

3.23 DFT with transform length $N = 8$

$$\begin{aligned}\bar{x}(k) &= \sum_{n=0}^7 x(n)W^{nk} = \sum_{n=0}^3 x(2n)W^{2nk} + \sum_{n=0}^3 x(2n+1)W^{(2n+1)k} \quad k = 0, 1, \dots, 7 \\ &= \sum_{n=0}^3 x(2n)W^{(2n)k} + W^k \sum_{n=0}^3 x(2n+1)W^{2nk}\end{aligned}$$

Now, $W^{2nk} = e^{-jn\pi(2nk)/8} = e^{-jn\pi(nk)/4} = W'^{nk}$ where $W' = e^{-j2\pi/4}$. Hence, the DFT of length 8 can be partitioned into two DFTs of length 4, the computation can now be partitioned by

$$\begin{aligned}\bar{x}(k) &= DFT\{x(zn)\} + W^k DFT\{x(zn+1)\} \\ &= DFT\{x_{\text{even}}(n')\} + W^k DFT\{x_{\text{odds}}(n')\}\end{aligned}$$

where $n' = 0, 1, 2, 3$, and $x_{\text{even}}(n') = x(2n')$ and $x_{\text{odds}}(n') = x(2n'+1)$.