# 7. COUPLED FORMS

7.1 The poles and zeros are

| | $s_p$ | $s_z$ |
|---|---|---|
| 1 | -1.29527884953387 ± 3.05120447353959i | 0 ± 7.92170416987671i |
| 2 | -0.32085309025735 ± 4.11059683569423i | 0 ± 5.46102941479079i |
| 3 | -2.16492805122451 | |

$G$ = 2.160765326714776 e+03

The $Q$ values are: $Q_1 = 1.2795524$, $Q_2 = 6.4252146$, and $Q_3 = 0.5$.

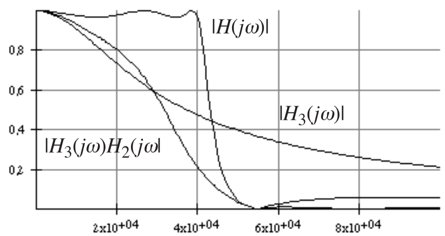
We place the section in the following order $Q_3$, $Q_2$, $Q_1$.

The zeros are paired in the following way: Begin with section with highest $Q$ value, i.e., $s_{z2}$ is placed in $Q_3$-section since $s_{z2}$ are the closest zero. Then the zero $s_{z1}$ is placed in the $Q_2$-section. We get the following order: $(s_{p3})(s_{z1}, s_{p2})(s_{z2}, s_{p1})$.

DC gain in the first, $(s_{p3})$-section is $H_3(s) = \frac{G_1 2164.9281}{(s + 2164.9281)}$, select there $G_1 = 1$ so that it can be realized with an $RC$ sections with a voltage follower at the output. In some cases it may be advantageous to use a positive amplifiers with gain > 1 and with a high impedance input.

The $Q$ values are here relatively low and we select to realize sections with 1-OP sections, i.e., there is only one op amp in each section and its output coincide with the output of the section. If 2- or 3-OP sections are used, also the internal outputs of the amplifiers must be scaled.

We determine successively the gain constant in the sections. The gain constant in the second, $(s_{z2}, s_{p1})$-section is determined so that the maximum of the magnitude function from the input of the filter to the output of the sections is = 1. This can be done by plotting the magnitude function using MATLAB with different values for $G_2$. We get $G_2 = 0.36842$.



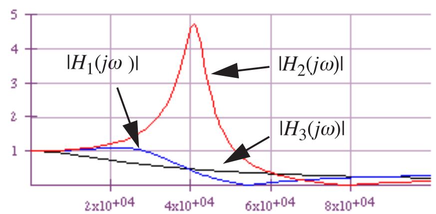

$$|H_3(s)H_2(s)| = \left|\frac{2164.9281}{(s + 2164.9281)} \cdot \frac{G_2(s^2 + 29822842)}{(s^2 + 2590.5577s + 1.0987596 \times 10^9)}\right|_{\substack{max \\ s = j\omega}}$$

Next the gain constant in the next section, i.e., $(s_{z1}, s_{p2})$-section, is determined so that the maximum of the magnitude function is 1. The scaled the transfer function is

$$H(s) = \frac{2164.9281}{(s + 2164.9281)} \cdot \frac{0.36842(s^2 + 29822842)}{(s^2 + 2590.5577s + 1.0987596 \times 10^9)} \cdot \frac{G_3(s^2 + 62753397)}{(s^2 + 6417.0618s + 1.6999953 \times 10^9)}$$

where $G_3 = 0.2709$.

The figure shows the magnitude functions for the scaled sections. Note that the gain in the last section ($H_1$) is very large while the gain in the other sections varies relatively little in the passband. At $\omega = 10^4$ (approx. at the passband edge), the gain in the last section is approx. 4.7 and approx. 0.5 in the two other sections. This means that the signals and also the signal-to-noise ratio in the filter and varies and






the dynamic range is reduced. However, here the reduction is relatively small, but it may be very large in, for example, narrowband filters with high $Q$ values.

7.2 We get from the specification $N \geq 2.74$ and the poles are

$s_{p1}$ = –11004.280625988333 +19059.97314495765*i

$s_{p2}$ = –22008.561251976673 $Q = 0.5$

$s_{p3}$ = –11004.280625988345 – 19059.973144957643*i $Q = 1$

A PF2 section is suitable since the $Q$ value is low.

$H$ = 1.06604358*10[13]/(($s^2$+22008.56 $s$+484376768)*($s$+22008.56))

We place a first-order section first with $R_2$ = 10 kΩ and $C_2$ = 4.543686 nF

The denominator for the second-order PF2 section is

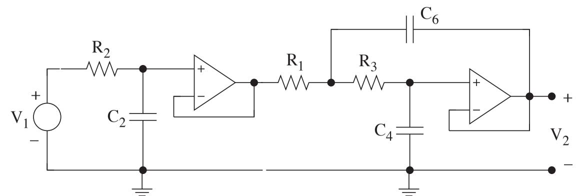
$D(s) = s^2 +(1/(C_6R_3)+1/(C_6R_1))s +1/(C_6C_4R_3R_1)$

We can select PF2 sections since the $Q$ factors are small, but it would have been better to select PF1 sections. We get with $1/(C_6C_4R_3R_1) = r_p^2$ and $\frac{1}{C_6}\left(\frac{R_1 + R_3}{R_1R_3}\right) = \frac{r_p}{Q}$ => $R_1 + R_3 = \frac{1}{C_4 r_p Q}$

=> $R_1 = \frac{1}{C_4 r_p Q} - R_3$ Inserting this into $R_1R_3 = \frac{1}{C_4C_6r_p^2}$ yields $R_3^2 - \frac{R_3}{C_4C_6r_p} + \frac{1}{C_4C_6r_p^2} = 0$ and

$$\left(R_3 + \frac{1}{2}\sqrt{\frac{C_6 - 4C_4Q^2}{C_4^1C_6r_p^2Q^2}} - \frac{1}{2C_4r_pQ}\right)\left(R_3 - \frac{1}{2}\sqrt{\frac{C_6 - 4C_4Q^2}{C_4^1C_6r_p^2Q^2}} - \frac{1}{2C_4r_pQ}\right) = 0$$

Hence, we must have $\frac{C_6}{C_4} \geq 4Q^2$ in order to get real valued resistance. We select $C_4$ = 2 nF and $C_6$ = 10 nF which yields $R_3$ = 6279.2 Ω and $R_1$ = 16439 Ω and $R_3$ = 16439 Ω and $R_1$ = 6279.2 Ω, respectively. If we select $C_6 = 4Q^2C_4$ we get $R_1 = R_3$.




7.3 Maximally flat => Butterworth and $A_{max}$ = 0.1 dB, $A_{min}$ = 25 dB and $f_s/f_c$ = 20/7 = 2.857 yields the filter order $N \geq 4.5307$

We get:

$s_{p1,5}$ = -19794.19 ± 60920.254*i $Q_1 = 1.618034$

$s_{p2,4}$ = -51821.86 ±37650.788*i $Q_2 = 0.61803$

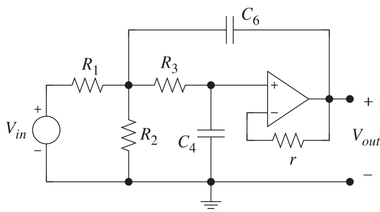
$s_{p3}$ = -64055.346 $Q_3 = 0.5$

$H$ = 1.0783926 10[24]/(($s^2$+39588.38 $s$+4103087326)*($s^2$+103643.7 $s$+4103087326)*($s$+64055.35))

We select to use PF2 sections, shown above, to realize the complex poles since the $Q$ values are low.

We select first the capacitance values (possibly after measure them), and according to Problem 7.2: $C_6/C_4 >= 4Q^2$

We select $C_4$ = 1 nF for the section that realizes $s_{p1,5}$ and $C_6 = 4Q^2C_4$ = 10.4721 nF which yield a



unique solution for $R_3$. We get $R_3 = \frac{1}{2C_4 r_p Q} = 1494.483\ \Omega$ and $R_1 = 8153.95\ \Omega$

We get for the second section that realizes $s_{p2,4}$ with $C_4 = 1$ nF and $C_6 = 1.527864$ nF => $R_3$ = 12629.97 Ω and $R_1$ = 12629.97 Ω. For the real pole: $H(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{RC}\frac{1}{s + \frac{1}{RC}}$

We have $s_p + 1/RC = 0$ and with $C = 1$ nF we get $R = 15611.5\ \Omega$

7.4

7.5

7.6 Suggest a suitable realization for the HP filter in Problem 2.31.
The poles and $Q$ values of the HP filter are

$s_{p1} = -1.96668803$ — $Q_1 = 0.5$
$s_{p2,3} = -0.53953363 \pm j\ 1.13067467$ Mrad/s — $Q_{23} = 1.161008$
$s_{p4,5} = -0.09292360 \pm j\ 0.9581225$ Mrad/s — $Q_{45} = 5.179621$

and the zeros are
$s_{z1,2} = \pm j\ 0.40530726$ Mrad/s
$s_{z3,4} = \pm j\ 0.612090254$ Mrad/s
$s_{z5} = 0$ Mrad/s

The filter is of low order and the $Q$ values are not very high. Hence, we may use the cascade form.
We first pair the pole pair with highest $Q$ value with its closest zero pair in section #1: $s_{p4,5}$ and $s_{z3,4}$
Next we the pair in section #2: $s_{p2,3}$with $s_{z1,2}$ and finally in section #3: $s_{p1}$and $s_{z5}$.
Section #1 is a HP-notch section which can be realized by, e.g., NF1 or PF2 sections
Section #2 is a HP-notch section which can be realized by, e.g., NF1 or PF2 sections or perhaps better with NF2 or PF1 sections which has better sensitivity properties
Section #3 is a BP section which can be realized by, e.g., NF1 or PF2 sections

7.7

7.8

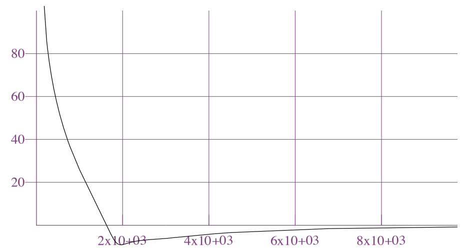
7.9 Partition $H(s)$ into three sections and order these according with increasing $Q$ values. We have

$r_{p0} = 500$ — $Q_0 = 0.5$

$r_{p1} = \sqrt{3.510^6} = 1870.828693387$ — $Q_1 = r_{p1}/400 = 4.6770717334674$

$r_{p2} = \sqrt{7.610^6} = 2756.809750418$ — $Q_2 = r_{p2}/2300 = 1.1986129349644$

The attenuation is shown below. Obviously this is not a standard highpass filter.




Hence, the order becomes: section 0, 2, and last section 1.
For the first section we get with $C = 10$ nF => $1/R_1 C = 500$ => $R_1 = 200$ kΩ.
Use PF2 sections, for section 2 and 3, since the $Q$ values are low. We select $C_3 = C_1+C_2 = 1$ nF, $R_4$ =



$4Q^2R_6$, in order to obtain unique resistors. We select $R_{61} = 2$ kΩ => $R_{41} = 175$ kΩ and for the third section we get with $R_{62}$= 2 kΩ => $R_{42} = 11.4934$ kΩ.
The error polynomial for PF2 HP is $E(s) = s^2 + (2Q^2 + 1)\ r_p\ s/Q + r_p^2$

7.10

7.11 They are characterized by very high sensitivity in the stopband. The reason is that two (large) signals from different signal paths should sum to a small output signal for frequencies in the stopband. Small relative errors in the two paths will then become a relatively large stopband signal.

7.12 Note that the last section is denoted $H_M(s)$ and the last node with $V_M$. The signal $V_0$ is

$$V_0 = -KV_i - f_1H_1(s)V_0 - f_2H_1(s)H_2(s)V_0 - \dots - f_MH_1(s)H_2(s)\dots H_M(s)V_0$$

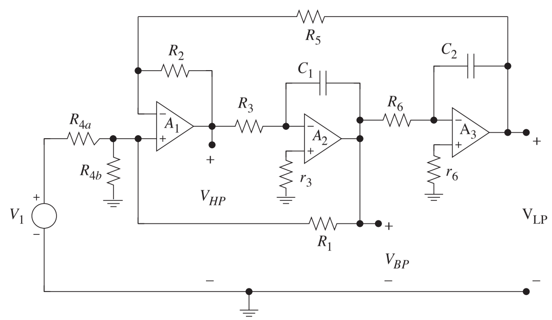
$$V_0 = -KV_i - \sum_{k=1}^{M} f_k\left(\prod_{j=1}^{k} H_j(s)\right)V_0$$ and

$$\frac{V_0}{V_i} = -\frac{K}{\sum_{k=1}^{M} f_k\left(\prod_{j=1}^{k} H_j(s)\right) + 1}$$ and if $V_M$ is taken as the output signal, we get

$$H(s) = \frac{V_M}{V_i} = -\frac{\prod_{j=1}^{M} H_j(s)}{\sum_{k=1}^{M} f_k\left(\prod_{j=1}^{k} H_j(s)\right) + 1}$$

The coefficient $f_k$ may be determined by minimizing the sensitivities using an optimization program.

7.13 We have $r_p = 3.00666$ krad/s and $Q = 7.516648$ and according to the above we have with $R_2 = R_3 = R_4 = R_5 = R = 10$ kΩ, $R_1 = 140.333$ kΩ, $C_1 = C_2 = C = 3.32595$ nF, $G_{HP} = -3$, $R_{4a} = 422.870$ kΩ and $R_{4b} = 10.2422$ kΩ. We get $G_{HP} = 14.0333 \cdot 3/126.861 = 0.331858$ and $G_{BP} = -42.1934 \cdot 3/126.861 = 0.9977866$.




The numerators for the BP and LP sections are $(-r_p/s)N_{HP}(s)$ and $(r_p/s)^2\ N_{HP}(s)$, respectively. The maximal gain of the LP section is $G_{LP} = r_p^2G_{HP} = 126.861$ and $G_{BP} = r_pG_{HP} = -42.1934$. The LP gain is thus to large why we use a voltage divider at the input. That is, $R_4$ is replaced with $R_{4a}$ and $R_{4b}$ where $R_{4a}//R_{4b} = R_4$ and $R_{4b}/(R_{4a} + R_{4b}) = 3/126.861$.

7.14 $s_{p1}, (s_{p1}^*) = -479.9438 \pm j13915.1038$ rad/s — $Q_1 = 14.505217$
$s_{p2}, (v_2^*) = -1327.3315 \pm j15532.3355$ rad/s — $Q_2 = 5.8722883$
$s_{p3}, (s_{p3}^*) = -1440.76978 \pm j18206.9763$ rad/s — $Q_3 = 6.3382487$

$s_{p4}$, $(s_{p4}{}^*) = -566.7471 \pm j20094.5532$ rad/s  $Q_4 = 17.735021$

$s_{z1}$, $(s_{z1}{}^*) = \pm j7521.0955$ rad/s

$s_{z2}$, $(s_{z2}{}^*) = \pm j25823.6$ rad/s

$s_{z3}$, $(s_{z3}{}^*) = 0$

$s_{z4}$, $(s_{z4}{}^*) = \infty$

$G = 4.4815\ 10^6 = 133.028$ dB

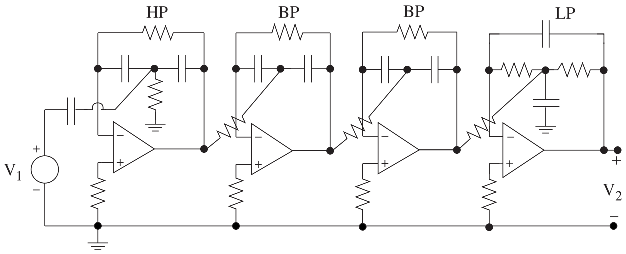



Figure 7.1. 8th-order BP filter

Figure 7.1 shows the final filter, which consist of four second-order sections of the type NF1. Figure 7.2 shows the order between the sections and how the poles and zeros have been combined so the signal dynamic becomes maximal. The order and the paring of the poles and zeros have been determined through complete search, i.e., all possible combinations have been evaluated. The difference between the worst and the best case increase for narrow band filters and with the section's $Q$ values. **Does the simple rule give the best solution?**

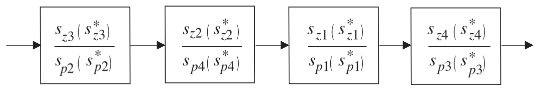



Figure 7.2. Sections with optimal ordering and paring of poles and zeros

7.15