

# Tracking of Animals Using Airborne Cameras

Clas Veibäck

- 1 Introduction
- 2 Background
- 3 Constrained Motion Model
- 4 Uncertain Timestamp Model
- 5 Mode Observations
- 6 Conclusions

# Introduction



- Tracking of animals

# Introduction



- Tracking of animals
- Overhead cameras

# Introduction



- Tracking of animals
- Overhead cameras
- Applicable to other target types

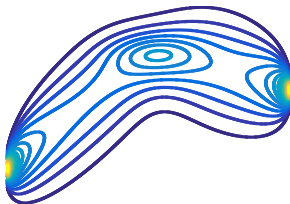
# Introduction



- Tracking of animals
- Overhead cameras
- Applicable to other target types
- Applications to demonstrate theory

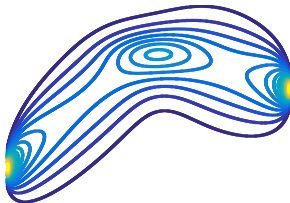
# Main Contributions

## Uncertain Timestamps



# Main Contributions

## Uncertain Timestamps



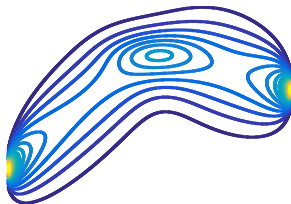
## Constrained Motion Model





# Main Contributions

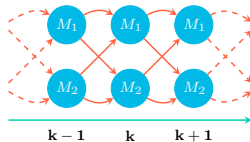
## Uncertain Timestamps



## Constrained Motion Model



## Mode Observations

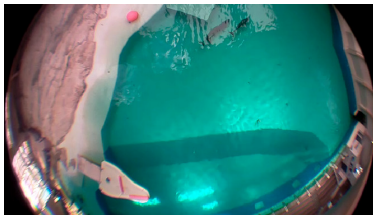


# Dolphin Application



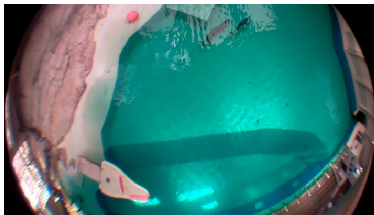
- Dolphinarium at Kolmården Wildlife Park

# Dolphin Application



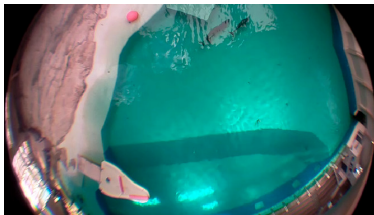
- Dolphinarium at Kolmården Wildlife Park
- Fisheye camera with occlusions

# Dolphin Application



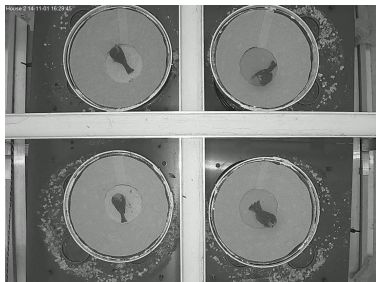
- Dolphinarium at Kolmården Wildlife Park
- Fisheye camera with occlusions
- Reflections and changing light conditions

# Dolphin Application



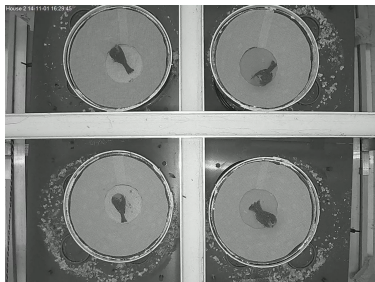
- Dolphinarium at Kolmården Wildlife Park
- Fisheye camera with occlusions
- Reflections and changing light conditions
- Constrained to basin

# Bird Application



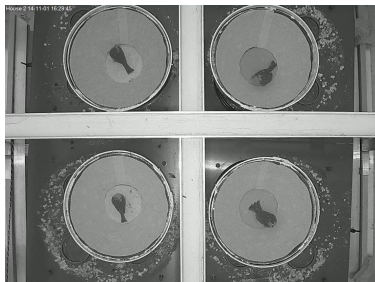
- Recording of birds in Emlen funnels

# Bird Application



- Recording of birds in Emlen funnels
- Detect take-off times

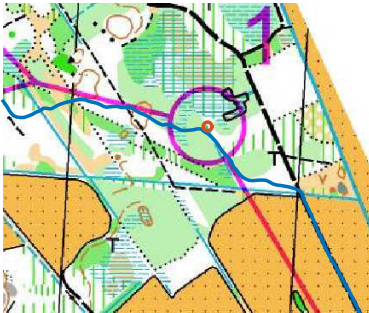
# Bird Application



- Recording of birds in Emlen funnels
- Detect take-off times
- Estimate take-off directions

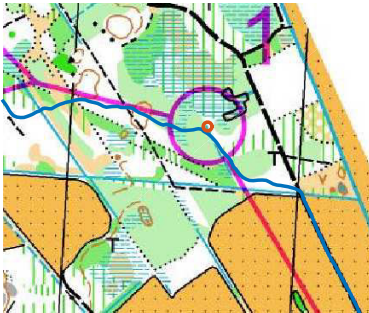


# Orienteering Application



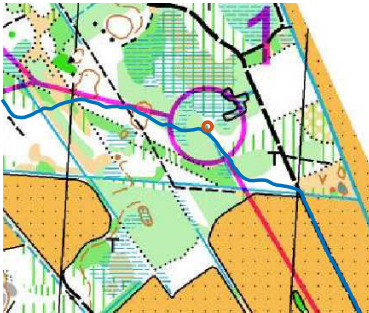
- GPS trajectory

# Orienteering Application



- GPS trajectory
- Control position known

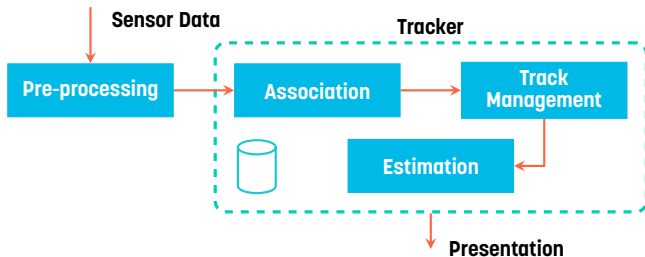
# Orienteering Application



- GPS trajectory
- Control position known
- Improve position estimate

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- 6 Conclusions

# Target Tracking



# Target Model

Linear Gaussian state-space model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k,$$

# Target Model

Linear Gaussian state-space model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, \quad \mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k),$$

# Target Model

## Linear Gaussian state-space model

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# Target Model

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# Target Model

## Linear Gaussian state-space model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k,$$

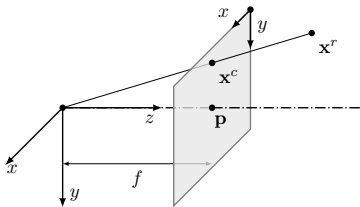
$$\mathbf{y}_j = \mathbf{H}_j \mathbf{x}_j + \mathbf{v}_j,$$

$$\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0).$$

$$\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k),$$

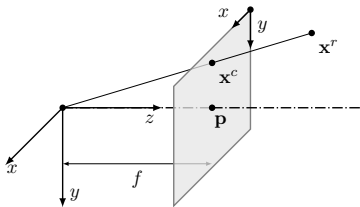
$$\mathbf{v}_j \sim \mathcal{N}(0, \mathbf{R}_j),$$

# Camera Model



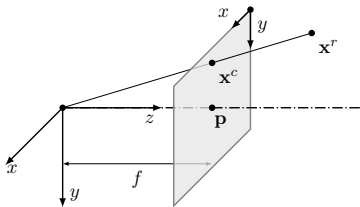
- Camera extrinsics

# Camera Model



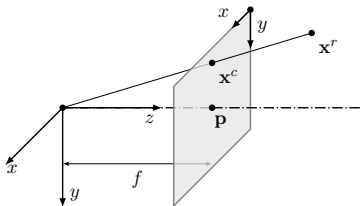
- Camera extrinsics
- Camera intrinsics

# Camera Model



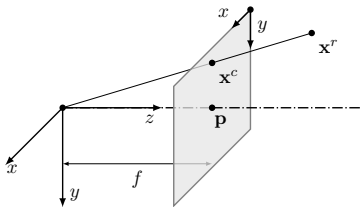
- Camera extrinsics
- Camera intrinsics
- Projection

# Camera Model



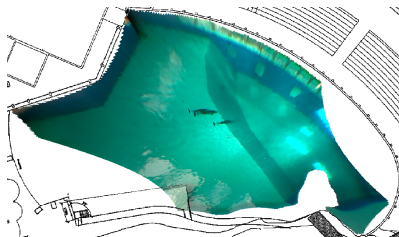
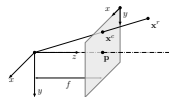
- Camera extrinsics
- Camera intrinsics
- Projection
- Perspective compensation

# Camera Model



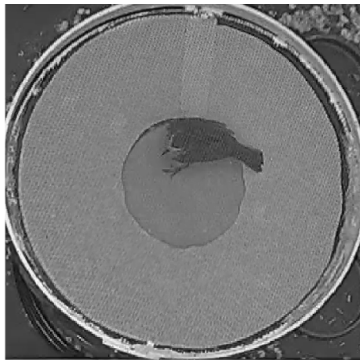
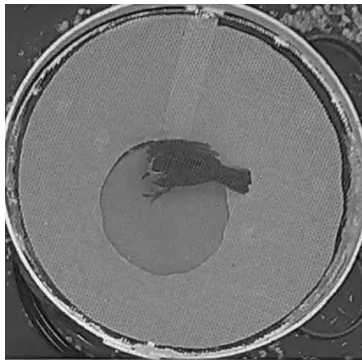
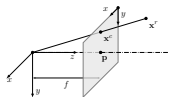
- Camera extrinsics
- Camera intrinsics
- Projection
- Perspective compensation
- Lens distortion

# Dolphin Camera





# Bird Camera



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# Constrained Motion Model



# Constrained Motion Model



- Targets constrained to region

# Constrained Motion Model



- Targets constrained to region
- Feasible predictions

# Constrained Motion Model



- Targets constrained to region
- Feasible predictions
- Similar behaviour

# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

- Nearly constant speed

$$\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix}$$



# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

- Nearly constant speed
- Influence by edges

$$w(\mathbf{x}, \mathbf{n}) = \frac{1}{\|\mathbf{p} - \mathbf{n}\|^2}$$

# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

- Nearly constant speed
- Influence by edges
- Avoid collision with edges

# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges

# Turning Model



$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \left( \beta_d + \beta_a (\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}(\mathbf{n})) \right) w(\mathbf{x}, \mathbf{n}) d\mathbf{n}$$

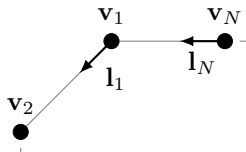
**Clockwise**  
**or**  
**Counterclockwise**

- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges
- Preferred direction

# Turning Model



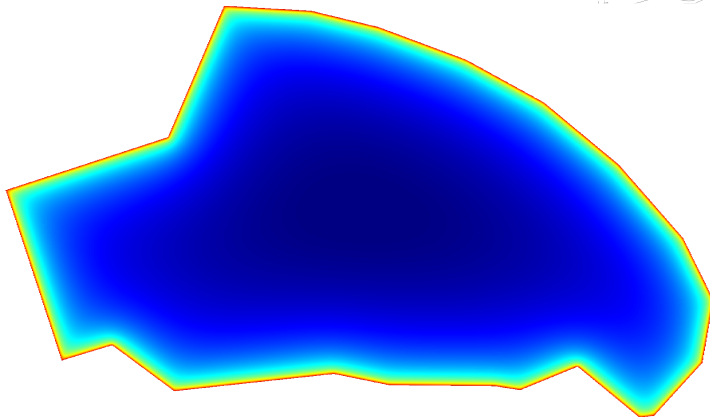
$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \sum_{i=1}^N (\beta_d + \beta_a(\dot{\mathbf{p}}_{\perp} \cdot \mathbf{l}_i)) w_i(\mathbf{x})$$



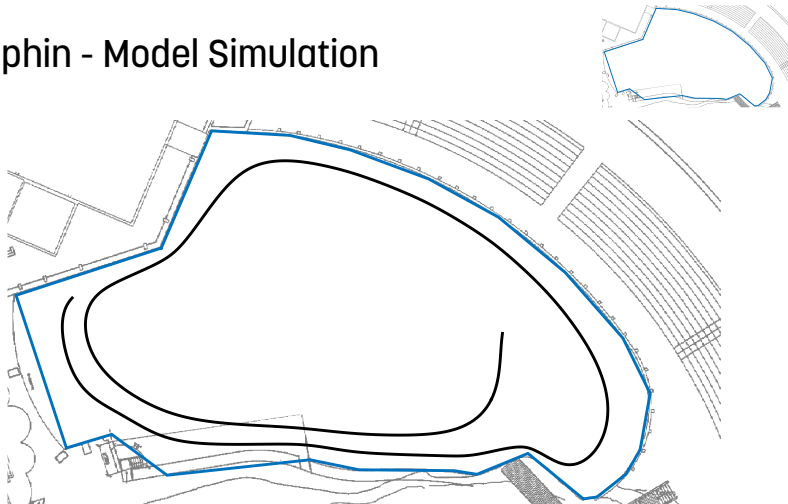
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- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges
- Preferred direction
- Polygon region

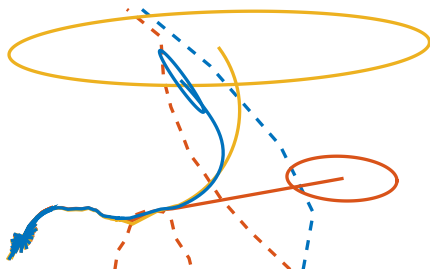
# Potential Field



# Dolphin - Model Simulation



# Dolphin - Model Comparisons



- - - Detection region
- - - Constraint region
- Constant Velocity model
- Coordinated Turn model
- Constrained Motion model

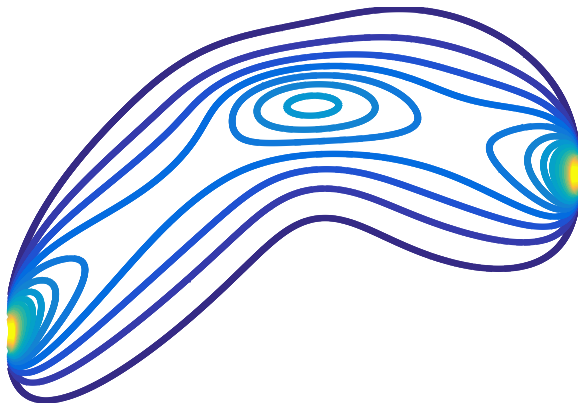


# Dolphin - Complete Framework

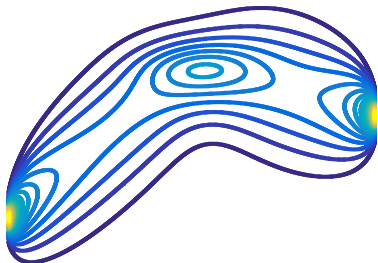


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# Uncertain Timestamp Model

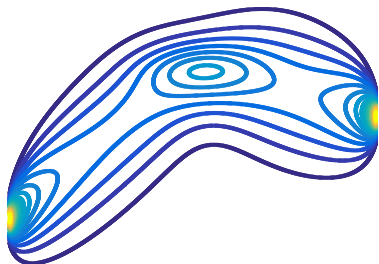


# Uncertain Timestamp Model



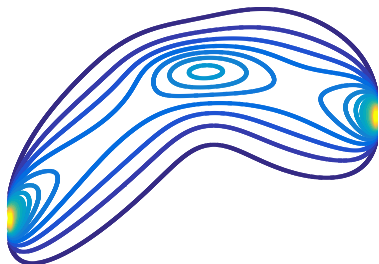
- Traditional measurements

# Uncertain Timestamp Model



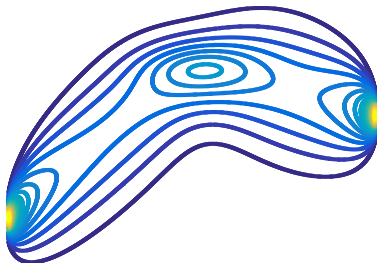
- Traditional measurements
- Observations sampled at an uncertain time

# Uncertain Timestamp Model



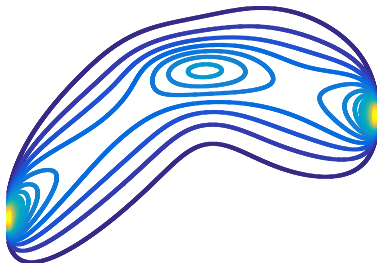
- Traditional measurements
- Observations sampled at an uncertain time
- Crime scene investigations

# Uncertain Timestamp Model



- Traditional measurements
- Observations sampled at an uncertain time
- Crime scene investigations
- Traces from animals

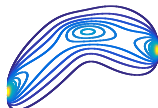
# Uncertain Timestamp Model



- Traditional measurements
- Observations sampled at an uncertain time
- Crime scene investigations
- Traces from animals
- Bottlenecks in mines



# Uncertain Timestamp Model



Consider a linear Gaussian state-space model,

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k,$$

$$\mathbf{y}_j = \mathbf{H}_j^y \mathbf{x}_j + \mathbf{v}_j^y,$$

$$\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0),$$

$$\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k),$$

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# Uncertain Timestamp Model



Consider a linear Gaussian state-space model,

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Extend the model with

$$\mathbf{z} = \mathbf{H}^z \mathbf{x}_\tau + \mathbf{v}^z, \quad \mathbf{v}^z \sim \mathcal{N}(0, \mathbf{R}^z), \quad \tau \sim p(\tau).$$

# Simple Uncertain Time Scenario



Consider the simple model,

$$\begin{aligned}x_k &= x_{k-1} + w_k, & w_k &\sim \mathcal{N}(0, Q), \\y_j &= x_j + v_j^y, & v_j^y &\sim \mathcal{N}(0, R^y)\end{aligned}$$

for  $k \in \{1, \dots, N\}$  and two measurements  $y_1$  and  $y_N$ .

# Simple Uncertain Time Scenario



Consider the simple model,

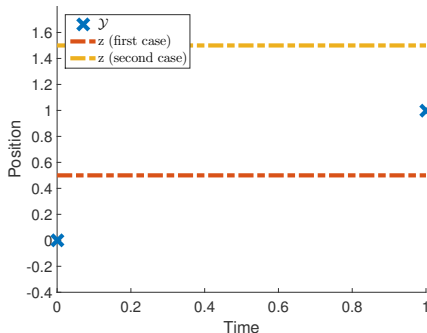
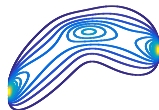
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for  $k \in \{1, \dots, N\}$  and two measurements  $y_1$  and  $y_N$ .

Extend the model with

$$z = x_\tau + v^z, \quad v^z \sim \mathcal{N}(0, R^z), \quad \tau \sim p(\tau).$$

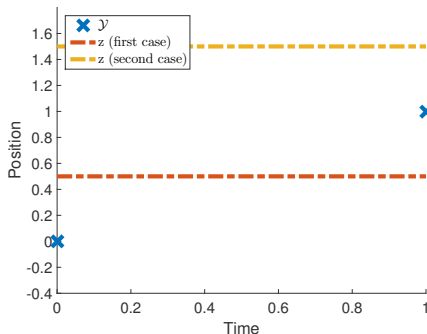
# Simple Uncertain Time Scenario



The measurements are

- $y_1 = 0$ ,
- $y_N = 1$ .

# Simple Uncertain Time Scenario



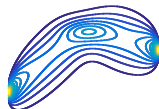
The measurements are

- $y_1 = 0$ ,
- $y_N = 1$ .

The two cases of observations are

- $z = 0.5$  with flat prior.
- $z = 1.5$  with non-flat prior.

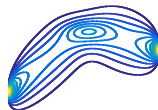
# Posterior Distributions - Time



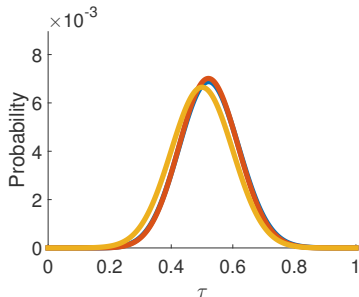
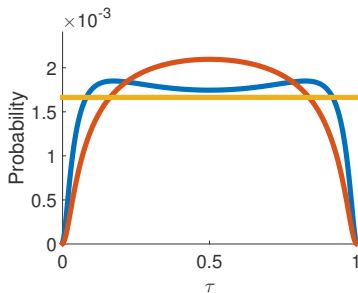
The posterior distribution of the uncertain time is

$$w_\tau \triangleq p(\tau|\mathcal{Y}, \mathbf{z}) \propto p(\tau)\mathcal{N}(\mathbf{z}|\hat{\mathbf{z}}_\tau, \mathbf{S}_\tau).$$

# Posterior Distributions - Time



—  $p(\tau|\mathcal{Y}, z)$   
—  $\max_{\mathcal{X}} p(\mathcal{X}, \tau|\mathcal{Y}, z)$   
—  $p(\tau)$





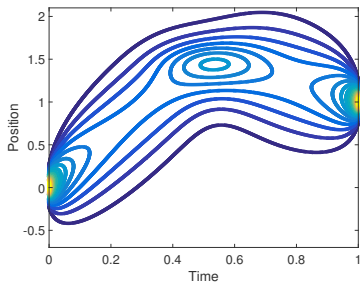
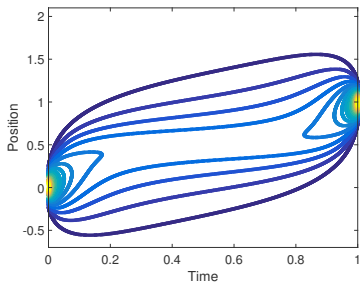
# Posterior Distributions - States



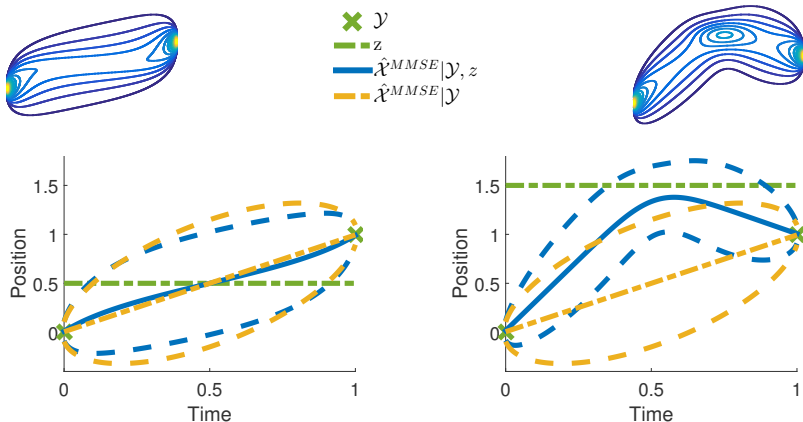
The posterior distribution of the state is

$$p(\mathbf{x}_k | \mathcal{Y}, \mathbf{z}) = \sum_{\tau=1}^N w_{\tau} \cdot \mathcal{N}(\mathbf{x}_k | \hat{\mathbf{x}}_k^{\tau}, \mathbf{P}_k^{\tau}).$$

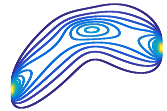
## Posterior Distributions - States



# Estimators - Minimum Mean Squared Error

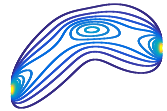


# Orienteering



- Adjusted trajectory

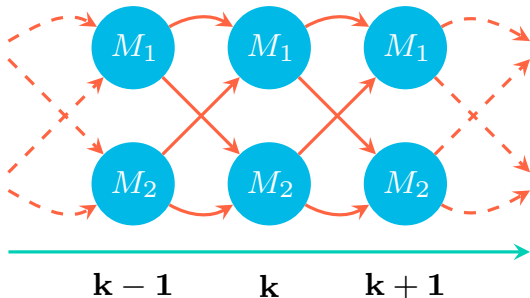
# Orienteering



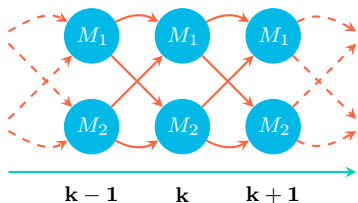
- Adjusted trajectory
- Single control point

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# Mode Observations



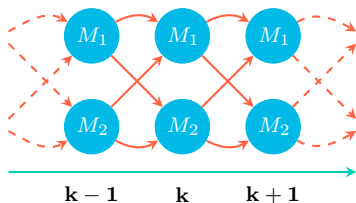
# Mode Observations



- Jump Markov model

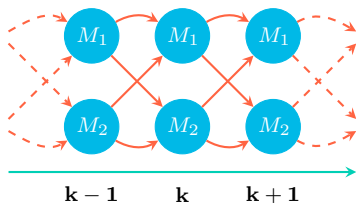


# Mode Observations



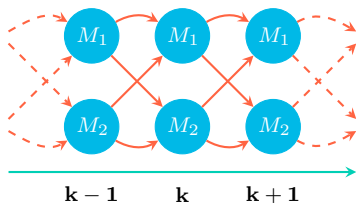
- Jump Markov model
- Model target behaviour

# Mode Observations



- Jump Markov model
- Model target behaviour
- Probability of switching

# Mode Observations



- Jump Markov model
- Model target behaviour
- Probability of switching
- Direct mode observation

# Jump Markov Model

The jump Markov linear model is

$$\begin{aligned}\mathbf{x}_k &= \mathbf{F}_k(\delta_k)\mathbf{x}_{k-1} + \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k(\delta_k)), \\ \mathbf{y}_j &= \mathbf{H}_j(\delta_j)\mathbf{x}_j + \mathbf{v}_j, & \mathbf{v}_j &\sim \mathcal{N}(\mathbf{0}, \mathbf{R}_j(\delta_j)).\end{aligned}$$



# Jump Markov Model

The jump Markov linear model is

$$\begin{aligned}\mathbf{x}_k &= \mathbf{F}_k(\delta_k)\mathbf{x}_{k-1} + \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k(\delta_k)), \\ \mathbf{y}_j &= \mathbf{H}_j(\delta_j)\mathbf{x}_j + \mathbf{v}_j, & \mathbf{v}_j &\sim \mathcal{N}(\mathbf{0}, \mathbf{R}_j(\delta_j)).\end{aligned}$$





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Augmented is the direct mode observation,

$$\mathbf{z}_k \sim p(\mathbf{z}_k | \delta_k).$$



## Posterior Distribution - Mode Sequence

The posterior probability of a mode sequence  $\{\delta_j^i\}_{j=1}^k$  is computed recursively by

$$w_k^i \propto w_{k-1}^i \cdot \prod_k^{\delta_k^i, \delta_{k-1}^i} \cdot \mathcal{N}(\mathbf{y}_k \mid \hat{\mathbf{y}}_k^i, \mathbf{S}_k^i) \cdot p(\mathbf{z}_k \mid \delta_k^i).$$





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## Posterior Distribution - State

The posterior distribution of the state is given by

$$p(\mathbf{x}_k | \{\mathbf{y}_j\}_{j=1}^k, \{\mathbf{z}_j\}_{j=1}^k) = \sum_{i=1}^{|\mathcal{S}|^k} w_k^i \cdot \mathcal{N}(\mathbf{x}_k | \hat{\mathbf{x}}_k^i, \mathbf{P}_k^i).$$

# Bird - Model

The modes are  $\mathcal{S} = \{s, f\}$ .





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The state-space model is

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(\delta_k)), \\ \mathbf{y}_k &= \begin{pmatrix} \mathbf{h}(\mathbf{p}_k) \\ b_k^{\delta_k} \end{pmatrix} + \mathbf{v}_k, & \mathbf{v}_k &\sim \mathcal{N}(\mathbf{0}, \mathbf{R}), \end{aligned}$$

# Bird - Model Extension

The direct mode observation is the radial position

$$z_k = \sqrt{x_k^2 + y_k^2},$$



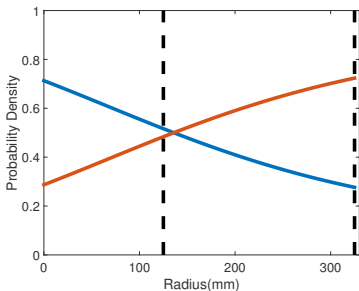


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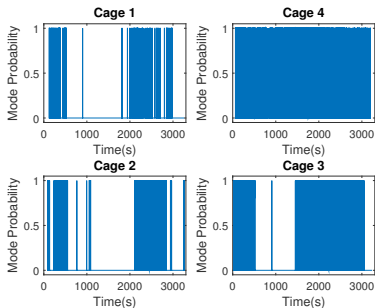
$$z_k = \sqrt{x_k^2 + y_k^2},$$

where  $p(z_k | \delta_k)$  is given by:





## Bird - Results

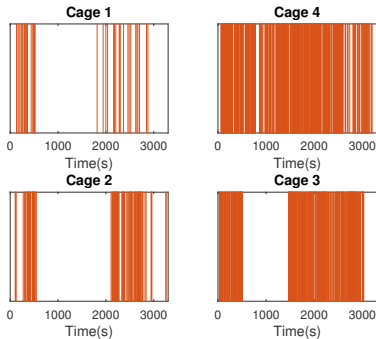


- Estimated modes





## Bird - Results



- Estimated modes
- Extracted takeoffs

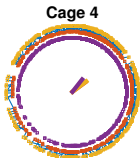


## Bird - Results

Cage 1



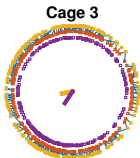
Cage 4



Cage 2



Cage 3

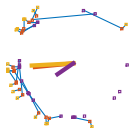


- Estimated modes
- Extracted takeoffs
- Takeoff directions

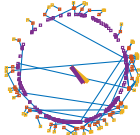


## Bird - Results

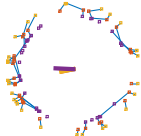
**Cage 1**



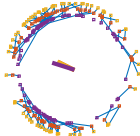
**Cage 4**



**Cage 2**

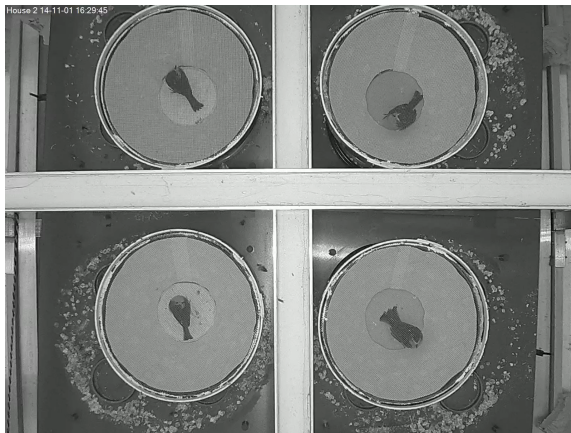


**Cage 3**



- Estimated modes
- Extracted takeoffs
- Takeoff directions
- Matched takeoffs

# Bird - Complete Framework



- 1 Introduction
- 2 Background
- 3 Constrained Motion Model
- 4 Uncertain Timestamp Model
- 5 Mode Observations
- 6 Conclusions

# Conclusions

Theory is presented on

- a constrained motion model

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Theory is demonstrated to work in applications

Questions?

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