3.15 Since $z = e^{j\omega t}$, or $\omega = \frac{1}{jT}\ln z$, we change the derivation with respect to $\frac{\partial}{\partial z}$ to $\frac{\partial}{\partial \omega}$ by

$$\frac{\partial}{\partial z} = \frac{d\omega}{dz}\cdot\frac{\partial}{\partial\omega} = \frac{d\left(\frac{1}{jT}\ln z\right)}{dz}\cdot\frac{\partial}{\partial\omega} = \frac{1}{jTz}\cdot\frac{\partial}{\partial\omega} = \frac{je^{-j\omega T}}{T}\cdot\frac{\partial}{\partial\omega}.$$

The transfer function can be written as $H(z) = |H(z)|e^{j\Phi(z)}$

The phase function is therefore $\Phi(z) = -j[\ln H(z) - \ln|H(z)|]$.

We compute $z\frac{d}{dz}\ln H(z)$.

$$z\frac{d}{dz}\ln H(z) = z\frac{d}{dz}\ln[|H(z)|e^{j\Phi(z)}] = z\left\{\frac{d}{dz}\ln|H(z)| + j\frac{d}{dz}\Phi(z)\right\}$$

$$= z\left\{\frac{\frac{d}{dz}|H(z)|}{|H(z)|} + j\frac{d}{dz}\Phi(z)\right\}$$

and

$$z\frac{d}{dz}\ln H(z) = \frac{z}{|H(z)|}\left\{\frac{d}{dz}|H(z)| + j|H(z)|\frac{d}{dz}\Phi(z)\right\}$$

$$= \frac{e^{j\omega T}}{|H(e^{j\omega T})|}\left\{\frac{je^{-j\omega T}}{T}\frac{\partial}{\partial\omega}|H(e^{j\omega T})| + \frac{e^{-j\omega T}}{T}|H(e^{j\omega T})|\frac{\partial}{\partial\omega}\Phi(e^{j\omega T})\right\}$$

$$= \frac{1}{|H(e^{j\omega T})|T}\left\{|H(e^{j\omega T})|\frac{\partial}{\partial\omega}\Phi(e^{j\omega T}) - j\frac{\partial}{\partial\omega}|H(e^{j\omega T})|\right\}$$

Since $|H(e^{j\omega T})|$ and $\Phi(e^{j\omega T})$ are real functions, $\frac{\partial}{\partial\omega}|H(e^{j\omega T})|$ and $\frac{\partial}{\partial\omega}\Phi(e^{j\omega T})$ are therefore real functions, we obtain

$$Re\left\{z\frac{d}{dz}\ln H(z)\right\} = Re\left\{\frac{1}{|H(e^{j\omega T})|T}|H(e^{j\omega T})|\frac{\partial}{\partial\omega}\Phi(e^{j\omega T}) - j\frac{\partial}{\partial\omega}|H(e^{j\omega T})|\right\}$$

$$= \frac{1}{|H(e^{j\omega T})|T}\left\{|H(e^{j\omega T})|\frac{\partial}{\partial\omega}\Phi(e^{j\omega T})\right\} = \frac{1}{T}\frac{\partial}{\partial\omega}\Phi(e^{j\omega T}).$$

Comparing with the defination of the group delay, we obtain

$$\tau_g(\omega T) = -\frac{\partial}{\partial\omega}\Phi(\omega T) = -TRe\left\{z\frac{d}{dz}\ln H(z)\right\} \text{ for } z = e^{j\omega T}.$$