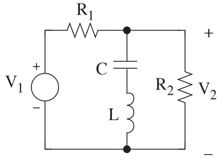
8. IMMITANCE SIMULATION

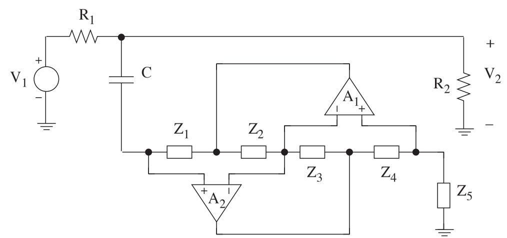
8.1 The series resonance circuit to ground yields the transfer function

$$H(s) = \frac{1}{2}\left(\frac{s^2 + \frac{1}{LC}}{s^2 + \frac{Rs}{2L} + \frac{1}{LC}}\right)$$ Resonance at $\omega = \frac{1}{\sqrt{LC}}$. Note that is require a relatively large capacitance $C = 1$ μF since the resonance frequency is low.



$$L = \frac{1}{C\omega^2} = \frac{1}{1 \cdot 10^{-6}(100\pi)^2} = 10.13 \text{ H}$$

The inductor can be realized by using a PIC. We get the circuit shown below.

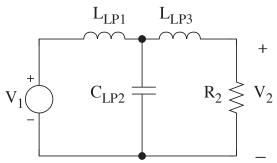


We select $Z_2 = Z_3 = R = 10$ kΩ and to reduce the influence of finite GB of the operational amplifiers, we select $Z_4 = 1/sC_4$ where $C_4 = 1$ μF

$$\omega_c C_4 R_5 = 1 \Rightarrow R_5 = \frac{1}{\omega_c C_4} = \frac{1}{100\pi \cdot 10^{-6}} = 3183 \ \Omega$$

$$Z_{in} = \frac{R_1}{\frac{1}{sC_4}} \cdot R_5 = sL \Rightarrow R_1 = \frac{L}{R_5 C_4} = \frac{10.13}{318.31 \cdot 10^{-6}} = 31.824 \text{ k‰}$$

8.2 The specification correspond to a HP filter of order $N = 3$.
We get the following normalized element values for the LP filter.
$L'_{LP1} = 1.3553$ $C'_{LP2} = 1.2740$ $L'_{LP3} = 1.7717$ $R_1' = 0$ and $R_2' = 1$.
The filter is, of course, not suitable as reference filter, since it is singly resistively terminated, i.e., it has high element sensitivity. Select $\Omega_c = 4$ krad/s and denormalize so that the band edge becomes 4 krad/s and $R_2 = 600$ Ω, i.e., select $R_0 = 600$ Ω.



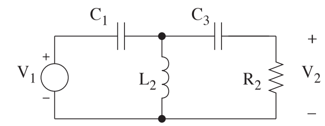
$$L = L_{LP}' \cdot \frac{R_0}{\Omega_c} \text{ and } C = C_{LP}' \cdot \frac{1}{R_0 \Omega_c}.$$

We get with the LP-HP transformation

$$C_3 = \frac{1}{\omega_I^2 L_3} = \frac{1}{\omega_I^2 L_{LP3}' \cdot \frac{R_0}{\Omega_c}} = \frac{1}{4000 \cdot 1.3553 \cdot 600} = 307 \text{ nF}$$

$$L_2 = \frac{1}{\omega_I^2 C_2} = \frac{1}{\omega_I^2 C_{LP2}' \cdot \frac{1}{R_0 \Omega_c}} = \frac{R_0}{\omega_I C_{LP2}'} = \frac{600}{4000 \cdot 1.2740} = 117.7 \text{ mH}$$

$$C_1 = \frac{1}{\omega_I^2 L_1} = \frac{1}{\omega_I^2 L_{LP1}' \cdot \frac{R_0}{\Omega_c}} = 235 \text{ nF}$$



The input impedance to a PIC is $Z_{in} = \frac{Z_1 \cdot Z_3}{Z_2 \cdot Z_4} \cdot Z_5$.

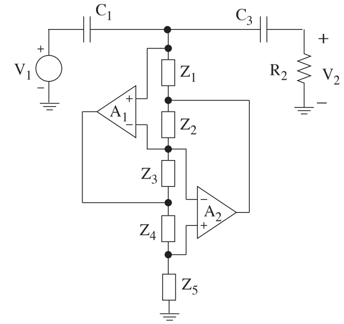
In order to simulate an inductor we select $Z_1 = R_1$, $Z_4 = 1/sC_4$ and $Z_5 = R_5$. In order to minimize the effect of finite GB of the operational amplifiers we select $Z_2 = Z_3 = R = 10$ kΩ and $\omega_c C_4 R_5 = 1$

$$R_5 = \frac{1}{\omega_c C_4} = \frac{1}{4000 \cdot 10 \cdot 10^{-9}} = 25\text{k}\Omega.$$

To realize the inductor we select

$$Z_{in} = sL_2 = \frac{R_1 R_3}{R_2} sC_4 R_5 = s117.7 \cdot 10^{-3} \qquad =>$$

$$R_1 = \frac{117.7 \cdot 10^{-3}}{25000 \cdot 10 \cdot 10^{-9}} = 471 \ \Omega$$

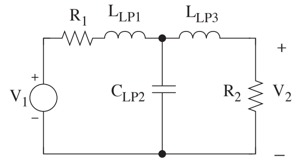


8.3

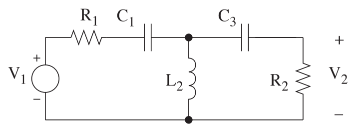
8.4 The specification correspond to a HP filter. First we transform the specification to a specification for a corresponding LP filter.
$\omega_c = 200\ 10^5\ 2\pi$ rad/s, $\omega_s = 50\ 10^3\ 2\pi$ rad/s. We select $\omega_I = \omega_c => \Omega_c = \omega_c$
We get for the LP filter $\Omega_s = \omega_I^2/\omega_s = (200 \cdot 10^3\ 2\pi)^2/(50 \cdot 10^3\ 2\pi) =$

$$\Omega_s = \frac{\omega_c^2}{\omega_s} = \frac{4\pi^2 \cdot 4 \cdot 10^{10}}{2\pi \cdot 5 \cdot 10^4} = 16\pi \cdot 10^5 \text{ and } \frac{\Omega_s}{\Omega_c} = 4 \text{ which}$$

yields $N = 3$.
A normalized LP T ladder, which has the lowest number of inductors, has the element values



$r = 1$
$L_{LP1}' = 2.0236$
$C_{LP2}' = 0.9941$
$L_{LP3}' = 2.0236$
The element values for the normalized HP filter (with passband edge $\omega_c = 1$) are after the LP →HP transformation



$$C_1' = \frac{1}{\omega_I^2 L_1'} = \frac{1}{1 \cdot 2.0236} = 0.49417$$

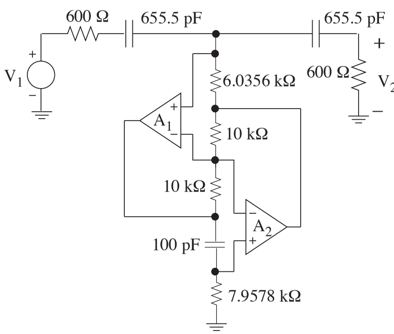
$$L_2' = \frac{1}{\omega_I^2 C_2'} = \frac{1}{1 \cdot 0.9941} = 1.005935$$

$C_3' = 0.49417$

Denormalization with respect to $R_0 = R_L = 600$ Ω and $\omega_I = \omega_c = 200 \cdot 10^5\ 2\pi$ rad/s yields

$$C_1 = C_1' \cdot \frac{1}{R_0 \omega_0} = 655.41 \text{ pF} = C_3$$

$L_2 = L_2' \cdot \frac{R_0}{\omega_0} = 480.2986$ □H. The grounded inductor can be replaced with a PIC with the input impedance $Z_{in} = \dfrac{Z_1 Z_3}{Z_2 Z_4} Z_5$ where we select $Z_4 = \dfrac{1}{sC_4}$, where $C_4 = 100$ pF and in order to minimize the influence of variations in GB of the operational amplifiers we select $Z_2 = Z_3 = R = 10$ kΩ. Determine R_5 and R_1:



$\hat{\omega} C_4 R_5 = 1 \Rightarrow \omega_c C_4 R_5 = 1 \Rightarrow$ =>

$R_5 = \dfrac{1}{\omega_c C_4} = \dfrac{1}{4\pi \cdot 10^5 \cdot 10^{-10}} = 7.9578$ k‰

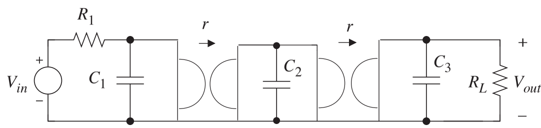
$Z_{in} = sL_2$ which yields $L_2 = \dfrac{R_1 R_3}{R_2} R_5 C_4$ =>

$R_1 = \dfrac{480.298610^{-6}}{795.7810^{-10}} = 6.0356$ k‰.

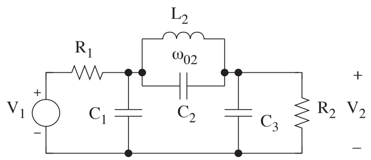
How can the A_1 amplifier be provided with bias currents?

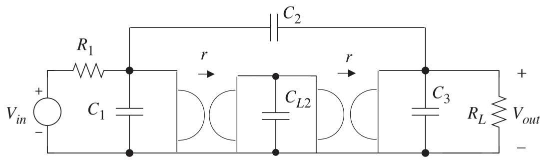
8.5

8.6 An inductor in a series arm can be realized with a capacitor in a shunt arm that is embedded between two identical gyrators according to the figure below. The chain matrix for a series impedance is $\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$ which is equivalent with a shunt admittance between two identical gyrators, i.e., $\begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix} = \begin{bmatrix} 1 & r^2 Y \\ 0 & 1 \end{bmatrix}$. Henc, $Z = r^2 Y$ and we get according to Figure. 8.7 that $C_2 = r^2/L_2$. The method is suitable for LC ladder with grounded inductors, e.g., in HP filters without finite zeros.

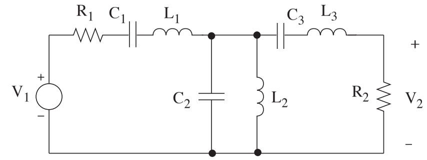


8.7 The filter can be realized in the same ways as in Problem 8.6. We get the same element values except for C_{L2} which becomes $C_{L2} = r^2/L_2$. If we use a PIC to realize the gyrator, i.e., we select $R_1 = R_2 = R_3 = R_5 = R = 10$ kΩ and regard C_4 as the load capacitor. We get $Z_{in} = R^2 C_4 s$, i.e., according to Eqs. (8.4) $Z_{in} = R^2 C_4 s = r^2 C_4 s$. The gyrator constant is $r = R$. Hence, we get $Z_{in} = L_2 s = R^2 C_4 s \Rightarrow C_4 = L_2/r^2 = 83.47 \cdot 10^{-3}/10^8 = 0.8347$ nF.



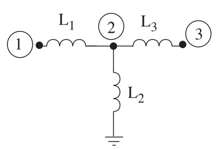


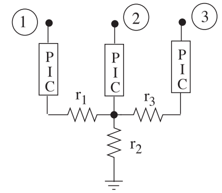
8.8 Realize the Cauer filter using Gorski-Popiel's method.
We extract the inductor network as shown in the figure (we interchange the position of L_3 and C_3 while this do not effect the frequency response). We replace the inductor network with a resistor network and place identical PICs at all external nodes, except for the ground node. We get the equivalent network shown to the below.
Using Antoniou's GIC we get



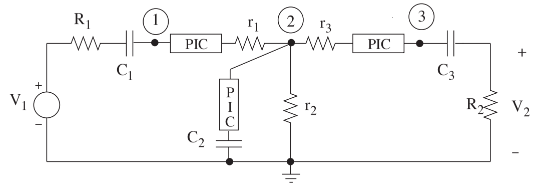
$K(s) = \dfrac{R_1 R_2 s C_4}{R_3}$ and

$r_i = L_i/K(s)$ for $i = 1, 2, 3$.



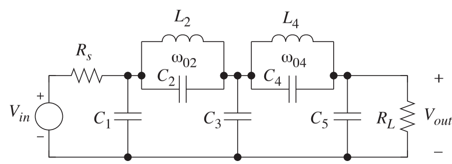


The final realization is.



8.9

8.10 A 5th-order Cauer filter is realized with a π ladder with the following element values meets the requirement.



$C_1 = 240.150$ pF

$C_2 = 28.1619$ pF
$L_2 = 113.5940$ μH
$\omega_{02} = 1.768037192185705$ e+07

$C_3 = 294.38$ pF

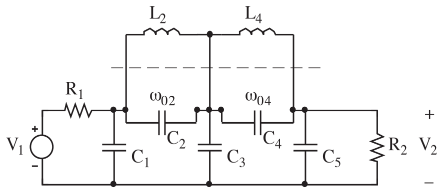
$C_4 = 78.787$ pF
$L_4 = 88.218$ μH
$\omega_{04} = 1.199481997139389$ e+07

$C_5 = 202.803$ pF

A π ladder with only two inductors is a good choice since only three PICs are required to separate the inductive network from the rest of the ladder. This is also true for the corresponding T ladder. For simple LP and HP filter it does not matter which ladder that is selected. For BP filter is the type of ladder more important.

Extract the inductors as shown in the figure below. We need three PICs with $K(s) = sRC$ => inductors becomes resistorswith the values L_2/RC and L_4/RC. Select RC to obtain suitable impedance in the network N_L. The PICs should be designed so that $\omega_c C_4 R_5 \approx 1$ and $R_2 = R_3$ (Note that this is not the terminating resistors) in order to minimize the effect of finite bandwidth of the operational amplifiers, i.e., we should select $\omega_c C_4 R_5 = 1$.

In this case we have ω_c = 8 Mrad/s => $C_4 R_5 = 1.25 \cdot 10^{-7}$. We select C_4 = 10 pF and $R_2 = R_3$ = 10 kΩ and we get R_5 = 12.5 kΩ. The loading resistors of the PICs (corresponding to the inductors) are $R_2 = sL_2/K(s) = L_2/C_4R_5 = L_2\omega_c =$ $= 0.11359 \cdot 10^{-3} \cdot 8 \cdot 10^6 = 908.7$ Ω.



$R_4 = L_4/C_4R_5 = L_4\omega_c = 8.82179 \cdot 10^{-5} \cdot 8 \cdot 10^6 = 7.05$ Ω.

Obviously this is to small resistors for the operational amplifiers. The resistors should be at least a few kΩ.

We get more reasonable values if we instead select

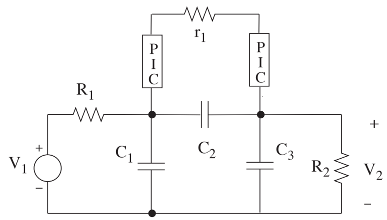
$R_2 = 200\, L_2/C_4R_5 = 200\, L_2\omega_c = 200 \cdot 908.7 = 18.174$ kΩ.

$R_4 = 200\, L_4/C_4R_5 = 200\, L_4\omega_c = 200 \cdot 7.05 = 1410$ Ω.

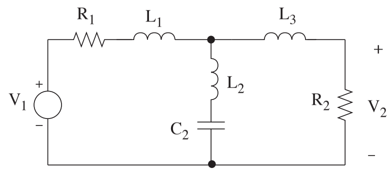
8.11 Realize the filter in Problem 12.8 using Gorski-Popiel's method.

We select $K(s) = \dfrac{R_1 R_2 s C_4}{R_3} = 7.78\ 10^{-5} s$ where

$R_1 = 787$ Ω, $R_2 = R_3 = 4.7$ kΩ, $C_4 = 100$ nF which gives the element values in the filter: $R_1 = R_2 = 1$ kΩ, $r_2 = 1.061$ kΩ, $C_1 = C_3 = 209.4$ nF, $C_2 = 33.04$ nF. It is important that the PICs are identical, otherwise the PIC network that simulate an inductor will not be reciprocal and may even be come active and the sensitivity is deteriorated.

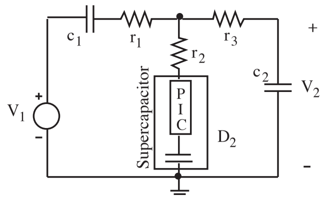


8.12 The corresponding T ladder have the elements $R_1 = R_2 = 1$ kΩ, $L_1 = L_3 = 209.4$ mH, $L_2 = 33.06$ mH and $C_2 = 83.47$ nF. By multiplying all impedances with $K(s)$ a new ladder structure with the same transfer function but where inductors are replaced with resistors, capacitors replaced with supercapacitors, and resistors replaced with capacitors.



We select $K(s) = k/s$. If k is selected too small, the resistors that correspond to small inductors will be small and the capacitance that correspond to small resistors will be too large.

We select $k = 10^5$ which yields reasonable element values. We get $c_1 = c_3 = R_1K(s)$ => $c_1 = c_2 = 10$ nF and $r_1 = r_3 = sL_1K(s) = 0.2094 \cdot 10^5 = 20.940$ kΩ, $r_2 = sL_2K(s) = 0.03306 \cdot 10^5 = 3.306$ kΩ, and $\dfrac{1}{sC_2}\dfrac{k}{s} = \dfrac{1}{D_2 s^2}$ where $D_2 = C_2/k = 8.347\ 10^{-13}$ Fs.



We get the element values in the PIC, that realizes the supercapacitor with $D_2 = 1.6694\ 10^{-7}$ Fs, with $C_1 = C_1 = C$, $R_2 = R_3 = 10$ kΩ, we get $D_2 = C^2R_4$ which with $C = 10$ nF, yields $R_4 = 8.347$ kΩ.

8.13

8.14 For a GIC with $Z_1 = 1/sC_1$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = 1/sC_5$

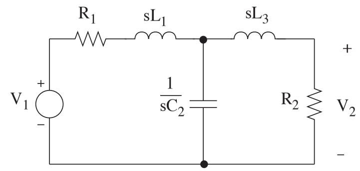
$$Z_{in}(s) = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5 = \frac{\frac{1}{sC_1} R_3}{R_2 R_4} \frac{1}{sC_5} = \frac{R_3}{R_2 R_4 C_1 C_5} \frac{1}{s^2} = \frac{1}{Ds^2}$$

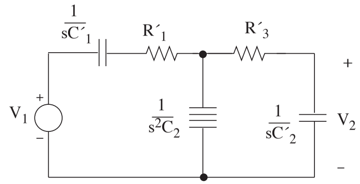
We select $R_4C_5 = 1/\omega_{critical} = 1/10^3 = 10^{-3}$ where we assume that $\omega_{critical} = 10^6$ rad/s

$$\frac{R_3}{R_2 R_4 C_1 C_5} = \frac{1}{D} \Rightarrow \frac{R_3}{R_2 C_1 \omega_{critial}} = \frac{1}{D} \Rightarrow R_3/C_1R_2 = R_4C_5 \cdot 10^6 = 10^3$$

For $D = 10^{-6}$ Fs, we select, for example, $R_3 = R_2 = 10$ kΩ => $C_1 = 1$ pF

8.15 We multiply all elements with $1/s$.





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