Tracking of Animals Using Airborne Cameras

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Introduction

- Tracking of animals

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- Tracking of animals
- Overhead cameras

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- Overhead cameras
- Contributions

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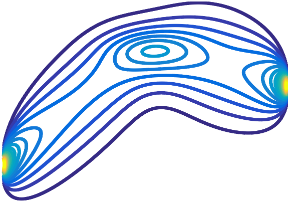
- Tracking of animals
- Overhead cameras
- Contributions
- Applications

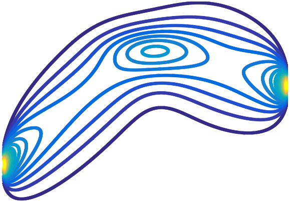
Main Contributions

Uncertain Timestamps



Main Contributions

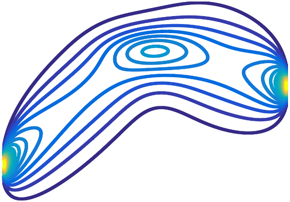
Uncertain Timestamps



Constrained Motion Model

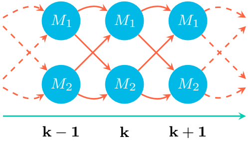
Main Contributions

Uncertain Timestamps



Constrained Motion Model

Mode Observations



Dolphin Application

- Dolphinarium at Kolmården Wildlife Park

Dolphin Application

- Dolphinarium at Kolmården Wildlife Park
- Fisheye camera with occlusions

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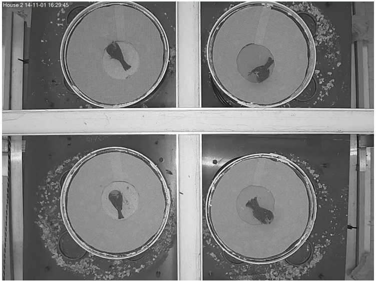
- Fisheye camera with occlusions
- Reflections and changing light conditions

Dolphin Application

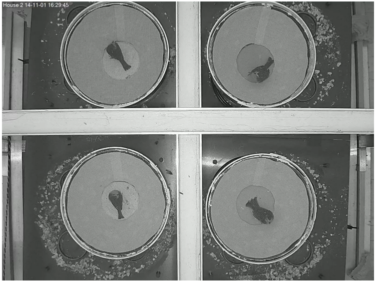
- Dolphinarium at Kolmården Wildlife Park
- Fisheye camera with occlusions
- Reflections and changing light conditions
- Constrained to basin

Bird Application



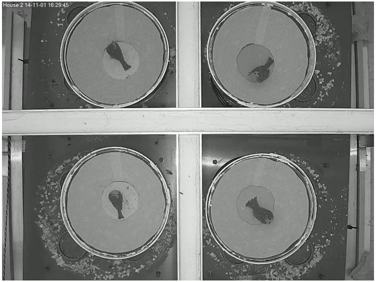
- Recording of birds in Emlen funnels

Bird Application



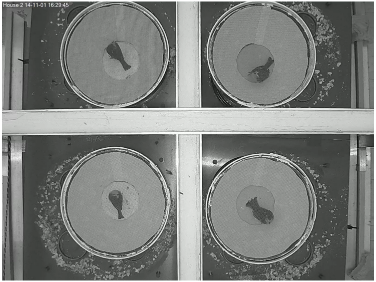
- Recording of birds in Emlen funnels
- Detect take-off times

Bird Application



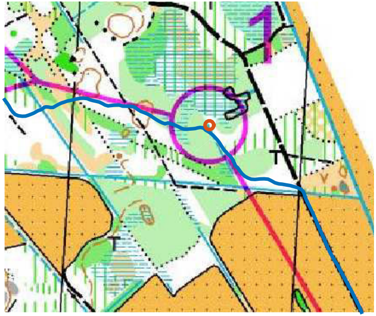
- Recording of birds in Emlen funnels
- Detect take-off times
- Estimate take-off directions

Bird Application



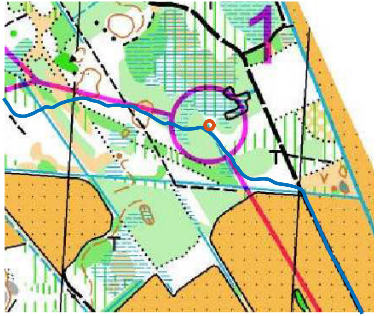
- Recording of birds in Emlen funnels
- Detect take-off times
- Estimate take-off directions
- Stationary and flight modes

Orienteering Application



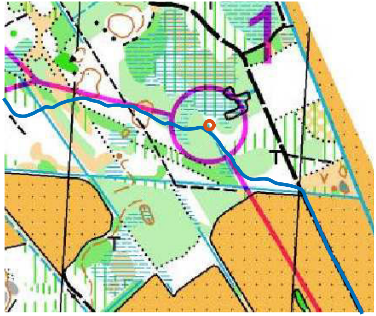
- GPS trajectory

Orienteering Application



- GPS trajectory
- Control position known

Orienteering Application



- GPS trajectory
- Control position known
- Improve position estimate

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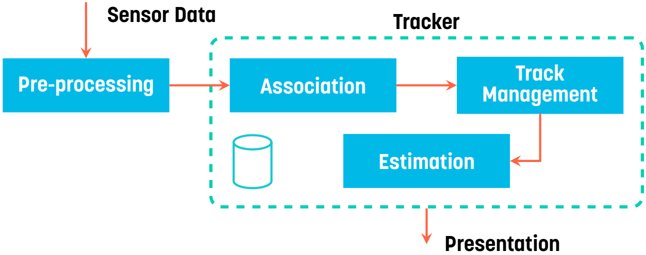
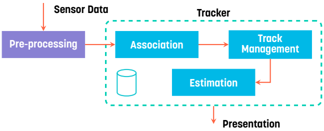
Target Tracking

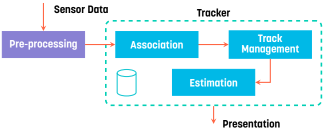


Pre-processing



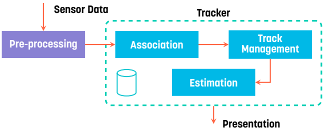
- Raw Sensor Data

Pre-processing



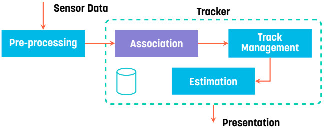
- Raw Sensor Data
- Signal and Image Processing

Pre-processing



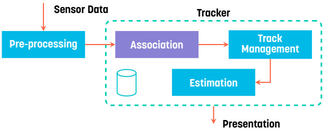
- Raw Sensor Data
- Signal and Image Processing
- Detections

Association

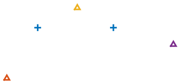


- Targets

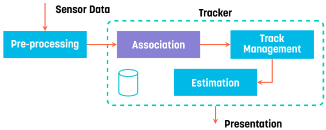
Association

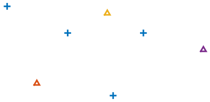


- Targets
- Detections



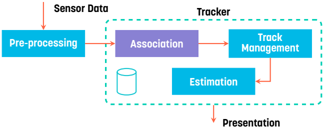
Association

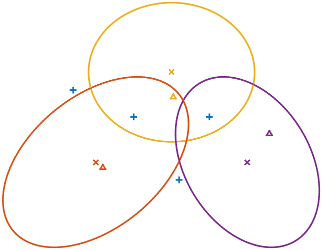




- Targets
- Detections
- False and Missed Detections

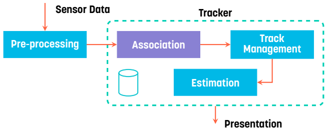
Association





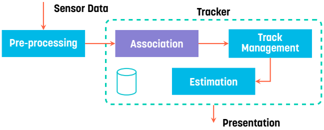
- Targets
- Detections
- False and Missed Detections
- Tracks

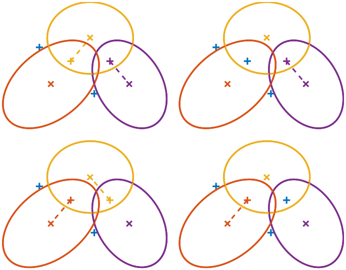
Association



- Targets
- Detections
- False and Missed Detections
- Tracks
- Hypothesis

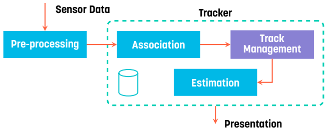
Association

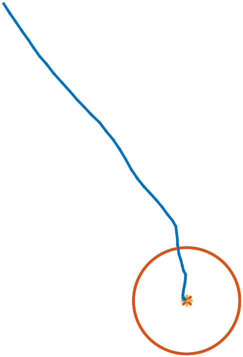




- Targets
- Detections
- False and Missed Detections
- Tracks
- Hypothesis
- Multiple Hypotheses

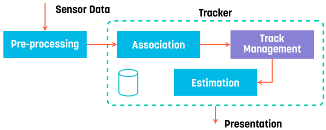
Track Management

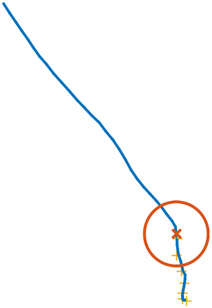




- New track

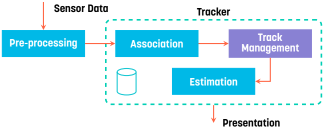
Track Management

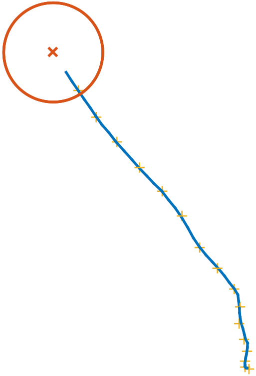




- New track
- Confirmed track

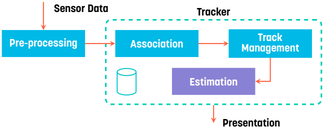
Track Management

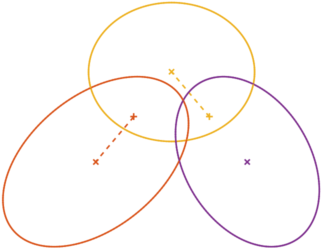




- New track
- Confirmed track
- Dead track

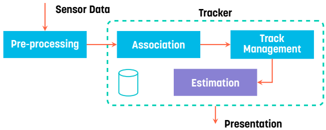
Estimation

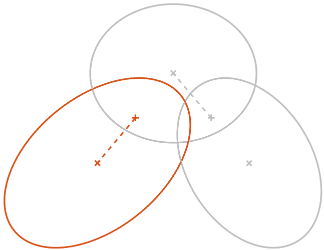




- Given association hypothesis

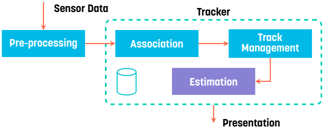
Estimation

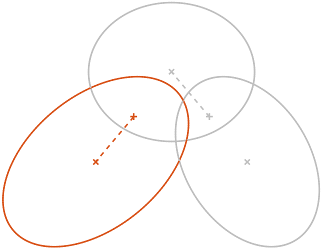




- Given association hypothesis
- Properties of each target

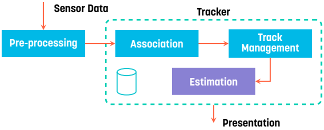
Estimation





- Given association hypothesis
- Properties of each target
- Over time using model

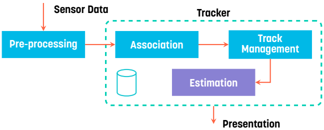
Estimation



Linear Gaussian state-space model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, \qquad \mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k),$$
$$\mathbf{y}_j = \mathbf{H}_j \mathbf{x}_j + \mathbf{v}_j, \qquad \mathbf{v}_j \sim \mathcal{N}(0, \mathbf{R}_j),$$
$$\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0).$$

Estimation



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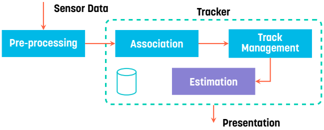

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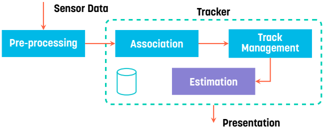

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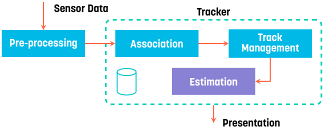
Estimation



Linear Gaussian state-space model

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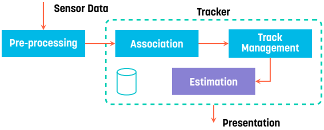
Estimation



Linear Gaussian state-space model

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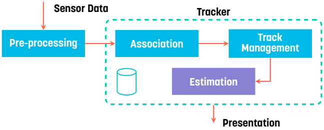
Estimation



Linear Gaussian state-space model

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(0, \mathbf{Q}_k), \\ \mathbf{y}_j &= \mathbf{H}_j \mathbf{x}_j + \mathbf{v}_j, & \mathbf{v}_j &\sim \mathcal{N}(0, \mathbf{R}_j), \\ \mathbf{x}_0 &\sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0). \end{aligned}$$

Posterior distribution

$$p(\mathbf{x}_k | \mathbf{x}_0, \mathbf{y}_1, \ldots, \mathbf{y}_k)$$

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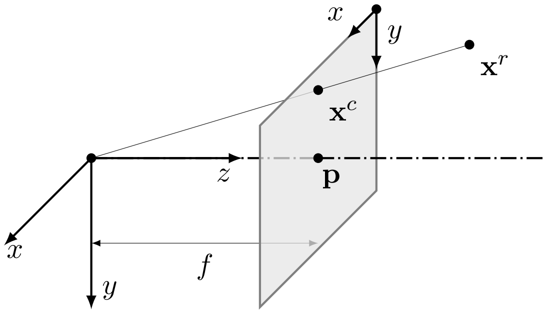
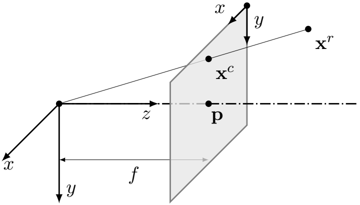
Camera Sensor

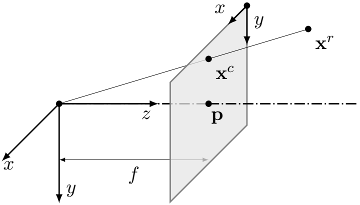


Camera Model



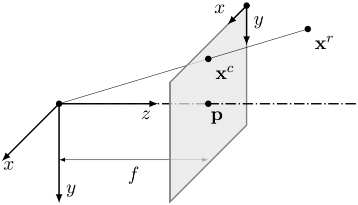
- Camera extrinsics

Camera Model



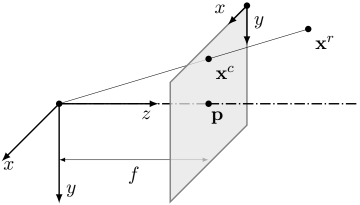
- Camera extrinsics
- Camera intrinsics

Camera Model



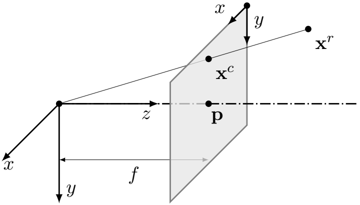
- Camera extrinsics
- Camera intrinsics
- Projection

Camera Model



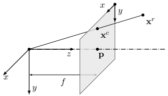
- Camera extrinsics
- Camera intrinsics
- Projection
- Perspective compensation

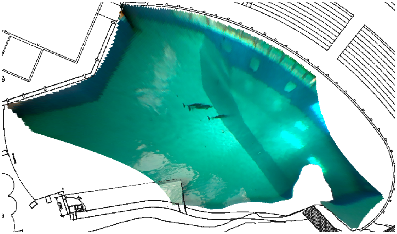
Camera Model



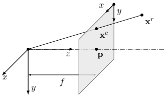
- Camera extrinsics
- Camera intrinsics
- Projection
- Perspective compensation
- Lens distortion

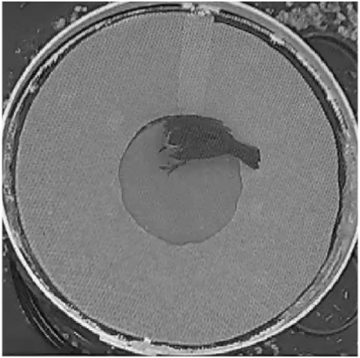
Dolphin - Camera Model



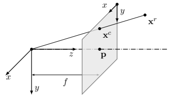


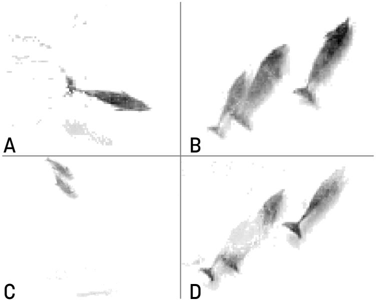
Bird - Camera Model





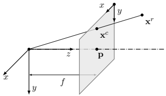
Foreground Segmentation

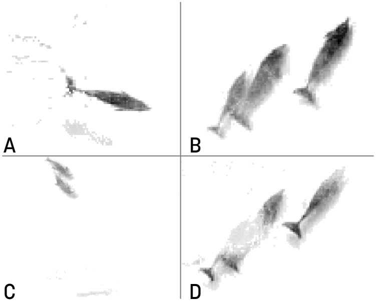




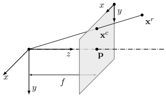
- Gaussian-mixture background model

Foreground Segmentation

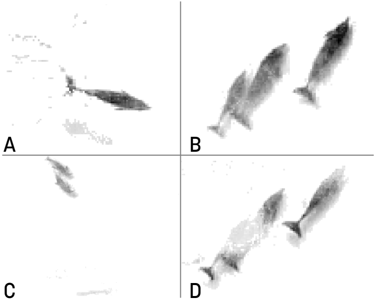




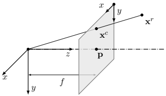
- Gaussian-mixture background model
- Handles changing light conditions



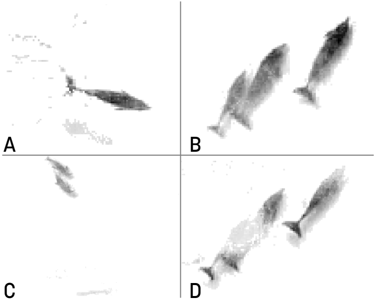
Foreground Segmentation



- Gaussian-mixture background model
- Handles changing light conditions
- Handles reflections



Foreground Segmentation



- Gaussian-mixture background model
- Handles changing light conditions
- Handles reflections
- Degree of confidence

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Constrained Motion Model

Constrained Motion Model

- Targets constrained to region

Constrained Motion Model

- Targets constrained to region
- Feasible predictions

Constrained Motion Model

- Targets constrained to region
- Feasible predictions
- Similar behaviour

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n}) \big) \Big) w(\mathbf{x},\, \mathbf{n}) \, d\mathbf{n}$$

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n})\big)\Big) w(\mathbf{x},\, \mathbf{n})\, d\mathbf{n}$$

- Nearly constant speed

$$\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \dot{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix}$$

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n})\big)\Big) w(\mathbf{x},\, \mathbf{n})\, d\mathbf{n}$$

- Nearly constant speed
- Influence by edges

$$w(\mathbf{x},\, \mathbf{n}) = \frac{1}{\|\mathbf{p} - \mathbf{n}\|^2}$$

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n}) \big) \Big) w(\mathbf{x},\, \mathbf{n})\, d\mathbf{n}$$

- Nearly constant speed
- Influence by edges
- Avoid collision with edges

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n}) \big) \Big) w(\mathbf{x},\, \mathbf{n})\, d\mathbf{n}$$

- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges

Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a\big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n})\big)\Big) w(\mathbf{x},\ \mathbf{n})\, d\mathbf{n}$$

- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges
- Integrate along edge

Turning Model

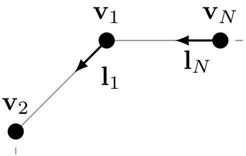
$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \int_{\mathbf{N}} \Big(\beta_d + \beta_a \big(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}(\mathbf{n})\big)\Big) w(\mathbf{x},\, \mathbf{n})\, d\mathbf{n}$$

Clockwise
or
Counterclockwise

- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges
- Integrate along edge
- Preferred direction

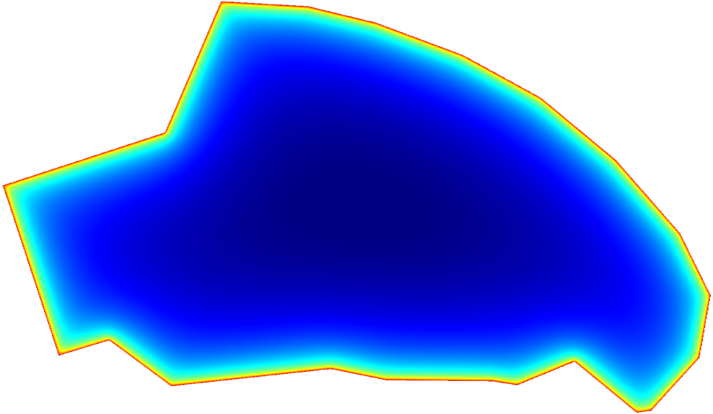
Turning Model

$$\omega(\mathbf{x}) = d_r(\mathbf{x}) \sum_{i=1}^{N} \big(\beta_d + \beta_a(\dot{\mathbf{p}}_\perp \cdot \mathbf{l}_i)\big) w_i(\mathbf{x})$$

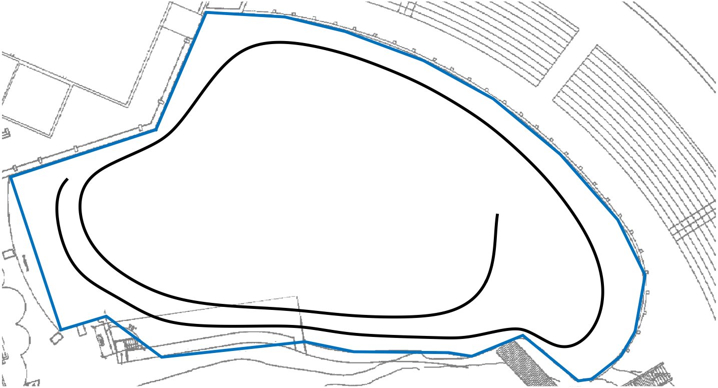


- Nearly constant speed
- Influence by edges
- Avoid collision with edges
- Align with edges
- Integrate along edge
- Preferred direction
- Polygon region

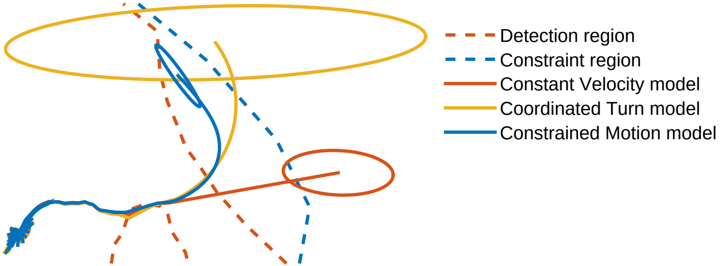
Potential Field



Dolphin - Model Simulation



Dolphin - Model Comparisons



Dolphin - Complete Framework

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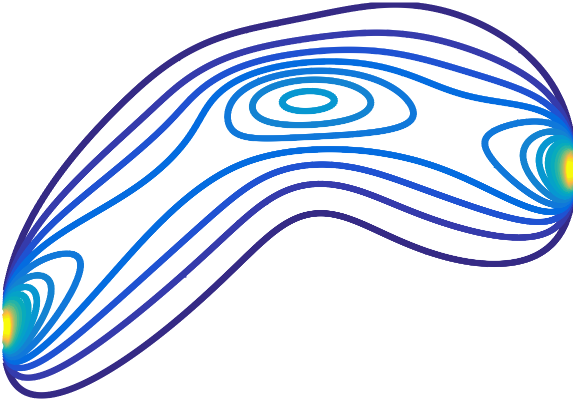
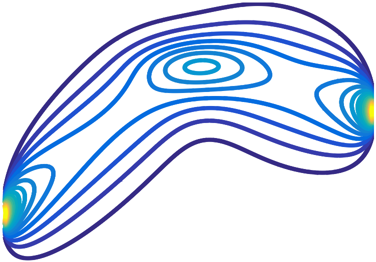
Uncertain Timestamp Model

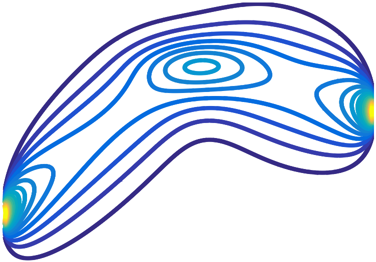


Uncertain Timestamp Model



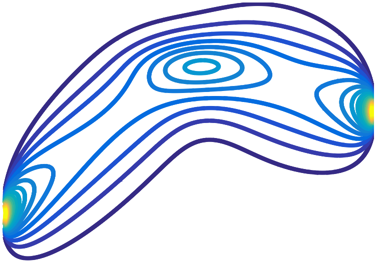
- Traditional measurements

Uncertain Timestamp Model



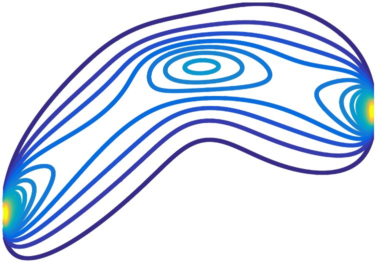
- Traditional measurements
- Observations sampled at an uncertain time

Uncertain Timestamp Model



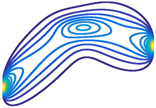
- Traditional measurements
- Observations sampled at an uncertain time
- Crime scene investigations

Uncertain Timestamp Model



- Traditional measurements
- Observations sampled at an uncertain time
- Crime scene investigations
- Traces from animals

Uncertain Timestamp Model



Consider a linear Gaussian state space model,

$$
\begin{aligned}
\mathbf{x}_k &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, & \mathbf{w}_k &\sim \mathcal{N}(0, \mathbf{Q}_k), \\
\mathbf{y}_j &= \mathbf{H}_j^y \mathbf{x}_j + \mathbf{v}_j^y, & \mathbf{v}_j^y &\sim \mathcal{N}(0, \mathbf{R}_j^y), \\
\mathbf{x}_0 &\sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0), & &
\end{aligned}
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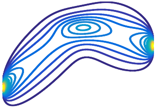
Uncertain Timestamp Model



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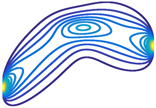
\end{aligned}
$$

Extend the model with

$$
\mathbf{z} = \mathbf{H}^z \mathbf{x}_\tau + \mathbf{v}^z, \qquad \mathbf{v}^z \sim \mathcal{N}(0, \mathbf{R}^z), \qquad \tau \sim p(\tau).
$$

Uncertain Timestamp Model



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Uncertain Timestamp Model

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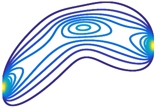

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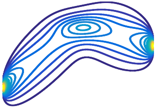

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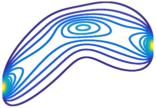
Simple Uncertain Time Scenario

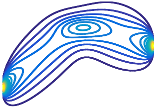


Consider the simple model,

$$x_k = x_{k-1} + w_k, \qquad w_k \sim \mathcal{N}(0, Q),$$
$$y_j = x_j + v_j^y, \qquad v_j^y \sim \mathcal{N}(0, R^y)$$

for $k \in \{1, \ldots, N\}$ and two measurements y_1 and y_N.

Simple Uncertain Time Scenario



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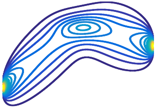
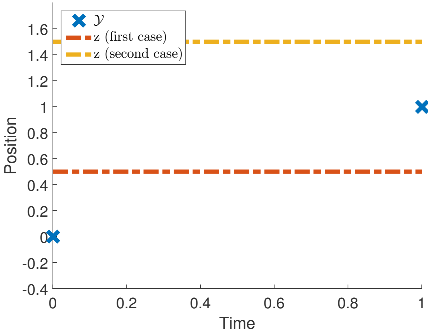
Extend the model with

$$z = x_\tau + v^z, \qquad v^z \sim \mathcal{N}(0, R^z), \qquad \tau \sim p(\tau).$$

Simple Uncertain Time Scenario

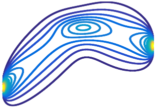


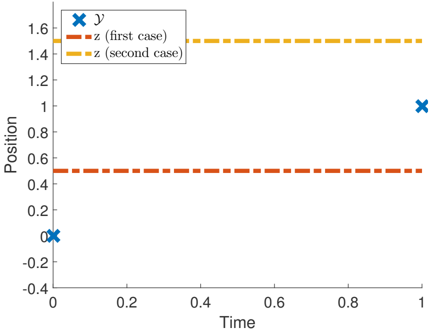


The measurements are

- $y_1 = 0,$
- $y_N = 1.$

Simple Uncertain Time Scenario





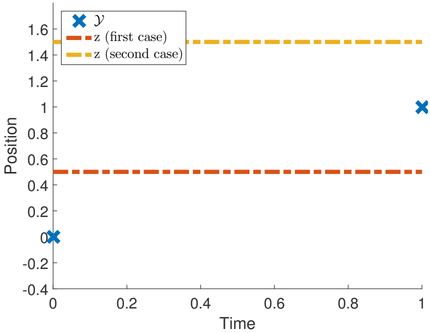
The measurements are

- $y_1 = 0$,
- $y_N = 1$.

The two cases of observations are

- $z = 0.5$ with flat prior.

Simple Uncertain Time Scenario



The measurements are

- $y_1 = 0$,
- $y_N = 1$.

The two cases of observations are

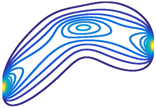
- $z = 0.5$ with flat prior.
- $z = 1.5$ with non-flat prior.

Posterior Distributions - Time

The posterior distribution of the uncertain time is

$$w_\tau \triangleq \underbrace{p(\tau|\mathcal{Y}, \mathbf{z})}_{\text{Posterior}} \propto \underbrace{p(\tau)}_{\text{Prior}} \underbrace{\mathcal{N}(\mathbf{z}|\, \hat{\mathbf{z}}_\tau,\, \mathbf{S}_\tau)}_{\text{Likelihood}}.$$

Posterior Distributions - Time



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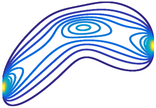

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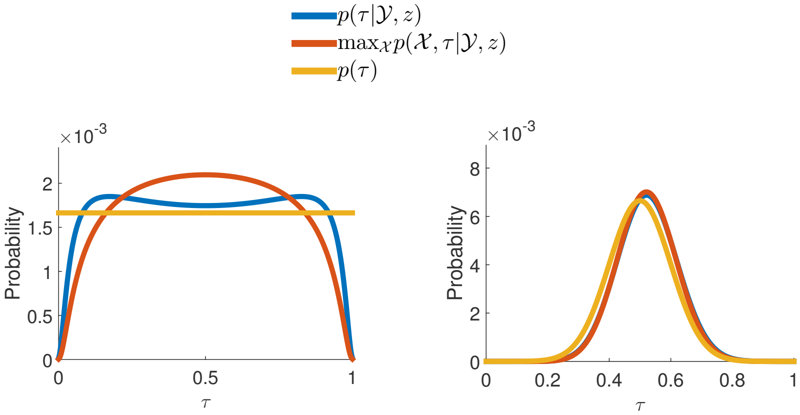
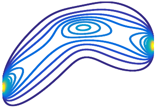
Posterior Distributions - Time



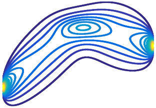
Posterior Distributions - States



The posterior distribution of the state is

$$p(\mathbf{x}_k|\mathcal{Y}, \mathbf{z}) = \sum_{\tau=1}^{N} w_\tau \cdot \mathcal{N}(\mathbf{x}_k|\hat{\mathbf{x}}_k^\tau, \mathbf{P}_k^\tau).$$

Posterior Distributions - States



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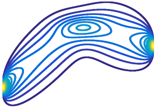

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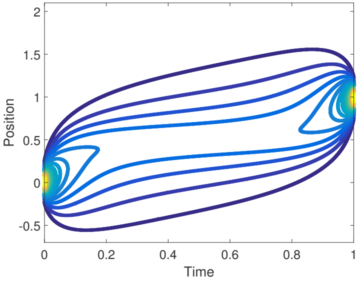
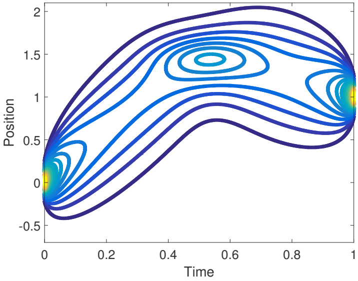
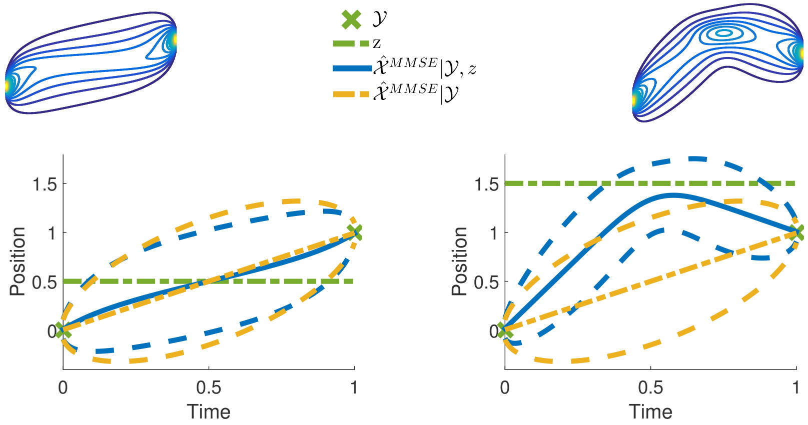
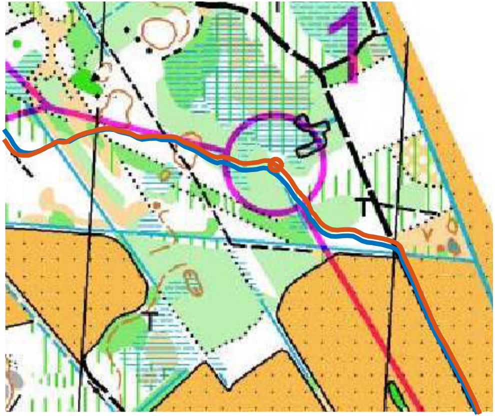




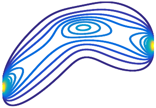
Estimators - Minimum Mean Squared Error



Orienteering - Results



- Adjusted trajectory



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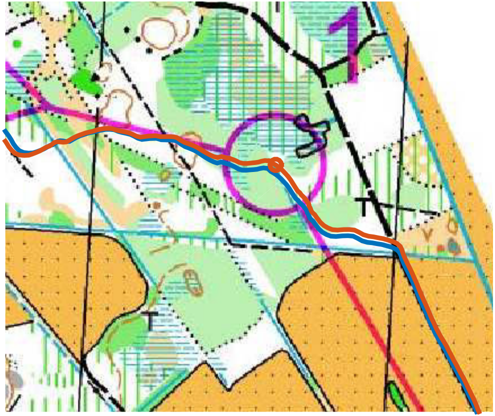
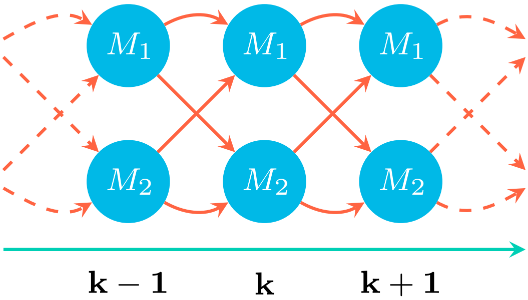


- Adjusted trajectory
- Single control point

Mode Observations



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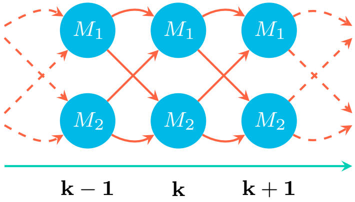
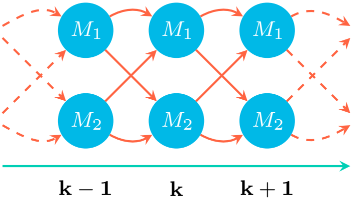


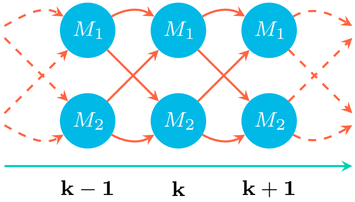
- Jump Markov model

Mode Observations



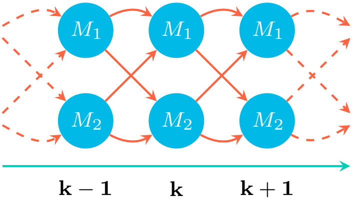
- Jump Markov model
- Model target behaviour

Mode Observations



- Jump Markov model
- Model target behaviour
- Probability of switching

Mode Observations



- Jump Markov model
- Model target behaviour
- Probability of switching
- Direct mode observation

Jump Markov Model

The jump Markov linear model is

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\mathbf{x}_k &= \mathbf{F}_k(\delta_k)\mathbf{x}_{k-1} + \mathbf{w}_k, \qquad & \mathbf{w}_k &\sim \mathcal{N}(\mathbf{0},\ \mathbf{Q}_k(\delta_k)),\\
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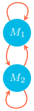

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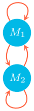
$$p(\delta_k \mid \delta_{k-1}) = \mathbf{\Pi}_k^{\delta_k, \delta_{k-1}}, \qquad \delta_k \in \mathcal{S}.$$



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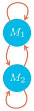
Augmented is the direct mode observation,

$$\mathbf{z}_i \sim p(\mathbf{z}_i \mid \delta_i).$$



Posterior Distribution - Mode Sequence

The posterior probability of a mode sequence $\{\delta_j^i\}_{j=1}^k$ is computed recursively by

$$\underbrace{w_k^i}_{\substack{\text{New}\\\text{Weight}}} \propto \underbrace{w_{k-1}^i}_{\substack{\text{Old}\\\text{Weight}}} \cdot \underbrace{\mathbf{\Pi}_k^{\delta_k^i,\delta_{k-1}^i}}_{\substack{\text{Transition}\\\text{Probability}\\\text{Matrix}}} \cdot \underbrace{\mathcal{N}(\mathbf{y}_k \,|\, \hat{\mathbf{y}}_k^i,\, \mathbf{S}_k^i)}_{\text{Likelihood}} \cdot \underbrace{p(\mathbf{z}_k \,|\, \delta_k^i)}_{\substack{\text{Mode}\\\text{Observation}\\\text{PDF}}}.$$

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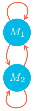

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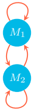

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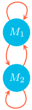

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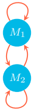

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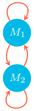

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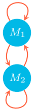

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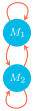

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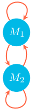
The direct mode observation is the radial position

$$z_k = \sqrt{x_k^2 + y_k^2},$$



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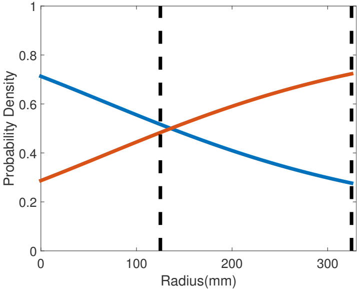
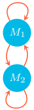
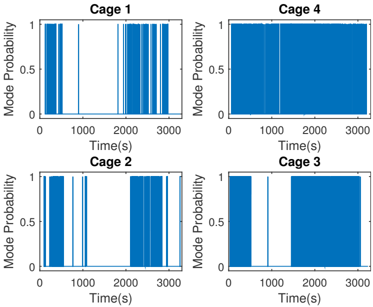
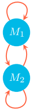




Bird - Results



- Estimated modes



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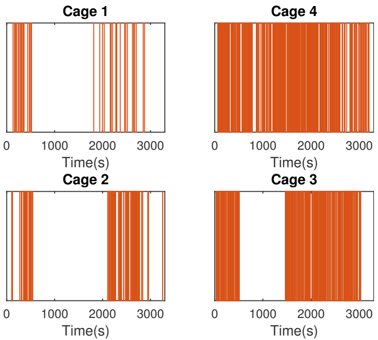

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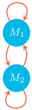

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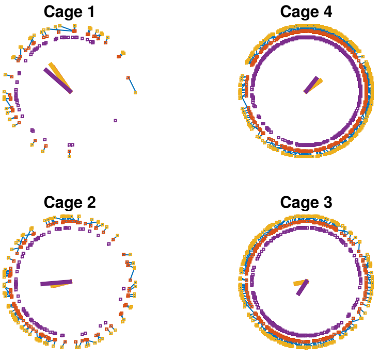

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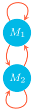

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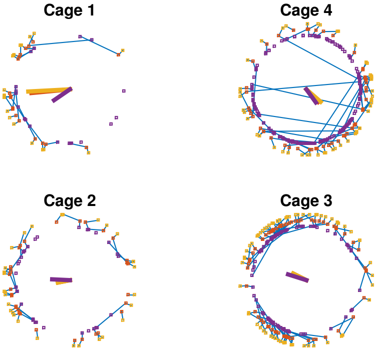
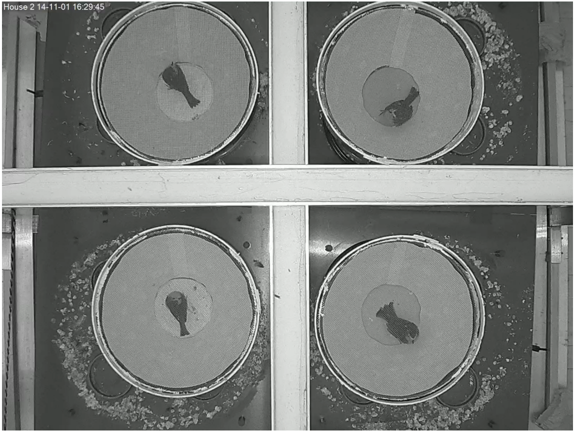


- Estimated modes
- Extracted takeoffs
- Takeoff directions
- Matched takeoffs

Bird - Complete Framework



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Conclusions

Theory is presented on

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Demonstration through various applications

Future Work

Possible future work is

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Thank you for listening!

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