5.18 a) The variance of the quantization noise for System #1 is

$$\sigma_1^2 = Q_1^2/12 \text{ where } Q_1 = 2^{-17}$$

The variance of the quantization noise for System #2 is

$$\sigma_2^2 = Q_2^2/12 \text{ where } Q_2 = 2^{-13}$$

We assume that the average values are = 0. The power spectrum after the A/D converter is

$$S_2 = \sigma_2^2$$

since the noise is white. A lowpass filter with cutoff angle $\omega_c T$ is placed after the converter in order to remove noise that should be aliased into the passband after the decimation. The variance of the noise after the lowpass filter is

$$\sigma_3^2 = r(0) = \frac{1}{2\pi}\int_0^{2\pi} S_2\, e^{j0\omega T}\, d\omega T = \frac{1}{2\pi}\int_0^{2\pi} S_2\, d\omega T =$$

$$= \frac{2}{2\pi}\int_0^{\omega_c T} \sigma_2^2\, d\omega T = \frac{2}{2\pi}\,\sigma_2^2\,\frac{2\pi f_{sample}}{2 f_{sample2}} = \sigma_2^2\,\frac{f_{sample}}{f_{sample2}}$$

($r(\tau)$ is the autocorrelation function). We assume that the noise has zero mean. We must have

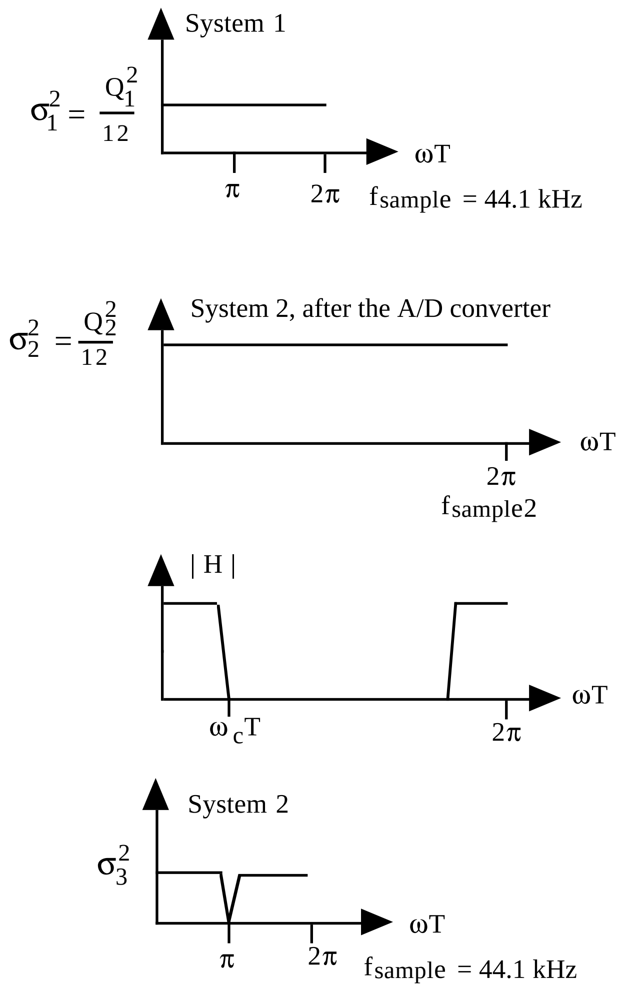
$$\omega_c T \ge \frac{2\pi f_{sample}}{2 f_{sample2}}$$

so that the audio signal will not be attenuated by the lowpass filter. If the quantization noise is to be less then or equal to the noise in System #1, we must have $\sigma_3^2 \le \sigma_1^2$

Hence, $\dfrac{Q_2^2}{12}\dfrac{f_{sample}}{f_{sample2}} \le \dfrac{Q_1^2}{12}$ and

$$f_{sample2} \ge \frac{Q_2^2}{Q_1^2} f_{sample} = 2^8 f_{sample};$$

$$f_{sample2} \ge 256 \cdot 44.1 \text{ kHz} = 11.264 \text{ MHz}$$



b) Decimation with a factor 1024 yields a sample frequency of

$$f_{sample2} \geq 1024 \cdot 44.1 \text{ kHz} = 45.1584 \text{ MHz}$$

Decimation can be done in steps of 2 and the decimation filters can work at the lower sample frequency. The filter orders are almost the same for all stages. We need 10 decimation stages. The workload for one stage is only half of that of the preceding stage. The workload of the whole decimation filter, counted from the output, is

$$N[2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^9] = N\,2^9 \frac{1 - 2^{-10}}{1 - 2^{-1}} \approx N\,2^{10} = 1024\,N$$

where N is the workload of a single decimation filter working at the lowest sample frequency.

c) System #1: $\dfrac{f_{sample} - f_c}{f_c} = \dfrac{44.1 - 20}{20} = \dfrac{24.1}{20} = 1.205$

System #2: $\dfrac{f_{sample2} - f_c}{f_c} = \dfrac{11264 - 20}{20} = \dfrac{11224}{20} = 561.2$

d) We assume that the acceptable ripple in the passband is about 0.1 dB and the required stopband attenuation is about 50 dB in both cases.

We get

$N_{System1}$ =

$N_{System2}$ =