Session 3

A-invariant subspace, Controllable subspace, (A, B)-invariant subspace, Controllability Subspace

Reading Assignment

Rugh, chapter 18. Sections 1-4 in Bo's report.

Exercises

Exercise 3.1 Use `/home/fulinsys/linsysII96/sessions/geom/` to calculate $\mathcal{V}^*$ and $\mathcal{R}^*$ for the system

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = 0$$

Reproduce the calculations manually.

Exercise 3.2 Show that $\langle A+BF \mid \mathcal{B} \rangle = \langle A \mid \mathcal{B} \rangle$ (Feedback does not change controllability.)

Exercise 3.3 Considering the discrete time system $x_{k+1} = Ax_k + Bu_k$ and the recursive algorithm for $\mathcal{V}^*_{(A,B)}(\mathcal{S})$, show that

$$\mathcal{V}_i = \{x \mid \exists u_0, \ldots, u_{i-1} \text{ such that } x_k \in S \ \forall k = 0, \ldots, i\}$$

Conclude that if $S = \text{Ker}C$ then $x \in \mathcal{V}_i \iff \mathcal{O}_i x \in \text{Im}T_i$ where

$$\mathcal{O}_i = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^i \end{bmatrix}, \quad T_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{i-1}B & \cdots & CAB & CB \end{bmatrix}$$

Exercise 3.4 If $\{A, B\}$ is controllable and $b \in \text{Im}B$ then there is an F such that $\{A + BF, b\}$ is controllable. (Hence, it is theoretically sufficient with one control signal plus feedback.)

Exercise 3.5 Rugh 18.9

Exercise 3.6 Show that if CB is invertible then $\text{Ker}C$ is (A, B)-invariant.

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 3.7 18.7

Exercise 3.8 Gunnar Bengtsson and Sture Lindahl introduced a standard multivariable example based on a boiler model by Karl Eklundh:

x_1 = drum pressure (bar)

x_2 = drum liquid level (m)

x_3 = drum liquid temperature ($^\circ$ C)

x_4 = riser wall temperature ($^\circ$ C)

x_5 = steam quality (%)

$$A = \begin{bmatrix} -0.129 & 0 & 0.396 * 10^{-1} & 0.250 * 10^{-1} & 0.191 * 10^{-1} \\ 0.329 * 10^{-2} & 0 & -0.779 * 10^{-4} & 0.122 * 10^{-3} & -0.621 \\ 0.718 * 10^{-1} & 0 & -0.100 & 0.887 * 10^{-3} & -0.385 * 10^{1} \\ 0.411 * 10^{-1} & 0 & 0 & -0.822 * 10^{-1} & 0 \\ 0.361 * 10^{-3} & 0 & 0.350 * 10^{-4} & 0.426 * 10^{-4} & -0.743 * 10^{-1} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0.139 * 10^{-2} \\ 0 & 0.359 * 10^{-4} \\ 0 & -0.989 * 10^{-2} \\ 0.249 * 10^{-4} & 0 \\ 0 & -0.534 * 10^{-5} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

You may use `/home/fulinsys/linsysII96/sessions/geom/boiler.m` to include the matrices. What can be said about controllability and observability? Find $\mathcal{V}^*$.