Session 4

Relations State Space — Geometric Theory, Transmission Zeros — Geometric Theory, Disturbance decoupling, Eigenvalue assignment

Reading Assignment

Rugh, chapter 19. Section 6 in Bo's report.

Exercises

Exercise 4.1 The following system was used in a paper (IEEE AC-27 1241-1243, 1982) to discuss multivariable root loci.

$$A = \begin{bmatrix} -4 & 7 & -1 & 13 \\ 0 & 3 & 0 & 2 \\ 4 & 7 & -4 & 8 \\ 0 & -1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 0 \\ -2 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & -5 & 2 & -2 \\ 8 & -14 & 0 & 2 \end{bmatrix}$$

Determine the poles, zeros, and zero directions, ie x_0 and u_0 defined in lecture 2, Theorem 17.13+. Derive what happens with the closed loop poles for output feedback $u = -kIy$ as $k \to \infty$. Calculate also matrix V with $\mathrm{Im} V = \mathcal{V}^*$ and a "friend" F. Investigate what happens for the feedback $u_1 = u_2 = -k(y1 + y2)$ by looking at the system $(A, B * [1, 1]^T, [1, 1] * C)$.

Exercise 4.2 Rugh 19.3

Exercise 4.3 Rugh 19.4

Exercise 4.4 Rugh 19.7

Exercise 4.5 Use feedforward and feedback to reduce the effect of the disturbance v in the output y.

$$\dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v$$
$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x$$

(draw a figure).

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 4.6 Calculate the zeros for the system defined by

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Find the transfer function and show that the system

$$\begin{aligned} \dot{w} &= -w + [\,-1 \quad 1\,]\, y \\ u &= -4w + [\,-3 \quad 3\,]\, y + [\,0 \quad 1\,]\, \frac{dy}{dt} \end{aligned}$$

is a left inverse.

Exercise 4.7 Regard the system

$$\begin{aligned} (s+1)x_1 &= u_1 \\ (s+2)x_2 &= u_2 \\ (s+3)x_3 &= x_1 + u_1 + u_2 \\ y_1 &= x_3 \\ y_2 &= x_1 + x_2 \end{aligned}$$

Determine $\mathcal{R}^*$, $\mathcal{V}^*$, zeros and zero-directions. What happens with the poles of the system in case of the feedback $u = -ky$ when $k \to \infty$?