Session 5

Polynomial Matrix Descriptions, Stable Coprime Factorizations, Youla Parametrization

Reading Assignment

Copies from Kailath, chapter 8. Notes on the Youla Parametrization.

Exercises

Exercise 5.1 Show that Fuhrmann Strict Equivalence of the two left and right MFD's $\{D_L(s), N_L(s)\}$, $\{D_R(s), N_R(s)\}$ means that both MFD's are minimal.

Exercise 5.2 Find, if possible, matrices that will relate the system matrices (SISO)

$$\begin{bmatrix} a(s) & b(s) \\ c(s) & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a(s) & c(s) \\ b(s) & 0 \end{bmatrix}$$

Under what conditions are they FSE?

Exercise 5.3 Kailath Exercise 8.2.3a

Exercise 5.4 Kailath Exercise 8.2.4

Exercise 5.5 Kailath Exercise 8.2.9

Exercise 5.6 Kailath Exercise 8.3.9

Exercise 5.7 It was proved in the lecture (theorem 1) that FSE implies identical system behaviour. Does the opposite implication hold?

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 5.8 Apply the PMD state realization algorithm to

a. $\dot{y} = \dot{u}$. Verify that the state description has the same system behavior.

b. the PMD defined by

$$\begin{aligned} P(s) &= \begin{bmatrix} s^2 & 0 \\ 0 & s \end{bmatrix}, \quad Q(s) = \begin{bmatrix} s^2 & 1 \\ 1 & 1 \end{bmatrix} \\ R(s) &= \begin{bmatrix} 0 & s^2 \\ 0 & 1 \end{bmatrix}, \quad W(s) = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Exercise 5.9 Consider the PMD

$$\begin{aligned} P(s) &= \begin{bmatrix} s^3 & 0 \\ s^2 & s \end{bmatrix}, \quad Q(s) = I, \\ R(s) &= \begin{bmatrix} s & 1 \\ s^2 & 0 \end{bmatrix}, \quad W(s) = 0 \end{aligned}$$

If we consider $\{P(s), R(s)\}$ as a right MFD, we have a naturally associated controller form. Find a state similarity transformation giving the realization obtained by the standard PMD algorithm.