Wave digital filter theory is based on a scattering parameter formalism. A one-port can be described by the reflectance function, defined as

\[ \frac{b}{v} = S \]

where \( a \) is the incident wave, \( b \) is the reflected wave, and \( R \) is a positive real constant, called port resistance.

The steady-state voltage waves are defined as

\[
\begin{align*}
I_R - a & \quad \sqrt{b} \\
I_R + a & \quad \sqrt{v}
\end{align*}
\]

The one-port network can be described by the incident and reflected waves instead of voltages and currents.
Example

Determine the reflectance for a pure reactance.

\[ \frac{y + z}{y - z} = s \]

Reflectance is an allpass function for a pure reactance.

The voltage waves are described by:

\[
\begin{align*}
I_R - A &= B \\
I_R + A &= V
\end{align*}
\]

We get the impedance is described by:

\[ I Z = \Lambda \]

and the impedance is described by:

\[ I R - A = B \]

The voltage waves are determined for an impedance \( z \).
It is not possible to directly use reference filters with lumped circuit elements, since nonsequentially computable algorithms are obtained. Instead certain classes of transmission line filters can be mapped to classical filters. Fortunately, some of these filter structures can be mapped to classical filters. The abundant knowledge of lumped element filters can make full use of these filter structures and we can make full use of the structures with lumped circuit elements and use reference filters with lumped circuit elements.
Transmission lines are often referred to as unit elements. In each direction, the characteristic impedance and time delay are the propagation time.

\[
\begin{bmatrix}
Z_I & 0 \\
Z_A & (Z/Z_s) \tanh \frac{t}{2Z_s} \\
\end{bmatrix}
\begin{bmatrix}
1 \\
\frac{V}{Z} \\
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
\frac{I}{Z} \\
\end{bmatrix}
\]

A lossless transmission line can be described as a two-port by the chain matrix. A special case of filter networks with distributed circuit elements is a common sense.
Obviously, a transmission line cannot be described by poles and zeros since the elements in the chain matrix are not rational functions in $s$.

Wave digital filter design involves synthesis of such reference filters. Wave digital filters imitate voltage signals by means of incident and reflected voltage waves.

Wave digital filters imitate reference filters built out of resistors and loss-

Wave digital filter design involves synthesis of such reference filters. Commensurate-length transmission line filters constitute a special case of

Computable digital filter algorithms can be obtained if the reference filter is designed using only such transmission lines.

Commensurate-length transmission line filters constitute a special case of distributed element networks that can easily be designed by mapping them to a lumped element structure.
This mapping involves Richards' variable which is defined as

\[
\left[ \begin{array}{cc}
\frac{2}{\omega} & -1 \\
\frac{2}{\omega} & 1
\end{array} \right] \left[ \begin{array}{cc}
0 & Z \\
\frac{1}{Z} & 1
\end{array} \right] \frac{\frac{2}{\omega} - i\eta}{1} = \left[ \begin{array}{c}
i \\
i \eta
\end{array} \right]
\]

Notice the similarity between the bilinear transformation and Richards' variable. Substituting Richards' variable into the chain matrix yields

\[\left( \frac{2}{1}\right) \tan = \zeta\]

The real frequencies in the \(s\)- and \(\eta\)-domains are related by

\[\text{where Richards' variable is a dimensionless complex variable} \]

\[\left( \frac{2}{1}\right) \text{tanh} = \frac{1 + 2s^2}{1 - 2s^2} \sqrt{\frac{\omega}{\eta}}\]

This mapping involves Richards' variable which is defined as...
The chain matrix above has element values that are rational functions in Richards' variable, except for the square-root factor. Fortunately, this factor can be handled separately during the synthesis. The synthesis procedures (programs) used for lumped element design can therefore be used with small modifications in the synthesis of commensurate-length transmission line filters. The transmission line filters of interest are, with a few exceptions, built using only one-ports.
The input impedance of the transmission line, with characteristic impedance \( Z_0 \), loaded with an impedance \( Z_2 \) is

\[
\begin{align*}
Z_{in} & = \frac{\gamma Z + R}{\gamma Z + Z_2} = (\gamma)^{u_1} Z \quad \text{(short-circuited)} \quad \gamma = \frac{0}{Z'} \\
& = \frac{\gamma Z + R}{\gamma Z + Z_2} = (\gamma)^{u_1} Z \quad \text{(open-ended)} \quad \gamma = \frac{\infty}{Z'} \\
& = \frac{\gamma Z + R}{\gamma Z + Z_2} = (\gamma)^{u_1} Z \quad \text{(matched termination)} \quad \gamma = \frac{Z}{Z'}
\end{align*}
\]

The input impedance of a lossless transmission line with characteristic impedance \( Z \) that is terminated by an impedance \( R = 0 \) is

\[
\begin{align*}
0Z' \frac{Z + 0Z}{Z + Z_2} = \frac{I}{I} = (\gamma)^{u_1} Z
\end{align*}
\]
Transmission Line Filters

\[ \frac{1 + 2s \varepsilon}{1 - 2s \varepsilon} = \gamma \]
The input impedance to an open-circuited unit element (a $Y$-domain capacitor) with $Z_0 = R$ is

$$Z_{in} = \frac{R}{R - \frac{1}{Z}}$$

and

$$Z_{in} = \frac{R}{R - \frac{1}{Z}}$$

The input impedance to a short-circuited unit element (a $Y$-domain inductor) with $Z_0 = R$ is

$$Z_{in} = \frac{R}{R - \frac{1}{Z}}$$

We get the reflectance

$$\frac{H}{\mathcal{Y}} = (\mathcal{H})^u_1Z$$

The input impedance to an open-circuited unit element (a $Y$-domain capacitor) with $Z_0 = R$ is

$$Z_{in} = \frac{R}{R - \frac{1}{Z}}$$

Wave-Flow Building Blocks
The reflectance is

and

The reflectance for a unit element termi-

Hence, an input signal to such a unit element

\[ 0 = \frac{R + \frac{u_1}{Z}}{R - \frac{u_1}{Z}} = (\mathcal{H})S \]

\( R \) (matched) is

\( R \)

The reflectance for a unit element termi-

and

\[ z^{-1} = (z)S \]

The reflectance is
The corresponding wave-flow graphs to a short-circuit an open-circuit are described by a wave-flow graph called an *adaptor*. In order to interconnect different wave-flow graphs, it is necessary to obey Kirchhoff’s laws at the interconnection. Generally, at a point of connection, the incident waves are partially transmitted and reflected. Transmission and reflection at the connection point are described by Kirchhoff’s laws at the interconnection.
Symmetric Two-Port Adaptor

The symbol for the symmetric two-port adaptor that corresponds to a connection of two ports.

The incident and reflected waves for the two-port are:

\[ \begin{align*}
    \tilde{Z}_I &= I_A \\
    \tilde{Z}_L &= I_I
\end{align*} \]

According to Kirchhoff's laws, we have:

\[ \begin{align*}
    \tilde{Z}_I R - \tilde{Z}_L &= \tilde{Z}_B \\
    \tilde{Z}_I R + \tilde{Z}_L &= \tilde{Z}_V
\end{align*} \]

and

\[ \begin{align*}
    I_I R - I_A &= I_B \\
    I_I R + I_A &= I_V
\end{align*} \]

The symbol for the symmetric two-port adaptor that corresponds to a connection of two ports.
The adaptor coefficient \( a \) is usually written on the side corresponding to port 1. As can be seen, the wave-flow graph is almost symmetric.

Note that \( a = 0 \) for \( R_2 = 0 \). The adaptor degenerates into a direct connection of the two ports and the incident waves are not reflected at the point of interconnection.

For \( R_2 = 0 \) we get \( a = 1 \) and the incident wave at port 1 is reflected and multiplied by -1 while for \( R_2 = \infty \) we get \( a = -1 \) and the incident wave at port 1 is reflected and multiplied by 1.

\[
\begin{align*}
\frac{R_1 + a R_2}{R_1 - a R_2} &= a \\
(1 - a) v_1 + a v &= R_2 b \\
(1 - a) v_1 + R_2 v &= a b
\end{align*}
\]
Design of Wave Digital Filters