Session 6

H_∞ norm, frequency-domain descriptions, Bode's gain and phase relation, Performance limitations

Reading Assignment

Copies from KJÅ-AK, Zhou Glover Doyle, paper by KJÅ

Exercises

Exercise 6.1 Describe all stabilizing controllers for $G(s) = \frac{4-2s}{(s-1)(s-3)}$ using Youla factorization.

Exercise 6.2 Evaluate the H_∞ norm of H_1, H_2, and H_2H_1 for $H_1(s) = \frac{3s}{s^2+s+3}$ and $H_2(s) = \frac{3s}{s^2+s/2+2}$.

Exercise 6.3 Determine the performance limitations for $G(s) = \frac{4-2s}{(s-1)(s-3)}$.

Exercise 6.4 Determine a minimum phase $G_m(s)$ and corresponding all-pass factor(s) $B_i(s)$ for the Rosenbrock example,

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

Evaluate the performance limitations for $G_2(s) = G(s)\frac{1}{s+2}$.

Exercise 6.5 Consider a dynamical system consisting of a small tank followed by a pipe with radius r. Assume that the mixing in the tank is complete and immediate, i.e. the residence time is negligible. Assume further that the flow through the pipe is laminar and stationary, and that the flow can be described by the circular symmetric velocity profile $v(r)$,

$$v(r) = v_0\left[1 - (r/r_0)^2\right]$$

The input to the system is the concentration of a tracer in the tank, and the output signal is its concentration averaged over a cross-section area at the distance ℓ from the tank. Determine the output as a convolution of a weighting function $h(\tau)$ and the input.

Draw the Bode diagram for $\ell/v_0 = 1$.

Exercise 6.6 Determine if $G(s) = (1-s)/(s+1)^2$ is minimum phase using the Bode relations.

Hand in problems

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