

3.4(a) The autocorrelation function $r(k) = \sum_{n=0}^{\infty} x(n)x^*(n+k)$, ($k \geq 0$).

$$\begin{aligned} Z\{r(k)\} &= \sum_{k=-\infty}^{\infty} r(k)z^{-k} = \sum_{k=0}^{\infty} \left(\sum_{n=0}^{\infty} x(n)x^*(n+k) \right) z^{-k} = \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} x(n)x^*(n+k)z^{-k} \right) = \sum_{n=0}^{\infty} x(n) \left(\sum_{k=0}^{\infty} x^*(n+k)z^{-k} \right) \\ &= \sum_{n=0}^{\infty} x(n)z^n [X^*(z^*)] = X\left(\frac{1}{z}\right)X^*(z^*) \end{aligned}$$

(b) The convolution $y(n) = \sum_{n=0}^{\infty} h(k)x(n-k)$.

$$\begin{aligned} Z\{y(n)\} &= \sum_{n=-\infty}^{\infty} y(n)z^{-n} = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} h(k)x(n-k) \right) z^{-n} \\ &= \sum_{k=0}^{\infty} h(k) \left(\sum_{n=0}^{\infty} x(n-k)z^{-n} \right) \\ &= \sum_{k=0}^{\infty} h(k)[z^{-k}X(z)] = X(z)H(z) \end{aligned}$$