alpha = 7$ and the slopes of the curve. There is a bias between the present aerodynamic model and the estimation results. This can be due to the fact that there are uncertainties in the present aerodynamic model. Also, system excitation was low for angles of attack between 10 to 14 degrees. Comparing the different approaches it is interesting to note that the EKF OE approach gives a closer resemblance to the present model which has been been built up from several numerical calculations, wind tunnel test and flight test during a period of more than 25 years. Concerning the derivatives in Table 5, the pitching moment, M, is resonably modelled by both methods. For the normal force, N, there are some discrepancies both in magnitude and sign. This can be due to the assumed linear time inavriant model structure for these parameters, because in reality there are some small variations during the wind-up turn.

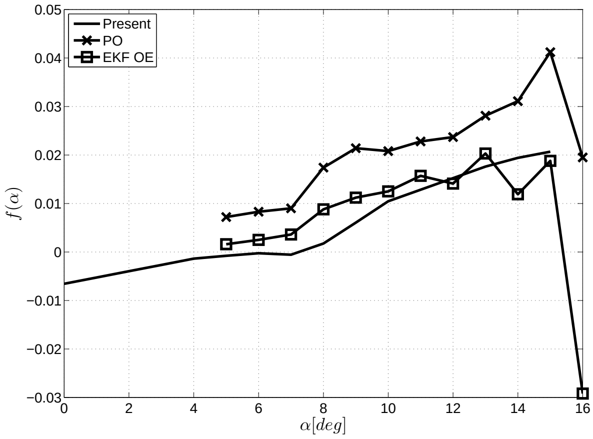

Fig. 6. Pitching moment as a function of α

Table 5. Aerodynamic derivatives.

	Present	PO	EKF OE
N_α	1.6574	0.9926	1.0035
N_q	0.0056	0.0139	0.0104
M_q	1.0038	0.8997	0.9449
N_{δ_e}	0.1985	-0.0059	-0.0021
N_{δ_c}	0.0053	-0.0014	0.0092
$N_{\delta_{LE}}$	-0.0022	-0.0058	-0.0094
M_{δ_e}	-0.4785	-0.4793	-0.4131
M_{δ_c}	0.1330	0.1935	0.1502
$M_{\delta_{LE}}$	-0.0015	-0.0501	-0.0297

5. CONCLUSIONS

Two approaches for direct prediction-error identification of unstable nonlinear systems have been presented, a directly parametrized observer approach and an approach based on the EKF. These methods have been validated on simulated data from an unstable nonlinear system. The validation shows that a good agreement is obtained for the PO approach for the given case, at least when the SNR is high, while the EKF OE approach gives less good and non-intuitive results.

The approaches have also been tested on real data from a flight test near the speed of sound. In general, both approaches show promising results since a good resemblance to the present aerodynamic model was found, but there are some biases in the results. These do not have to be entirely due to the estimators, but could to some extent come from the inaccuracies in the present aerodynamic model for the JAS 39 Gripen.

ACKNOWLEDGEMENTS

This research has been performed within the Vinnova industry excellence center LINK-SIC. Flight test data have been provided by Saab AB.

REFERENCES

P. D. Bruce and M. G. Kellett. Modelling and identification of nonlinear aerodynamic functions using b-splines. In *UKACC International Conference on Control'98, IEE Conference Publication, No. 455*, 1998.

F. E. C. Culick. What the Wright brothers did and did not understand about flight mechanics - in modern terms. In

37th AIAA/ASME/SAE/SAEE Joint Propulsion Conference and Exhibit, 2001.

U. Forssell and L. Ljung. Identification of unstable systems using output error and Box-Jenkins model structures. *IEEE Transactions on Automatic Control*, 45(1):131–147, Jan 2000.

A. Ghaffari, J. Roshanian, and M. Tayefi. Time-varying transfer function extraction of an unstable launch vehicle via closed-loop identification. *Aerospace Science and Tehnology, 11*, pages 238–244, 2007.

M. P. Horton. Real-time identification of missile aerodynamics using a linearised Kalman filter aided by an artificial neural network. *IEE Proc.-Control Theory Application, Vol. 144, No. 4*, 1997.

R. V. Jategaonkar. *Flight vehicle System Identification, A Time Domain Methodology.* American Institute of Aeronautics and Astronautics Inc., 2006.

R. V. Jategaonkar and F. Thielecke. Aircraft parameter estimation - a tool for development of aerodynamic databases. *Sādhanā, Vol. 25, Part 2*, pages 119–135, 2000.

T. Kailath, A. H. Sayed, and B. Hassibi. *Linear Estimation.* Prentice Hall PTR, 2000.

R. E. Kalman. A new approach to linear filtering and prediction problems. *ASME-Journal of Basic Engineering, 82 (Series D)*, pages 35–45, 1960.

V. Klein and E. A. Morelli. *Aircraft System Identification, Theory and Practice.* American Institute of Aeronautics and Astronautics Inc., 2006.

L. Ljung. *System Identification, Theory for the User.* Prentice Hall PTR, second edition, 1999.

A. C. Paris and O. Alaverdi. Nonlinear aerodynamic model extraction from flight-test data for the S-3B Viking. *Journal of Aircraft, Vol. 42, No. 1*, 2005.

M. B. Tischler and R. K. Remple. *Aircraft and Rotorcraft System Identification, Engineering Methods with Flight Test Examples.* American Institute of Aeronautics and Astronautics Inc., 2006.