MULTIRATE SYSTEMS

Transfer signals between two systems that operate with different sample frequencies

Implemented system functions more efficiently by using several sample rates (for example narrow-band filters).

Sample rate converters for changing the sample rate without affecting the information contained in the signal.

Converters for increasing and decreasing the sample frequency are usually called interpolators and decimators, respectively.

Potential advantages of multirate signal processing are reduced computational work load, lower filter order, lower coefficient sensitivity and noise, and less stringent memory requirements.

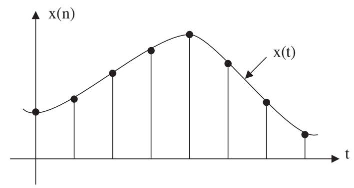
Disadvantages are more complex design, aliasing and imaging errors and a more complex algorithm.

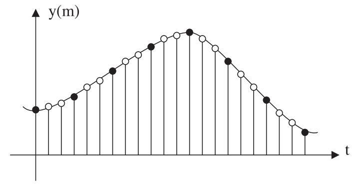
Multirate techniques are used today in many digital signal processing systems.

INTERPOLATION WITH AN INTEGER FACTOR

The process of increasing the sampling rate is called *interpolation*.

The aim is to get a new sequence corresponding to a higher sampling frequency, but with the same informational content, i.e., with the same spectrum as the underlying analog signal.



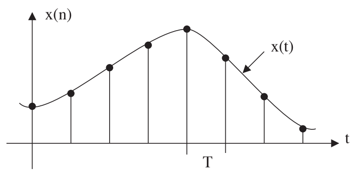


The interpolation process is essentially a two-stage process.

First step

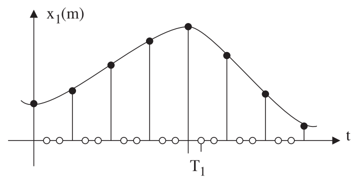
An intermediate sequence $x_1(m)$ is generated from the original sequence $x(n)$ by inserting zero-valued samples between the original sequence values to obtain the desired sampling rate.

$$x_1(m) = \begin{cases} x\left(\dfrac{m}{L}\right) & \text{if } m = 0, \pm L, \pm L, \dots \\ 0 & \text{otherwise} \end{cases}$$



The sample period for the new sequence is

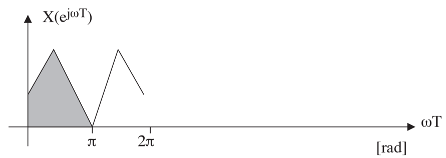
$$T_1 = T/L$$



The Fourier transform of $x_1(m)$ is

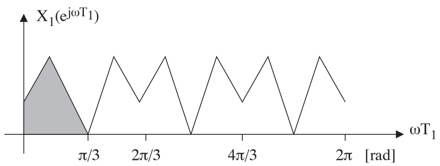
$$X_1\left(e^{j\omega T_1}\right) = \sum_{m=-\infty}^{\infty} x_1(m) e^{-j\omega m T_1} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega L T_1} = X(e^{j\omega T})$$

The original sequence $x(n)$ and the corresponding interleaved sequence $x_1(m)$ that has a three times higher sampling rate.



The spectrum of the sequence, $x_1(m)$, contains not only the baseband

$$\frac{-\pi}{T_1} < \omega < \frac{\pi}{T_1}$$

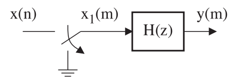


of the original signal, but also repeated images of the baseband.

Second step

An (ideal) lowpass filters are then used to remove the unwanted images

Obviously, it is not necessary to perform arithmetic operations involving the zeros in the input sequence $x_1(m)$.



Various schemes have been proposed to exploit this fact in order to reduce the computational work load.

Interpolation Using Wave Digital Filters

Interpolation can also be performed efficiently using FIR or IIR filters.

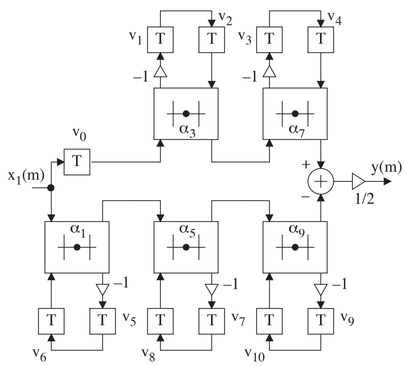
Lattice wave digital filters are particularly efficient for interpolation and decimation by a factor of two.

An IIR filter has a much smaller group delay than its corresponding linear-phase FIR filter, but the phase response is nonlinear.

Often this is of no concern, but in some cases the variation in group delay must be corrected. This can be done by placing an allpass filter in front of, or after, the lowpass filter (interpolator).

Example 4.16

Show that the interpolation filter (factor of two) can be made to operate at the lower (input) sampling rate.



The z-transform of the interleaved input sequence is

$$X_1(z) = X(z^2)$$

The transfer function of the original filter is

$$H(z) = z^{-1}H_1(z^2) + H_2(z^2)$$

Hence, the z-transform of the output is

$$Y(z) = H(z)X_1(z) = [z^{-1}H_1(z^2) + H_2(z^2)]X(z^2) =$$

$$= (z^{-1}H_1(z^2)X(z^2) + H_2(z^2)X(z^2))$$

Note that the term:

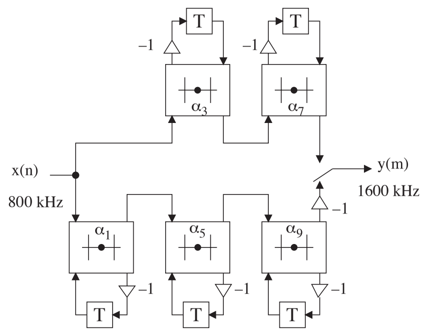
$H_2(z^2)\,X(z^2)$ corresponds to a sequence that is zero for every other sample.

Alternatively: This sequence can be obtained by filtering the input signal, $X(z)$, with a filter, $H_2(z)$, that operates at half the sampling rate.

Finally, the output of this filter is interleaved with zeros.

This scheme also applies to the first term, except for a unit delay of the sequence.

Thus, the output of the interpolator filter, $H(z)$, is obtained by summing the two sequences, but at each time instant only one of the two sequences has a nonzero value.



DECIMATION WITH AN INTEGER FACTOR

The process of reducing the sampling frequency is called *decimation*.

The relation between the Fourier transforms of the decimated signal and the original signal is

$$X_d(e^{j\omega T}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega T - 2\pi k)/M})$$

where M is the decimation factor.

The Fourier transform of the decimated signal consists of a sum of shifted replicas of the Fourier transform of the original signal.

Generally, aliasing takes place if the original signal bandwidth is larger than π/M.

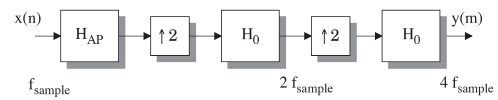
Although the aliasing can be removed by a bandlimiting digital filter, the decimated signal will no longer be the same.

INTERPOLATOR — CASE STUDY 3

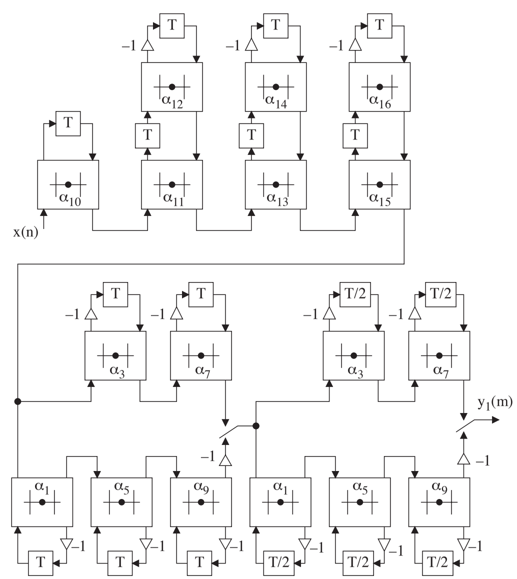
As the third case study, we choose an application with an interpolating wave digital filter.

The sampling frequency of the signal discussed in Example 4.15 shall instead be increased from 1.6 MHz to 6.4 MHz.

This can be done by interpolating the sampling rate in two steps. The interpolator has been cascaded with an allpass filter for equalizing the group delay.



We will for sake of simplicity use the bireciprocal lattice wave digital filter designed in Example 8.6 for both filters, although, only a ninth-order filter is required for the last stage.



The transfer function for the complete interpolator is

$$H_I(z) = \frac{1}{4}H_0(z)H_0(z^2)H_{AP}(z^4)$$

The adaptor coefficients in the allpass filter, which can be shortened to 9 bits, are

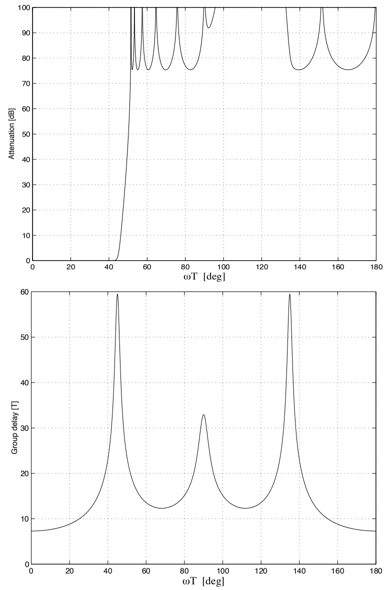
$\alpha_{10} = 0.53484764$

$\alpha_{11} = 0.28500882$ $\quad\quad$ $\alpha_{12} = -0.61671052$

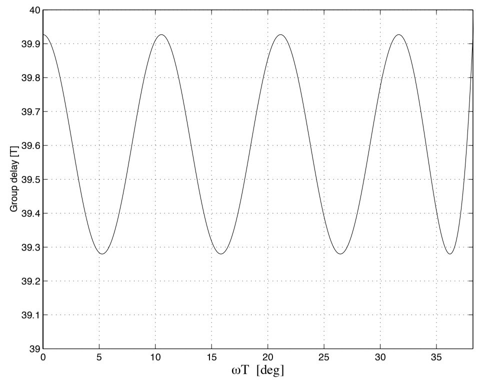
$\alpha_{13} = 0.28238053$ $\quad\quad$ $\alpha_{14} = -0.07952256$

$\alpha_{15} = 0.28518536$ $\quad\quad$ $\alpha_{16} = \ \ 0.50953672$

If instead a ninth-order filter, which represents a significantly lower work load, was used for the last stage more of the processing capacity could be allocated to the allpass filter to further reduce the variation in the group delay.



The group delay varies considerably in the passband 0 to 38.25°. The group delay of the interpolator is therefore equalized by adding a seventh-order allpass filter in front of the interpolator.



Also in this case the adaptor coefficients can be shortened to 9 bits.

The arithmetic work load is $35.2 \cdot 10^6$ adaptors/s corresponding to $35.2 \cdot 10^6$ multiplications/s and $105 \cdot 10^6$ additions/s and a significant number of operations for quantization, overflow detection and correction.