4 DIGITAL FILTERS

4.2 It is necessary and sufficient that the impulse response is symmetric or antisymmetric.

4.3 For the sake of simplicity let the filter order be odd and $N = 4$.

$$H(z) = h(0) + h(1)\, z^{-1} - h(1)\, z^{-2} - h(0)\, z^{-3}$$

Now, for $z = 1$ we have $H(1) = h(0) + h(1) - h(1) - h(0) = 0$
Hence, it is not possible to have a lowpass filter, with N = even, with antisymmetric impulse response since the filter has a zero at $z = 1$, i.e., inside the passband. For an even order filter, for example, $N = 5$, we have

$$H(\mathrm{z}) = h(0) + h(1)\, z^{-1} + h(2)\, z^{-2} - h(1)\, z^{-3} - h(0)\, z^{-4}$$

Now, $h(2)$ must be zero if the filter shall have an antisymmetric impulse response. Hence, also in this case we have a zero at $z = 1$. To summarize, a lowpass filter cannot have an antisymmetric impulse response.

4.4 The frequency response is

$$H(e^{j\omega T}) = a + b\, e^{-j\omega T} + c\, e^{-j2\omega T} + b\, e^{-j3\omega T} + a\, e^{-j4\omega T} =$$

$$= e^{-j2\omega T}\, [a\ \ e^{-j2\omega T} + b\, e^{j\omega T} + c + b\, e^{-j\omega T} + a\, e^{-j2\omega T}\,] =$$

$$= e^{-j2\omega T}\, [a\, cos(2\omega T) + c + b\ \ cos(\omega T)]$$

$$\Phi(\omega T) = arg\{\, [cos(2\omega T) - j\ sin(2\omega T)]\, \} \pm n\pi = arctan\{\frac{-sin(2\omega T)}{cos(2\omega T)}\} \pm n\pi$$

$\Phi(\omega T) = -\, 2\omega T \pm n\pi$ Linear phase

$\tau_g(\omega T) = -\dfrac{\partial\Phi(\omega T)}{\partial\omega} = -\, 2T$ Constant group delay

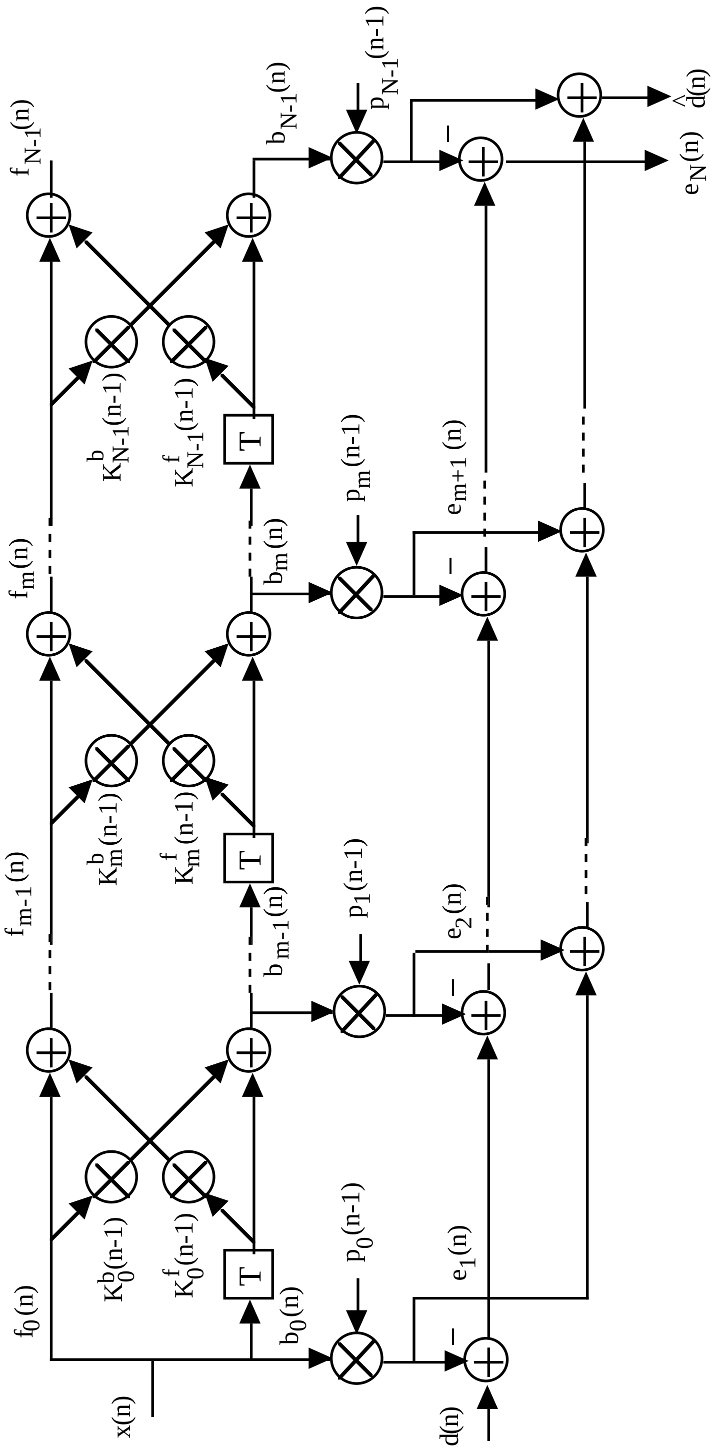
4.5 $H(e^{j\omega T}) = \dfrac{1}{4} + \dfrac{1}{2}\, e^{-j\omega T} + \dfrac{1}{4}\, e^{-j2\omega T} = \dfrac{1}{2}\, (1 + cos(\omega T))\, e^{-j\omega T}$

The magnitude function is $|H(e^{j\omega T})| = \dfrac{1}{2}\, (1 + cos(\omega T))$

The phase function is $\Phi(e^{j\omega T}) = -\, \omega T$

The group delay is $\tau(\omega T) = -\dfrac{\partial\Phi(e^{j\omega T})}{\partial\omega} = T$

4.6 a)



b)

4.7 Explore the symmetry and antisymmetry in the basis vectors, by using the linear-phase structure. See Eq.(4.15) and Fig. 4.6.

4.9 $A_{max} = -10\ log_{10}(1 - \rho^2) = 0.09883$ dB

4.10 The major factors are: the reflection coefficient, τ_g, element tolerances, and element losses.

4.11 The group delay is defined $\tau_{ga} = -\frac{\partial \Phi_a(\omega T)}{\partial \omega}$

and the group delay for the digital filter is defined $\tau_{gd} = -\frac{\partial \Phi_d(\omega T)}{\partial \omega}$

The relation between the phase of the analog and digital filter is

$$\Phi_d(\omega T) = \Phi_a(\omega_a) = \Phi_a(\frac{2}{T} tan(\frac{\omega T}{2}))$$

We get:

$$\tau_{gd}(\omega T) = -\frac{\partial \Phi_d(\omega T)}{\partial \omega} = -\frac{\partial \Phi_a(\omega_a)}{\partial \omega_a} \frac{\partial \omega_a}{\partial \omega} =$$

$$\tau_{gd}(\omega T) = \tau_{ga}(\omega_a) \frac{\frac{2}{T}}{cos^2(\frac{\omega T}{2})} \frac{T}{2} = \frac{\tau_{ga}(\frac{2}{T} tan(\frac{\omega T}{2}))}{cos^2(\frac{\omega T}{2})}$$

The group delay of the digital filter is distorted since the frequency axis is distorted according to Eq.(4.20) and because of the factor $cos^2(\frac{\omega T}{2})$ in the denominator.

4.17 $S = \frac{Z - R}{Z + R}$ where $Z = jX$ for a reactance. Hence,

$$S = \frac{jX - R}{jX + R} = \frac{(jX - R)}{(jX + R)} \frac{(-jX + R)}{(-jX + R)} = 1$$

4.25 If the input values are repeated L times, the corresponding spectrum is

$$X(e^{j\omega T}) = \sum_{m=-\infty}^{\infty} x_1(m)\, e^{-j\omega mT} = \sum_{n=-\infty}^{\infty} x(n) \sum_{k=0}^{L-1} e^{-j\omega kT}\, e^{-j\omega nT} =$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{1 - e^{-j\omega LT}}{1 - e^{-j\omega T}}\, e^{-j\omega nT} =$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega (L-1)T/2} \frac{sin(\omega LT/2)}{sin(\omega T/2)}\, e^{-j\omega nT}$$

Hence, the spectrum is weighted with a $\frac{sin(Lx)}{sin(x)}$ function that attenuates the unwanted images of the baseband, but it effects also the passband of interest. Compare this case with a zero-order-hold D/A converter. However, the attenuation is small and the computations must be done at the higher sample rate. Thus, the computational workload is much higher.

4.27 Interpolate the sample rate by using, for example, a lattice wave digital filter, with a factor 7. Decimate the sample rate by a factor 4 be retaining only every 4th sample.

4.28 a) Let $x(n)$ denote the input signal. We form a new input signal according to

$$x_i(m) = x(n) \text{ for } m = 2n \text{ and } = 0 \text{ otherwise.}$$

The interpolator is described by the difference equation

$$y(m) = \frac{1}{2}[x_i(m) + x_i(m-2)] + x_i(m-1)$$

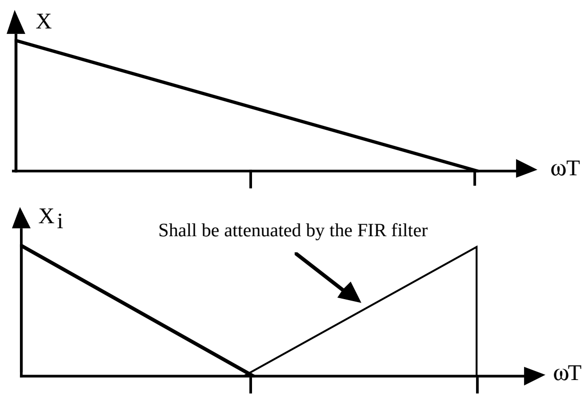
Only the first factor above contributes with an interpolated value for m = even since $x_i(m-1) = 0$. In the next sample interval m = odd. Hence only the second term contributes to the output, $x_i(m-1)$, $(= x(n))$, while both $x_i(m) = 0$ and $x_i(m-2) = 0$.

b) $H(z) = \frac{1}{2}[1 + z^{-2}] + z^{-1} = \frac{1}{2\,z^2}[z^2 + 2\,z + 1]$

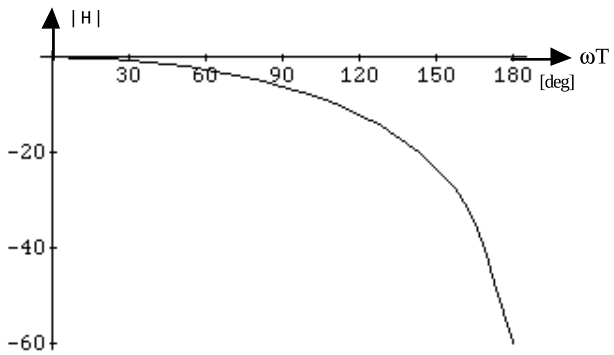
Selecting a unity gain for the filter we get $H(z) = \frac{z^2 + 2\,z + 1}{4\,z^2}$

c) A double pole for $z = 0$ and a double zero for $z = -1$

d) The magnitude of the Fourier transform is shown below for the original input signal, X, the new input signal, X_i and the magnitude function for the FIR filter.



e) Ideally the magnitude function shall be = 1 for $0 \le \omega T \le \frac{\pi}{2}$ and = 0 for $\frac{\pi}{2} < \omega T \le \pi$. The phase function shall be linear. Obviously, this is a poor FIR



filter since the attenuation of the unwanted image is very poor.

f) $h(m) = \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0, 0, 0, \ldots$

The filter is an FIR filter with length three. The filter order is 2.

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