3.1 The Fourier transform is given by $X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x(nT)e^{-jn\omega T}$

(a) $x(n) = a^n$ for $n \geq 0$ and = 0 otherwise

$$X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x(nT)e^{-jn\omega T} = \sum_{n=0}^{\infty} a^{nT}e^{-jn\omega T} = \sum_{n=0}^{\infty} (a^T e^{-j\omega T})^n$$

It is a geometric series, the convegence is given by Abels theorem

(i) $|a^T e^{-j\omega T}| < 1$, i.e. $|a^T| < 1$, $\sum_{n=0}^{\infty} (a^T a^{-j\omega T})^n$ is convergent

$$X(e^{j\omega T}) = \frac{1}{1 - a^T e^{-j\omega T}}.$$

(ii) $|a^T e^{-j\omega T}| > 1$, or $|a^T| > 1$, $\sum_{n=0}^{\infty} (a^T a^{-j\omega T})^n$ is not convergent.

(iii) $|a^T e^{-j\omega T}| = 1$, the convergence of $\sum_{n=0}^{\infty} (a^T a^{-j\omega T})^n$ is uncertain.

(b) $x(n) = -a^n$ for n < 0 and = 0 otherwise.

$$X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x(nT)e^{-jn\omega T} = \sum_{n=-\infty}^{-1} -a^{nT}e^{-jn\omega T} = \sum_{n=-\infty}^{-1} (a^T e^{-j\omega T})^n$$

$$= \sum_{n=1}^{\infty} (a^{-T} e^{j\omega T})^n$$

By using Abels theorem, we have

(i) $|a^{-T} e^{j\omega T}| < 1$, or $|a^T| > 1$, $-\sum_{n=1}^{\infty} (a^{-T} e^{j\omega T})^n$ is convergent and $X(e^{j\omega T}) = \frac{a^{-T}e^{j\omega T}}{1 - a^{-T}e^{j\omega T}}$.

(ii) $a^{-T} e^{j\omega T}| > 1$, i.e. $|a^T| < 1$, $-\sum_{n=1}^{\infty} (a^{-T} e^{j\omega T})^n$ is not convergent.

(iii) $|a^{-T} e^{j\omega T}| = 1$, the convergence of $-\sum_{n=1}^{\infty} (a^{-T} e^{j\omega T})^n$ is uncertain