4. FILTERS WITH DISTRIBUTED ELEMENTS

4.1 The chain matrix of two cascaded two-ports is obtained by multiplying the individual chain matrices.

$$K = \begin{bmatrix} \cosh(\gamma d_1) & Z_0 \sinh(\gamma d_1) \\ \frac{\sinh(\gamma d_1)}{Z_0} & \cosh(\gamma d_1) \end{bmatrix} \begin{bmatrix} \cosh(\gamma d_2) & Z_0 \sinh(\gamma d_2) \\ \frac{\sinh(\gamma d_2)}{Z_0} & \cosh(\gamma d_2) \end{bmatrix}$$ and we get

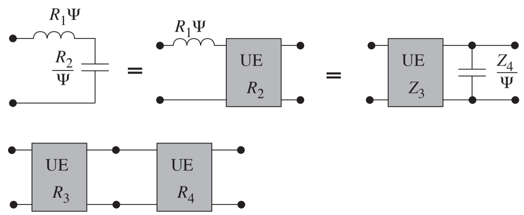
$$K = \begin{bmatrix} \cosh(\gamma(d_1+d_1)) & Z_0 \sinh(\gamma(d_1+d_1)) \\ \frac{\sinh(\gamma(d_1+d_1))}{Z_0} & \cosh(\gamma(d_1+d_1)) \end{bmatrix}$$

where we have used the fact that

$$\cosh(x_1+x_2) = \cosh(x_1)\cosh(x_2) + \sinh(x_1)\sinh(x_2)$$ and

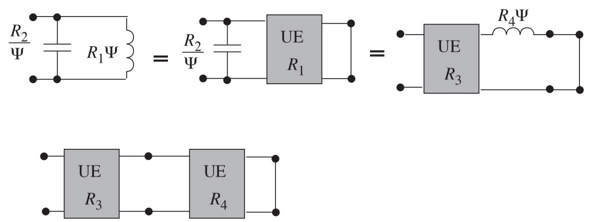
$$\sinh(x_1+x_2) = \sinh(x_1)\cosh(x_2) + \cosh(x_1)\sinh(x_2)$$

4.2



First we recognize that the input impedance to an open-ended lossless transmission line correspond to a capacitor in the Ψ-plane capacitor. Hence, we replace the capacitor as shown in the first step above. Next we use the first Kuroda identity in Table 4.1 to change the order of the series Ψ-inductor and the transmission line. We get the new port resistances $R_3 = R_1 + R_2$ and $R_4 = R_2 + R_2^2/R_1$. Again we identify the Ψ-capacitor with a open-ended transmission line.

4.3



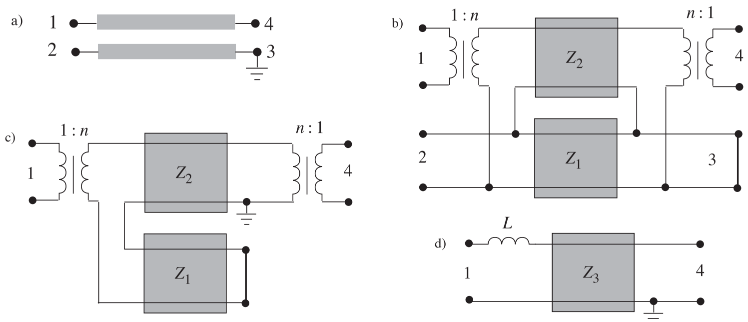
First we recognize that the input impedance to an short-ended lossless transmission line correspond to a Ψ-plane inductor. Hence, we replace the Ψ-inductor as shown in the first step above. Next we use the last Kuroda identity in Table 4.1 to change the order of the shunt Ψ-capacitor and the transmission line. We get the new port resistances $R_3 = R_1R_2/(R_1 + R_2)$ and $R_4 = R_1^2/(R_1 + R_2)$. Again we identify the Ψ-inductor with a short-circuited transmission line.

By repeatedly using Kuroda identities can any ladder structure be converted to a Richards' structure with the same input impedance. In fact, a Richards' structure can realize any reactance function.

4.4 From Example 4.5 we have for the original T ladder: $Z_1 = Z_3 = 3.85381 \cdot 50\ \Omega = 192.6905\ \Omega$ and $Z_2 = 0.3776909 \cdot 50\ \Omega = 18.884545\ \Omega$. We insert a UE between R_s and the first Ψ-inductor with port resistance R_s. Next we use the last Kuroda identity in Table 4.1 to change the order of the element. We get from the Table $R_s = \frac{Z_C Z_{UE}}{Z_C + Z_{UE}} = 50\ \Omega$ and $Z_1 = \frac{Z_{UE}^2}{Z_C + Z_{UE}} = 192.6905\ \Omega$. Solving yields $Z_{UE} = 242.6905\ \Omega$ and $Z_C = 62.97417\ \Omega$. Next we insert a UE between the last Ψ-inductor and the load resistor. We get using the first Kuroda identity $Z_{UE} = Z_3 + R_L = 242.6905\ \Omega$ and $Z_C = R_L + \frac{R_L^2}{Z_3} = 62.97417\ \Omega$. Ws get the following characteristic resistances from left to right using the notation in Figure 4.25: $Z_4 = 62.97417\ \Omega$, $R_4 = 242.6905\ \Omega$, $Z_5 = 18.884545\ \Omega$, $R_5 = 242.6905\ \Omega$, and finally, $Z_6 = 62.97417\ \Omega$. As expected there is symmetry in the elements and the element spread is the same as in Example 4.5

4.5

4.6 We perform the following simplifications steps: a) to d). We get $L = Z_1/n^2$ and $Z_3 = Z_2/n^2$. The circuit in d) is a series resonance circuit



4.7 We select a Chebyshev I filter of order N =5 and realize it with a π ladder with the element values $C_1 = C_5 = 1.7058$, $L_2 = L_4 = 1.2296$, and $C_3 = 2.5408$. We get after inserting two UE from each side. Four shunt Ψ-capacitor-UE sections and a final shunt Ψ-capacitor with the characteristic impedances $Z_1 = 2.5862$, $Z_2 = 0.4807$, $Z_3 = 0.3936$, $Z_4 = 0.4807$, and $Z_5 = 2.5862$. We select the length of the lines $l = \lambda/4$. It is also possible to select $l = \lambda/8$ which yields a shorter line. We have $v_p = 0.6c = 1.8\ 10^8$ m/s. We select $l = \lambda/4$ and get with $f_0 = v_p/\lambda_0 = 3$ GHz yields $l = v_p/4f_0 = 15$ mm.

4.8 A thid-order bandstop filter with center frequency 2.4 GHz and 3-dB bandwith is 50% can be realized wih a third-order lowpass filter with cutoff frequency at 1.7 GHz when we assume that $\tau = 1/4.8$ ns. A third-order Butterworth ladder filter (T type) with lumped element has the normalized element values: $R_s = R_L = 1$, $L_1 = L_3 = 1$, and $C_2 = 2$.

4.9 The normalized lowpass T ladder have the following elements $R_s = R_L = 1$, $L_1 = L_3 = 1.5963$ and $C_2 = 1.0967$. We get $\omega_{c1} = 0.9 \cdot 2.4 \cdot 10^9 \cdot 2\pi = 1.357168\ 10^{10}$ rad, $\omega_{c2} = 1.1 \cdot 2.4 \cdot 10^9 \cdot 2\pi = 1.6587609\ 10^{10}$ rad and $\Omega_c = \omega_{c2} - \omega_{c1} = 3.0159289\ 10^9$ rad. Denormalize the LP filter with $R = 50\ \Omega$, $L_{LP1} = RL_1/\Omega_c = 26.464463$ nH and $C_{LP2} = C_2/R\Omega_c = 7.2727178$ pF. $\omega_1^2 = \mathrm{sqrt}(\omega_{c1}\omega_{c2}) = 1.5004057\ 10^{10}$. We get for the bandpass filter $L_1 = L_3 = L_{LP1} = 26.464463$ nH, $C_1 = C_3 = 1/\omega_1^2 L_{LP1}$

= 0.16784916 pF, $C_2 = C_{LP2}$ = 7.2727178 pF, and $L_2 = 1/\omega_1^2 C_{LP2}$ = 0.61078147 nH.
This filter is obviously very difficult to implement with discrete components, since an inductor of 1 nH corresponds to a wire with a length of about 1 mm.

4.10 $Z_{in}(d) = \frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}Z_0$ where Z_0 = 50 Ω, d = 20 mm, Z_L = 30 + j60 Ω, f = 2.4 GHz,

$v_p = 0.6c = 1.8\ 10^8$ m/s, $\beta = 2\frac{\pi}{v_p} = 0.0838$ 1/mm ==>

$$Z_{in}(20) = \frac{30 + j60 + j50\tan(0.0838 \cdot 20)}{50 + j(30 + j60)\tan(0.0838 \cdot 20)}50 = 14.6929 - j26.6914\ \Omega$$

4.11 Synthesize a Richards' structure that realizes the reactance function $Z = \frac{12\Psi + 3\Psi^3}{2 + 13\Psi^2}$

Answer: The characteristic resistances are Z_1 = 1 Ω, Z_2 = 2 Ω, Z_3 = 3 Ω and the last UE is terminated with a short-circuit. The solution can be verified by

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Den = [2 0 13 0];
Num = [0 12 0 3];
[Z, KI, RL] = RICHARDS_REACTANCE(Num, Den)
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The reactance can also be realized with an inductor (4 H) in parallel with a series resonance circuit with C = 27/32 F and L = 4/27 H.

4.12 A 50 ohm transmission line is loaded with a complex impedance: Z_L = 30 + j60 Ω. the transmission line is operated at 2.4 GHz. Determine the length of this transmission line so that we have input impedance of the transmission line equal to: Z_{in} = 30 – j60 Ω.

$Z_L = 30 + j60$, $Z_0 = 50$, f = 2.4e9
$v = 0.6 * 3\ 10^8$, $\lambda = v/f$, $\beta = 2\pi i/\lambda$
$Z_{in} = 30 - j60$

Due to the equation 4.24 in the text book we have:
$Z_{in} = Z_0\ (Z_L + Z_0\tan(\beta d)) / (Z_0 + Z_L\tan(\beta d))$
By re-ordering this equation we find:
$d = (1/\beta)\mathrm{atan}(Z_0\ (Z_L - Z_{in})/(Z_L Z_{in} - Z_0^{\ 2}) = 14.9 * 10^{-3}$ meter
Plug in the values give the length (d):
d = 14.9 mm