Session 1

Matrix Fraction descriptions (MFD's), Hermite Form, Smith Form, Column/Row Degree

Reading Assignment

Rugh, chapter 16, Lecture notes from Lecture 7, Part I, MFD package for Maple, Polynomial tool-box for Matlab.

Exercises

Exercise 1.1 = Rugh 16.1

Exercise 1.2 Determine the right common factor between $(sI - A)$ and C and the left common factor between $(sI - A)$ and B for

$$A = \begin{bmatrix} -4 & -3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 2 & 0 & -1 \end{bmatrix}$$

Exercise 1.3 = Rugh 16.6 (new)

Exercise 1.4 = Rugh 16.8

Exercise 1.5 = Rugh 16.10 (16.9) In addition to the hints given by Rugh it is a good idea to also prove what is called "The Predictable-Degree Property of Column-Reduced Matrices":

Let $D(s)$ be a polynomial matrix of full normal column rank, and for any polynomial vector $p(s)$ let

$$q(s) = D(s)p(s)$$

Then $D(s)$ is column reduced if and only if

$$\deg q(s) = \max_{i:p_i(s)\neq 0} [\deg p_i(s) + c_i[D]]$$

where $p_i(s)$ is the ith entry of $p(s)$.

Exercise 1.6 Let

$$a(s) = s^3 + a_1 s^2 + a_2 s + a_3$$
$$A_c = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Show that the unimodular matrices

$$U(s) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & s \\ 1 & s + a_1 & s^2 + a_1 s + a_2 \end{bmatrix}$$
$$V(s) = \begin{bmatrix} 0 & -1 & s^2 \\ -1 & 0 & s \\ 0 & 0 & 1 \end{bmatrix}$$

will convert $sI - A_c$ to its Smith form. Generalize to the $n \times n$ matrix case.

Hand in problems

Solutions to the following problems should be handed in at the exercise session
Exercise 1.7 Consider the transfer function matrix

$$\begin{bmatrix} \frac{1}{s^2+\omega^2} & \frac{2\omega}{s(s^2+\omega^2)} \\ \frac{-2\omega}{s(s^2+\omega^2)} & \frac{s^2-3\omega^2}{s^2(s^2+\omega^2)} \end{bmatrix}$$

for a satellite (see Rugh p31–32). Obvious MFD's would be

$$\begin{bmatrix} s(s^2+\omega^2) & 0 \\ 0 & s^2(s^2+\omega^2) \end{bmatrix}^{-1} \begin{bmatrix} s & 2\omega \\ -2\omega s & s^2 - 3\omega^2 \end{bmatrix}$$
$$= \begin{bmatrix} s & 2\omega \\ -2\omega s & s^2 - 3\omega^2 \end{bmatrix} \begin{bmatrix} s(s^2+\omega^2) & 0 \\ 0 & s^2(s^2+\omega^2) \end{bmatrix}^{-1}$$

Determine a coprime right polynomial fraction.
Exercise 1.8 Theorem 16.27 can with obvious modifications be rephrased to work for properness instead of strict properness. Determine this modification and do the proof. Then consider the differentiator circuit

$$\begin{bmatrix} RCs + R/R_1 & 0 & -1 \\ 1 & -1/K & 0 \\ 1 & 1 & 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} V_{in}$$

where $y^T = \begin{bmatrix} V_c & V_{out} & RI \end{bmatrix}$. Calculate the transfer function from V_{in} to y and consider specifically the two cases $1/K \neq 0$ and $1/K = 0$. How is the properness related to the row-reduction?