TSRT09 – Control Theory

Lecture 1: Introduction, Regulator problem

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Course information

- Lecturer & examiner
  - Claudio Altafini (`claudio.altafini@liu.se`)
- Teaching & Lab assistant
  - Filipe Barbosa (`filipe.barbosa@liu.se`)
- Credits
  - 6 HP = 4.5 (exam) + 1.5 (lab)
- Course home page
  `https://isy.gitlab-pages.liu.se/rt/courses/TSRT09`
  - program, slides, exercises and lab material
  - old exams
- Lisam
  - lab sign-up
  - lecture videos from 2021

Organization of the course

- 12 lectures
  - live and in presence
- 12 exercise sessions
  - 9 regular exercise sessions + 3 computer sessions, all in presence
- 3 labs
  1. Multivariable regulator
     - where: RT3
     - register: Lisam
     - preliminary: "dugga"
     - evaluation: on the spot
  2. Robust control design for the JAS 39 Gripen
     - where: computer room (guidance + self work)
     - evaluation of preparatory tasks (before the lab); register in Lisam
     - evaluation of the lab: computer room
  3. Stability analysis and regulation of a nonlinear system
     - where: computer room
     - evaluation on the spot

Schema

TODAY

w4

w5

w6

w7

w8

w9

02-24/26: LAB1

03-03: LAB2 prep eval.

03-04/05: LAB2

03-06: LAB2 eval.

w10

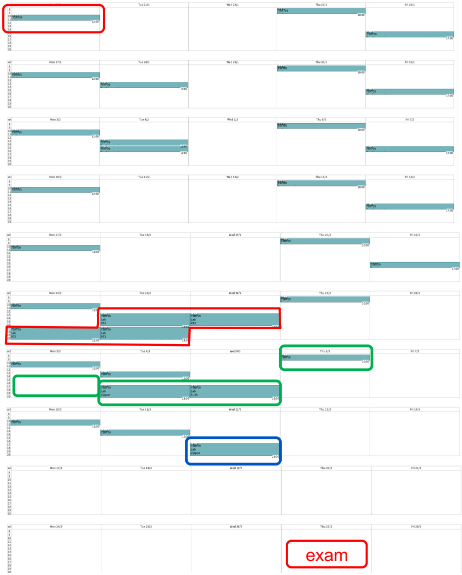
w11

03-12: LAB3

w12

03-27: exam

w13

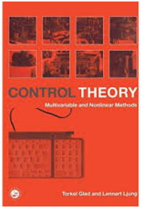


Course literature

- Textbook

Swedish

English

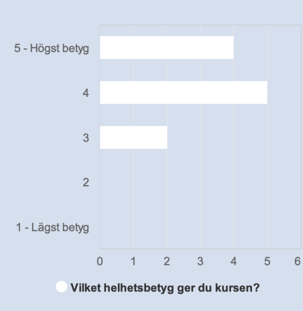


- Exercise compendium: see web page

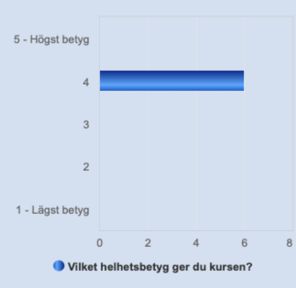
Feedback from students: overall evaluation

Vilket helhetsbetyg ger du kursen?

Year 2022
(in presence)



Year 2023
(in presence)



Year 2024
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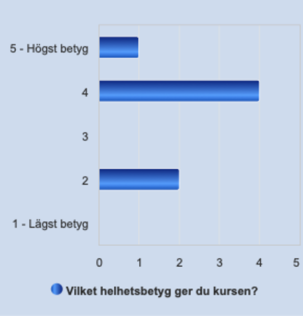


Feedback from students: lectures

- *Skulle uppskatta bättre struktur på föreläsningarna, med titlar innan text och ekvationer.*
- *Bra och pedagogiska föreläsningar med tydlig struktur!*
- *If the lectures were based on examples instead of a lot of theory I think it would be easier to understand the theory behind the examples.*
- *Jag gillade föreläsningarna men tycker att fler plottar kunde använts i exemplen för att förtydliga resultaten.*
- *Vet att folk har klagat på att kursen är för teoretisk. [...] Svårt att klaga när kursen heter "Reglerteori" känner jag.*
- *Föreläsningarna var väldigt svåra att hänga med i. Föreläsaren skrev väldigt snabbt och det var svårt att kunna anteckna i takt.*
- *It was good that the recorded lecture were available.*

Feedback from students: exercise sessions

- *Jag tycker att lektionerna var lärorika. Jag gillar att man fick en genomgång av relevant teori i början av dem och uppgiftsgenomgångarna var bra.*
- *Kopplat till lektionerna finns inget för mig att tillägga. De var väldigt bra!*
- *Jag tycker att för många uppgifter gicks igenom på tavla under lektionerna vilket lämnade lite få uppgifter till eget arbete. Personligen lär jag mig bättre när jag får försöka själv än när en uppgift gås igenom på tavla.*
- *The computer exercises (sess. 4. 6. 8) were not very useful. The questions were not very clear and the solutions were mostly unfamiliar Matlab functions that one did not know beforehand.*

Feedback from students: labs

- *The content of the laborations was very good.*
- *The labs were very useful to understand some topics of the course*
- *I think that lab 1 was very good but that the other two, especially the third, were less good.*
- *The labs were extremely time consuming. The schedule said 4 hours but all of them took at least 5 or 6 hours.*
- *Spent 51 hours on the labs in total, slightly too high so might need to be adjusted. Especially lab2 was a bit too heavy.*
- *Many had difficulty finishing lab1 and lab3 during the scheduled lab time, despite having finished all preparatory exercises.*
- *A big downside of this course was that all the labs ocurred during the three last weeks of the course.*

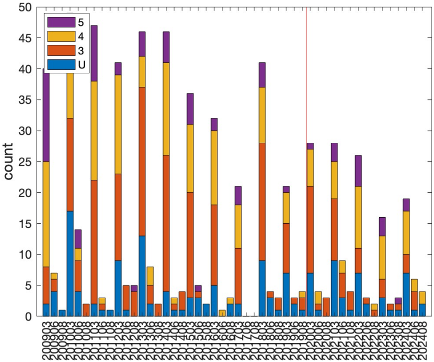
Feedback from students: exam

Year 2020

- *The layout of the exam should look more like the old versions and contain basic exercises without twists so the student can show his/her potential.*

Year 2023

- *I think the questions on the exam were excellent and that solving exercises closer to them during the course would've helped my learning outcome.*



Course outline

1. Fö 1: Introduction
2. Fö 2 – 4: Part I: Linear systems
   - Multivariable systems
   - Disturbance description
   - Kalman filter
3. Fö 4 – 7: Part II: Linear control theory
   - Closed-loop structure; Design methods
   - LQG-design
   - $\mathcal{H}_2$, $\mathcal{H}_\infty$ methods
4. Fö 8 – 11: Part III: Nonlinear control theory
   - Methods for nonlinear systems: stability, phase plane analysis
   - Self-sustained oscillations
   - Design via exact linearization
5. Fö 12: Part II again: Limitations in control design

Control Theory

"The science of controlling a system so that it achieves its goals in spite of disturbances and uncertainty."

- A very wide range of applications in all fields: electronic, mechanical, robotic, autonomous systems, bioengineering,...
- The fundamental concepts are not implementation-specific
- Basic Control course ("Reglerteknik"): linear SISO (Single-Input Single-Output) systems
- This course: linear MIMO (Multi-Input Multi-Output) systems
  + disturbances, uncertainties and nonlinearities

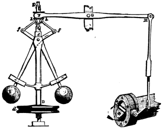
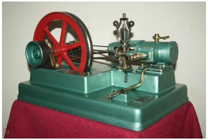
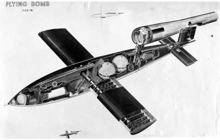
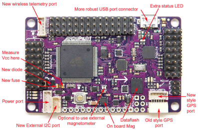


FIG. 4.—Governor and Throttle-Valve.







What are the systems one can control?

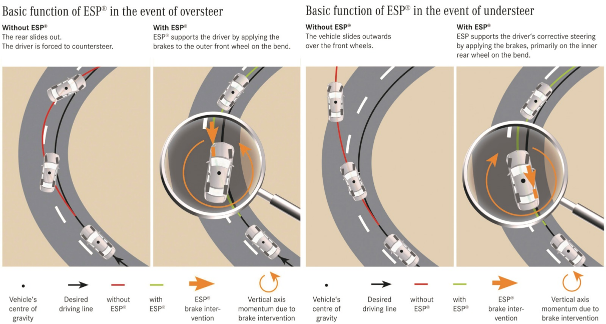
Basically all technologies around us

Example: Car:

- servosteering
- throttle
- brake
- gear box control
- ABS
- antispin, antiskid
- cruise control
- hybrid power train
- .....

Example: Anti-skid control in a Car

Example: Anti-skid control = ESP (Electronic Stability Program)



- no ESP

- with ESP

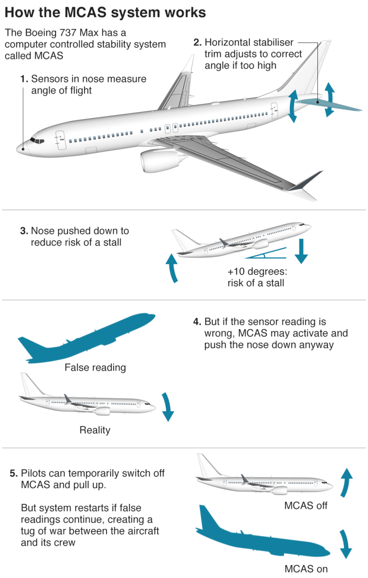
Example: Airplane

- stabilization
- cruise control, altitude control
- navigation
- fly-by-wire
- weapon systems
- ......

Airplane: Boeing 737Max crash

Example: effect of disturbance

- Control system: Manoeuvring Characteristics Augmentation System (MCAS)
- Aim: follow a correct pitch reference angle
- Disturbance: sensor misreading/failure
- Consequence: actuator inputs wrong control signal



Example: Autonomous systems

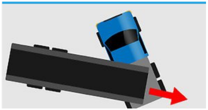
Example: quadrotors

Example: Autonomous systems

Example: autonomous reversing a truck and trailer

- 20 years ago (KTH)

- difficulty: no driver
- difficulty: unstable system (jack-knife) $\Leftrightarrow$ inverted pendulum



- now (LiU + Scania)

Example: high-speed trains

Instability in a dynamical system can have dramatic consequences

Example: lateral movement in train axles (vibration); becomes unstable above a certain velocity

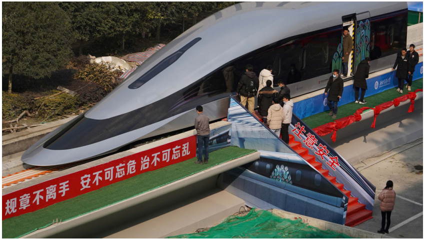
Example: high-speed trains (cont'd)

Instability was first discovered in the 1950s, when attempting to beat the train speed record (331 km/h)

Mastering this problem was an important step in developing high-speed trains.

Current record: 575 km/h (2007), Alstom train, France
(for MagLev: > 600 km/h, Japan)

In Sweden: current record 303 km/h (2008), Reginatåg.



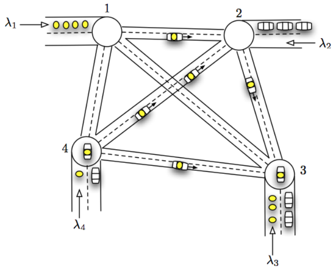
Example: Smartphone

- power control
- traffic/cell control
- DC/DC converter
- positioning (sensor fusion)
- ...

Example: Smart Urban Mobility

Example: Autonomous Mobility-on-Demand

- handles travel requests
- large-scale network
- routing and optimal service
- robustness and efficiency

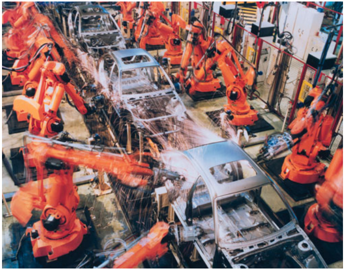


Example: Biomedical Engineering

- dialysis apparatus
- insuline pumps
- pacemakers
- anestesia
- avanced proteses
- haptics and telesurgery
- ......

Industrial production

- paper industry
- steel mills
- raffineries
- industrial robots
  - welding
  - assembling
  - painting
  - ...
- ......



Special in the course

- Advanced methods for control
    1. Multi-Input and Multi-Output (MIMO) systems.
    2. Systematic control design methods.
    3. Dealing with disturbances, uncertainty and nonlinearities.
    4. Basic limitations in control design.

- Computer exercise sessions and computer labs
    - become familiar with the software tools that implement the methodology
    - get the opportunity to work with problems of higher dimension (than those solvable by hand)

- Computer exam
    - conventional “hand calculations”
    - a computer assignment, typically the synthesis of a regulator with given specifications
    - use of the computer as an advanced calculator when needed

Lecture 1

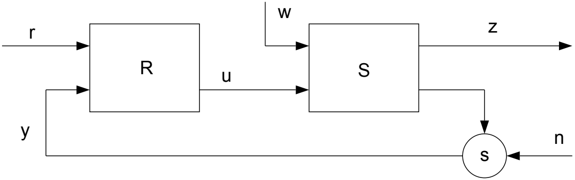
1. Introduction
2. Signal size, system gain
3. Singular values
4. Small gain theorem

In the book: Ch. 1 and 3.5

Regulator problem

Systems:
S: System to be controlled
R: Regulator

Signals:
z: Controlled variable
r: Reference signal
y: Measured signal
u: Control signal
w: System disturbance
n: Measurement error



Task: Choose u so that z follows r as close as possible, in spite of the presence of disturbances w, n, and of uncertainty in the system, while at the same time using reasonable values of u.

"Size" for a vector/matrix

- Norm of a vector

$$|z| = \sqrt{\sum_i z_i^2} = \sqrt{z^* z}$$

- Norm of a matrix

$$|A| = \sup_{z \neq 0} \frac{|Az|}{|z|} = \overline{\sigma}(A)$$

"Size" for a signal

- 2-norm

$$\|z\|_2^2 = \int_{-\infty}^{\infty} |z(t)|^2 \ dt$$

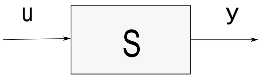
- ∞-norm

$$\|z\|_\infty = \sup_t |z(t)|$$

"Size" for a system

S could be:

1. Plant to be controlled
2. Regulator
3. Closed loop system



Gain of a system $\mathcal{S}$:

$$||\mathcal{S}|| = \sup_u \frac{||y||_2}{||u||_2}$$

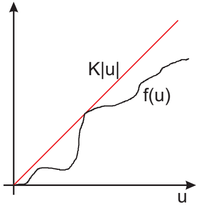
"Choose u so that the quotient between the size of output and that of the input is maximized".

Computation of system gain

Gain can be computed sufficiently easily for two types of systems:

Example 1: Static nonlinearities

$$y = f(u) \qquad \text{s.t.} \quad |f(u)| \leq K|u|$$



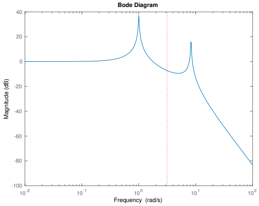
- Gain can vary with amplitude.
- Gain does not vary with frequency.

$$||\mathcal{S}|| = \sup_u \frac{||y||_2}{||u||_2} = K$$

Computation of system gain

Example 2: Single-Input Single-Output (SISO) linear systems

$$Y(s) = G(s)U(s) \quad \text{s.t.} \quad |G(i\omega)| \leq K \ \text{ for all } \omega$$



- Gain does not vary with amplitude.
- Gain varies with frequency.

$$||\mathcal{S}|| = \sup_u \frac{||y||_2}{||u||_2} = K$$

What is the "size" of $G(i\omega)$ if $G(i\omega)$ is a matrix?

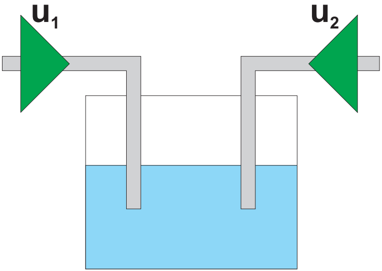
- SISO case: gain is associated to Bode plot of $G(i\omega)$
- MIMO case: the calculation of the gain is more difficult

$$G(i\omega) = \begin{bmatrix} G_{11}(i\omega) & G_{12}(i\omega) & \dots & G_{1m}(i\omega) \\ \vdots & \vdots & & \vdots \\ G_{p1}(i\omega) & G_{p2}(i\omega) & \dots & G_{pm}(i\omega) \end{bmatrix}$$

Question: Which Bode plot is of interest?

Question: How do we compute the gain of the system?

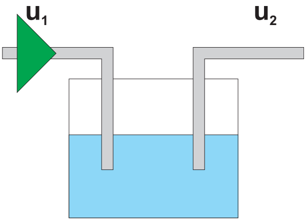
An example with two inputs

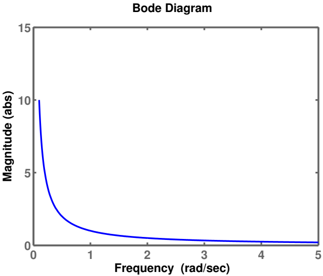


$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Regulation of the level x in a tank. Inflow into the tank is controlled from the two inputs u_1 and u_2 both of which can be positive or negative. $\Longrightarrow$ Multi-Input Multi-Output (MIMO) system.

An example with two inputs



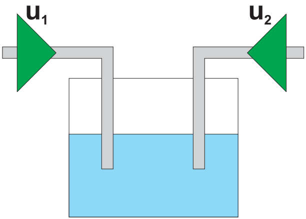


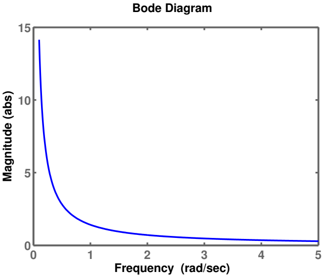
$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Consider first a regulation in which we disregard u_2 ($u_2 = 0$).

What will the gain be if we combine the contribution of u_1 and u_2?

An example with two inputs

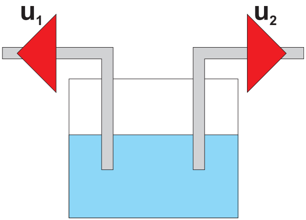


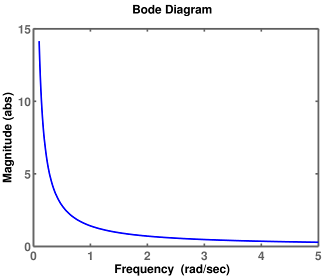


$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Let the two pumps work identically ($u_2 = u_1$): level changes faster.

An example with two inputs





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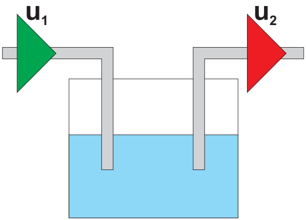
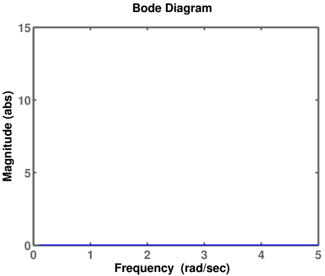
Let the two pumps work identically ($u_2 = u_1$): level changes faster.

An example with two inputs

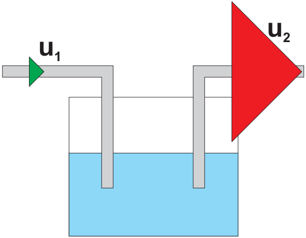


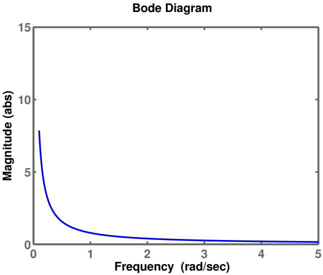


$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Let the pumps oppose each other ($u_2 = -u_1$): level does not change at all.

An example with two inputs

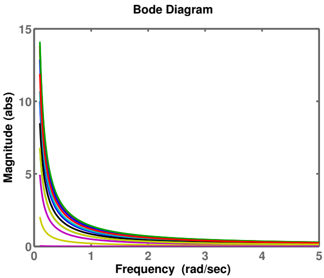




$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Consider arbitrary combinations (ex. $u_2 = -5.03 \cdot u_1$) between the two pumps.

An example with two inputs



$$\begin{cases} \dot{x} &= u_1 + u_2 \\ y &= x \end{cases} \qquad G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

"All" combinations lead to an "infinite number of Bode diagrams"...
Which one is the most relevant?

Computing the gain if $G(i\omega)$ is a matrix

MIMO system

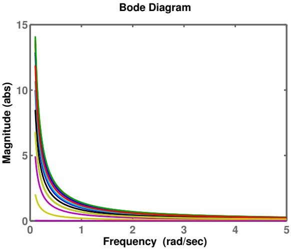
$$G(i\omega) = \begin{bmatrix} G_{11}(i\omega) & G_{12}(i\omega) & \dots & G_{1m}(i\omega) \\ \vdots & \vdots & & \vdots \\ G_{p1}(i\omega) & G_{p2}(i\omega) & \dots & G_{pm}(i\omega) \end{bmatrix}$$

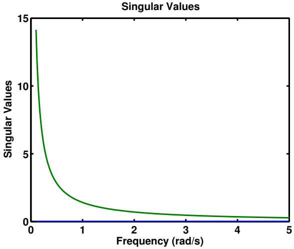
Question: Which Bode plot is of interest?

Question: How do we compute the gain of the system?

Answer: It is not the Bode plots that is of relevance, but the singular values of $G(i\omega)$.

An example with two inputs





$$\dot{x} = u_1 + u_2, \quad y = x$$

$$G(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \end{bmatrix}$$

Computing the gain if $G(i\omega)$ is a matrix. Summary

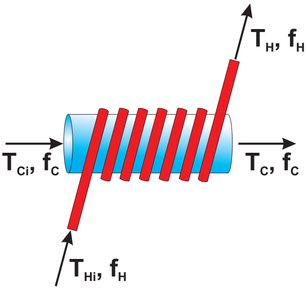
Important notions for gain of a MIMO system:

- $\underline{\sigma}(G(i\omega)) \leq \frac{|Y(i\omega)|}{|U(i\omega)|} \leq \overline{\sigma}(G(i\omega)) = |G(i\omega)|$
- $|G(i\omega)|=$ largest singular value of $G(i\omega)$
- $\|G\|_\infty=$ largest singular value of $G(i\omega)$ over all ω

$$\|G\|_\infty = \sup_\omega \overline{\sigma}(G(i\omega))$$

What matter are the singular values, not the Bode plots!

An example: heat exchanger



T_{H_i}, T_H = hot temp.
T_{C_i}, T_C = cold temp.
V_H, V_C = volumes
f_H, f_C = flows
β = heat exchange coeff.

$$V_C \frac{dT_C}{dt} = f_C(T_{C_i} - T_C) + \beta(T_H - T_C)$$
$$V_H \frac{dT_H}{dt} = f_H(T_{H_i} - T_H) - \beta(T_H - T_C)$$

Heat exchanger (cont'd)

$$\dot{x} = \begin{bmatrix} -(f_C+\beta)/V_C & \beta/V_C \\ \beta/V_H & -(f_H+\beta)/V_H \end{bmatrix} x + \begin{bmatrix} f_C/V_C & 0 \\ 0 & f_H/V_H \end{bmatrix} u$$
$$y = x$$

with $x = \begin{bmatrix} T_C \\ T_H \end{bmatrix}^T$ and $u = \begin{bmatrix} T_{C_i} \\ T_{H_i} \end{bmatrix}^T$.

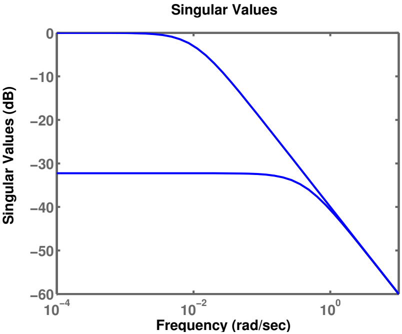
Use the numerical values $f_C = f_H = 0.01$ (m^3/min), $\beta = 0.2$ and $V_H = V_C = 1$ (m^3), which gives

$$\dot{x} = \begin{bmatrix} -0.21 & 0.2 \\ 0.2 & -0.21 \end{bmatrix} x + \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} u$$
$$y = x$$

Transfer function matrix

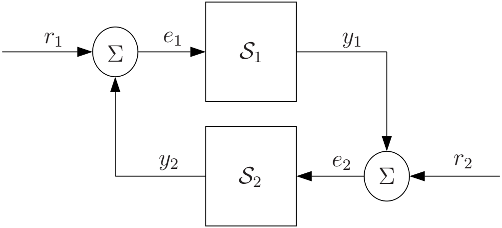
$$G(s) = \frac{0.01}{(s+0.01)(s+0.41)} \begin{bmatrix} s+0.21 & 0.2 \\ 0.2 & s+0.21 \end{bmatrix}$$

Singular values:



Small gain theorem

Input-Output stability: finite gain

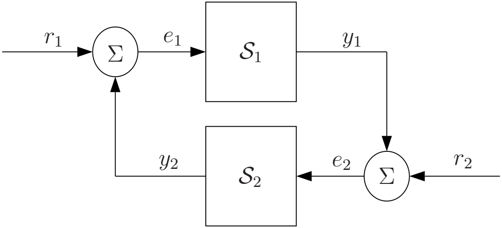


Theorem: Two stable systems $\mathcal{S}_1$ and $\mathcal{S}_2$ feedback coupled as in the figure give an interconnected system which is input-output stable if

$$||\mathcal{S}_2|| \cdot ||\mathcal{S}_1|| < 1$$

Small gain theorem

Input-Output stability: finite gain



Corollary: For linear systems, the criterion simplifies to

$$||\mathcal{S}_2\mathcal{S}_1|| < 1$$

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