4.11 The group delay is defined $\tau_{ga} = -\frac{\partial \Phi_a(\omega T)}{\partial \omega}$

and the group delay for the digital filter is defined $\tau_{gd} = -\frac{\partial \Phi_d(\omega T)}{\partial \omega}$

The relation between the phase of the analog and digital filter is

$$\Phi_d(\omega T) = \Phi_a(\omega_a) = \Phi_a(\frac{2}{T} tan(\frac{\omega T}{2}))$$

We get:

$$\tau_{gd}(\omega T) = -\frac{\partial \Phi_d(\omega T)}{\partial \omega} = -\frac{\partial \Phi_a(\omega_a)}{\partial \omega_a}\frac{\partial \omega_a}{\partial \omega} =$$

$$\tau_{gd}(\omega T) = \tau_{ga}(\omega_a)\frac{\frac{2}{T}}{cos^2(\frac{\omega T}{2})}\frac{T}{2} = \frac{\tau_{ga}(\frac{2}{T} tan(\frac{\omega T}{2}))}{cos^2(\frac{\omega T}{2})}$$

The group delay of the digital filter is distorted since the frequency axis is distorted according to Eq.(4.20) and because of the factor $cos^2(\frac{\omega T}{2})$ in the denominator.