Session 2

Relations between MFD's and state realizations. Poles and Zeros

Reading Assignment

Rugh, chapter 17.

Exercises

Exercise 2.1 Consider the right-MFD

$$N(s) = \begin{bmatrix} s(s+1) & s+2 \\ 2s+1 & 2 \end{bmatrix} \quad D(s) = \begin{bmatrix} s^3 - s - 1 & s(s+1) \\ s^2 + s + 1 & s+1 \end{bmatrix}$$

Write down a controller realisation by first making the MFD column-reduced. Is the realisation minimal? Determine the McMillan degree.

Exercise 2.2 Determine the zeros of a coprime right-MFD $N(s)D^{-1}(s)$ when $N^T(s)$ is on Hermite-form or Smith-form respectively. Consider especially the two different cases ($N(s)$ on Hermite-form)

$$D(s) = \begin{bmatrix} s+2 & 0 \\ 0 & s+3 \end{bmatrix} \quad N_1(s) = \begin{bmatrix} s+1 & 0 \\ 0 & s+1 \end{bmatrix} \quad N_2(s) = \begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix}$$

Would the zeros change by column operations on $N(s)$? What about row operations?

Exercise 2.3 Consider a stable (closed loop) single input single output system $G(s) = C(sI - A)^{-1}B$. Assume that $G(0) = -CA^{-1}B = 1$, i.e. the error coefficient $e_0 = 1 - G(0) = 0$. Show that $e_1 = -[dG/ds](0) = CA^{-2}B$ can be obtained as $e_1 = \sum_{i=1}^{m} 1/z_i - \sum_{i=1}^{n} 1/p_i$, where z_i are the zeros of $G(s)$ and p_i its poles.

Exercise 2.4 = Rugh 17.6

Exercise 2.5 = Rugh 17.7

Exercise 2.6 = Rugh 17.8

Exercise 2.7 Is it really possible to change the column degrees of a column reduced rightMFD using "state feed-back"? Can you use state-feedback to obtain $\hat{D}(s) = \begin{bmatrix} (s+1)^2 & 0 \\ 0 & (s+1)^2 \end{bmatrix}$ from $D(s) = \begin{bmatrix} (s+1) & 0 \\ 0 & (s+1)^3 \end{bmatrix}$? Compare the calm reflection on Rugh p324.

Exercise 2.8 = Rugh 17.10

Hand in problems

Solutions to the following problems should be handed in at the exercise session.

Exercise 2.9 From the right coprime polynomial fraction for the satellite in exercise 1.7, construct a minimal state space realization.

Exercise 2.10 Introduce the controller form realisation of $N(s)D^{-1}(s)$ with

$$D(s) = \begin{bmatrix} (s+2)^2 & 0 \\ 0 & (s+3)^2 \end{bmatrix} \quad N(s) = \begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix}$$

Determine a state feedback that places poles under the zeros. Investigate the observability of the closed system using PBH.

Exercise 2.11 One way to analytically investigate the poles after high gain output feedback is to use a minimal leftMFD:

$$A(s)y(s) = B(s)u(s), \qquad u(s) = k\left(v(s) - y(s)\right)$$

If we now regard both y and u as outputs we get

$$\begin{bmatrix} -A(s) & B(s) \\ kI & I \end{bmatrix} \begin{bmatrix} y(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} 0 \\ kv(s) \end{bmatrix}$$

or after unimodular row-operations

$$\begin{bmatrix} 0 & B(s) + (1/k)A(s) \\ kI & I \end{bmatrix} \begin{bmatrix} y(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} A(s)v(s) \\ kv(s) \end{bmatrix}$$

Nonzero y and u may exist for zero v, if $y(t) = y_0 e^{s_i t}$ and $u(t) = u_0 e^{s_i t}$, where the closed loop poles s_i are the zeros of $\det\left(B(s) + A(s)/k\right)$. Describe what happens to the poles as k goes to infinity.

What happens if $B(s)$ is a singular polynomial matrix? In particular, consider the example

$$A(s) = \begin{bmatrix} s^2 & 8 \\ 0 & s^2 \end{bmatrix} \qquad B(s) = (s+1)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Use Maple or Matlab to plot the root-locus. The identity $\det(I + uv^T) = \det(1 + v^T u)$ may be useful to understand what happens.