Uncertain Timestamp Model

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Problem Formulation

Traditionally in estimation, the information in measurements is assumed to be noisy, while the sampling times are assumed to be accurately known. However, there are many applications where observations are also uncertain in time.

Uncertain Timestamp Model

A traditional continuous-discrete linear Gaussian state-space model is considered

$$\begin{aligned}\mathbf{x}(t_0) &\sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0),\\ d\mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t)dt + d\boldsymbol{\beta}(t), \qquad \mathbf{E}[d\boldsymbol{\beta}(t)d\boldsymbol{\beta}^T(t)] = \mathbf{Q}dt,\\ \mathbf{y}_j &= \mathbf{H}_j^y\mathbf{x}(t_j^y) + \mathbf{e}_j^y, \qquad \mathbf{e}_j^y \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_j^y), \qquad j \in \mathcal{J},\end{aligned}$$

with known timestamps for the measurements $\mathcal{Y} = \{\mathbf{y}_j\}_{j\in\mathcal{J}}$. Observations $\mathcal{Z} = \{\mathbf{z}_i\}_{i\in\mathcal{I}}$ with uncertain timestamps $\mathcal{T} = \{\tau_i\}_{i\in\mathcal{I}}$ are augmented to the model as

$$\mathbf{z}_i = \mathbf{H}_i^z\mathbf{x}(\tau_i) + \mathbf{e}_i^z, \qquad \mathbf{e}_i^z \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_i^z), \qquad i \in \mathcal{I}.$$

The prior distribution of the timestamps is given by $p(\mathcal{T})$.

Posterior Distribution

The posterior distribution of the state is given by

$$p(\mathbf{x}(\cdot)|\mathcal{Y},\mathcal{Z}) = \int_{\mathcal{T}} p(\mathbf{x}(\cdot)|\mathcal{T},\mathcal{Y},\mathcal{Z}) \cdot p(\mathcal{T}|\mathcal{Y},\mathcal{Z})\, d\mathcal{T},$$

where the first factor is a Gaussian process, and the posterior distribution of the uncertain timestamps is given by

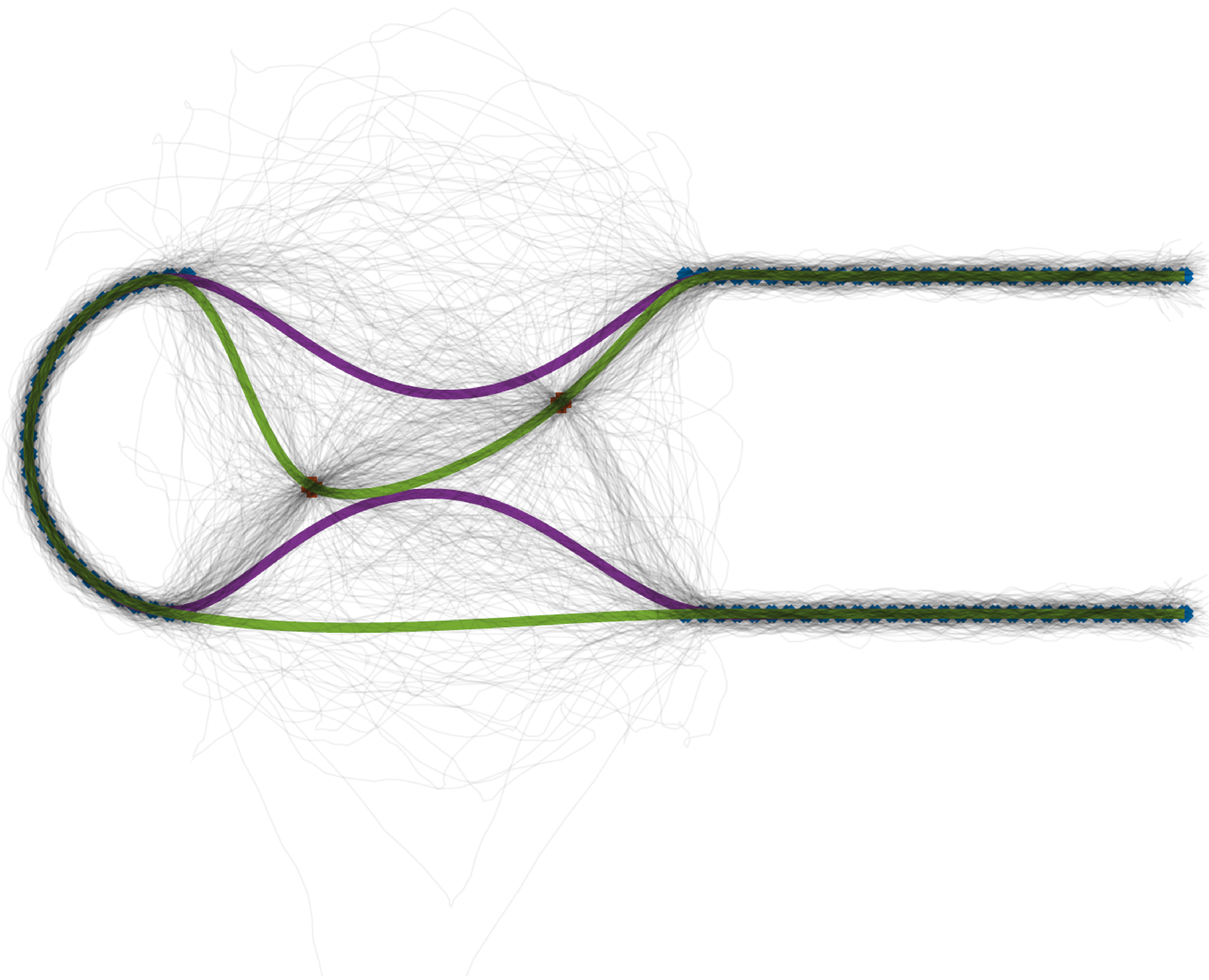
$$p(\mathcal{T}|\mathcal{Y},\mathcal{Z}) \propto \prod_{i\in\mathcal{I}} \left[p(\mathbf{z}_i|\mathcal{T}_{1:i},\mathcal{Y},\mathcal{Z}_{1:i-1}) p(\tau_i|\mathcal{T}_{1:i-1}) \right].$$

Approximate inference is achieved using

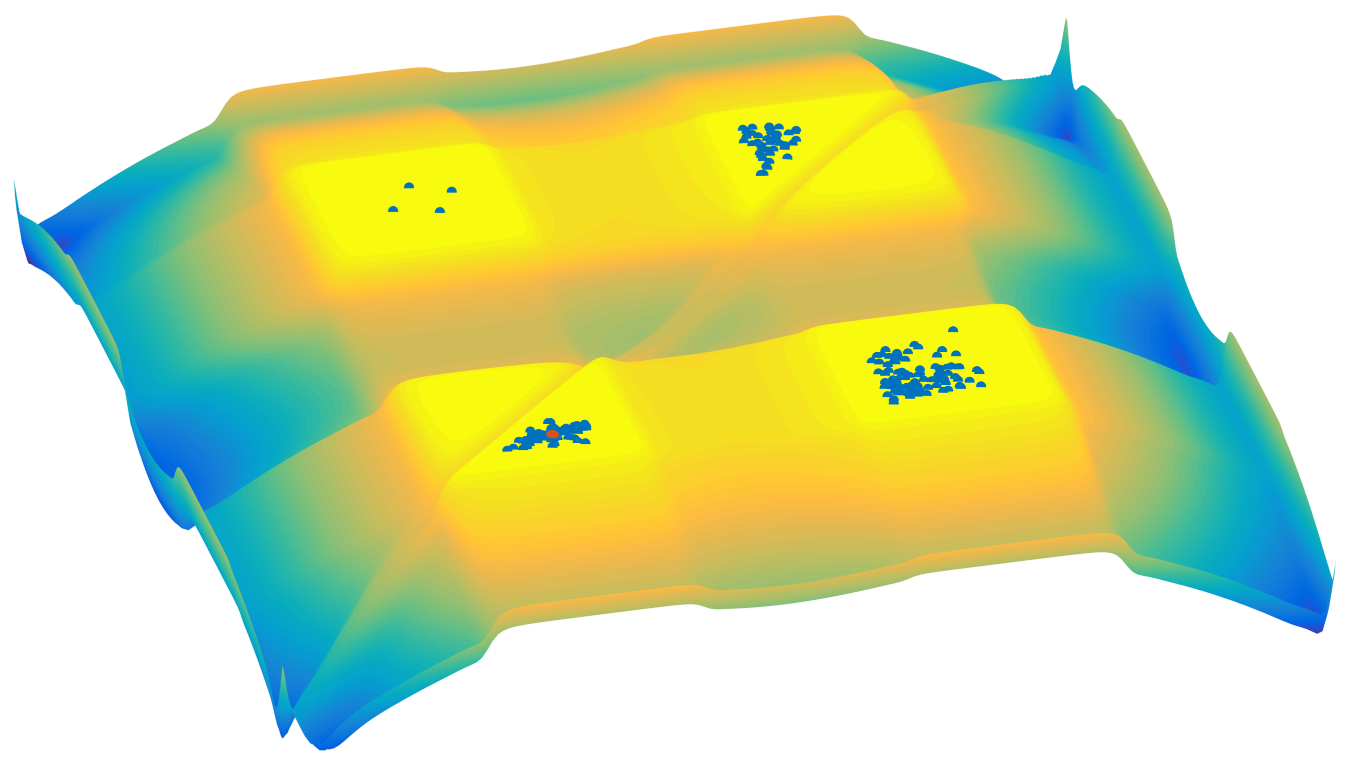
- Gibbs sampling
- Rauch-Tung-Striebel smoothing
- Expectation-Maximization algorithm
- Discrete timestamp approximation

Simple Scenario with Multiple Observations

A simple two-dimensional scenario is considered with a nearly constant velocity model and with two observations. There is an abundance of traditional measurements, except for two gaps, and two observations with uncertain timestamps close to the gaps. The figure shows sample trajectories from the posterior distribution, the MMSE estimate (purple) and the MAP estimate (green).



The figure shows the logarithm of the posterior distribution of the timestamps. The blue points are samples from a Gibbs sampler and the red dot is the *maximum a posteriori* estimate.

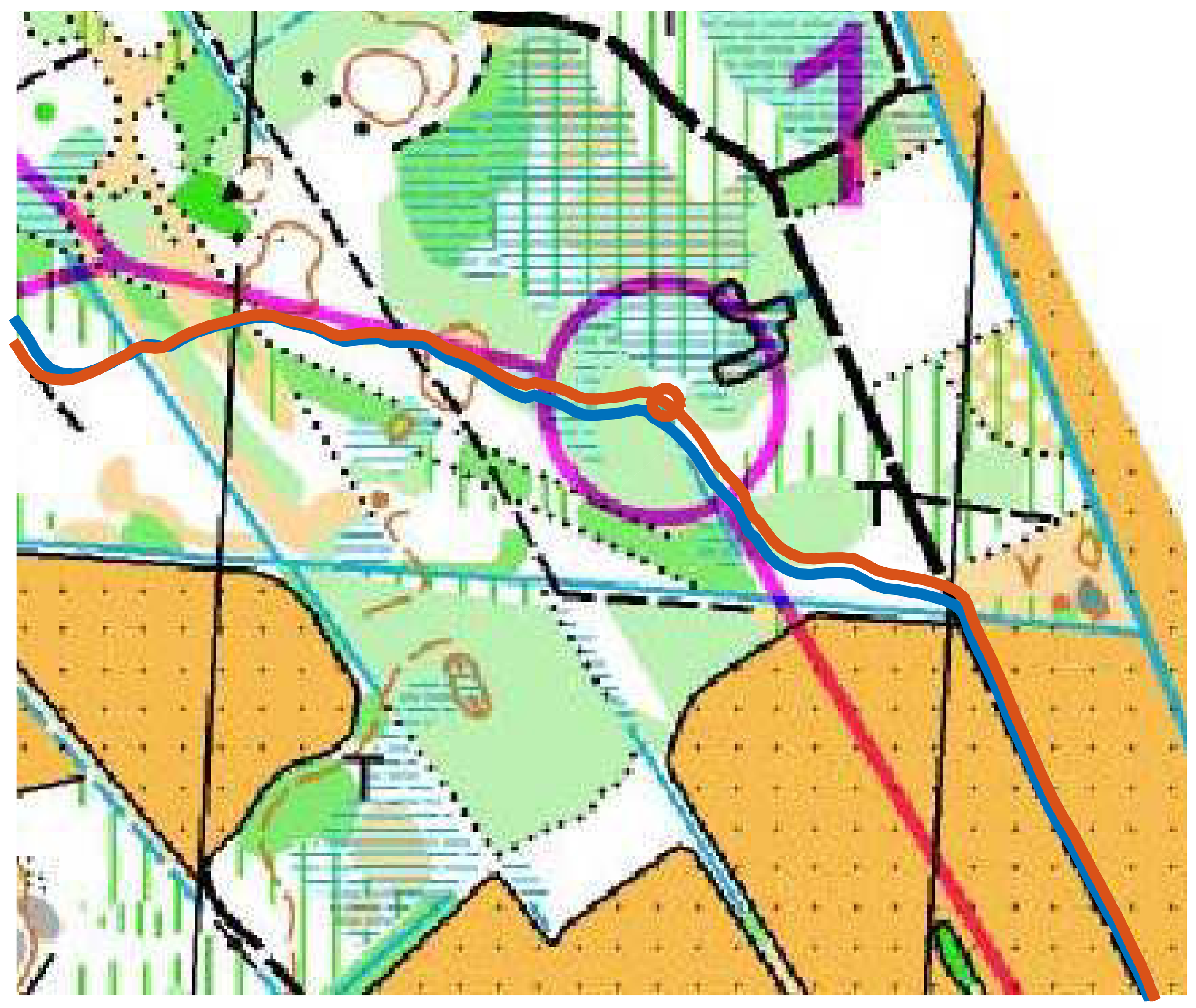


Applications

Tracking of Animals: In addition to traditional measurements, such as from radars or cameras, a trace left by an animal can be seen as an accurate observation of the position with an uncertain timestamp.

Crime Scene Investigations: The place of the crime often is known accurately, but not the time, and witness statements often contain uncertainty in both time and place, while surveillance cameras are precise in time.

Sprint Orienteering: A control point in sprint orienteering can be used as an observation with an uncertain timestamp to improve the GPS position of the sprinter.



Conclusions & Future Work

- The estimate is improved by the additional observations.
- Efficient methods for approximating the posterior distribution and MAP estimate are used.
- Consider corresponding filter and nonlinear models.
- Further analyse gain and accuracy of method.

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