5. BASIC CIRCUIT ELEMENTS

5.1 a

5.2 a) According to Theorem 5.1 we have

A system that initially is at rest and contains no stored energy is passive if the energy, w(t), which is supplied to the system, is always non-negative. That is, for all ports we have

$$w(t) = \sum_{\text{all ports}} Re\left\{\int_{-\infty}^{t} i^*(\tau)v(\tau)d\tau\right\} \geq 0 \qquad \forall t \tag{5.1}$$

and for a resistor we have $v(t) = R\,i(t)$. We get

$$w(t) = Re\left\{\int_{-\infty}^{t} i^*(\tau)Ri(\tau)d\tau\right\} = Re\left\{\int_{-\infty}^{t} Ri^2(\tau)d\tau\right\} \geq 0$$

for all t and om $R > 0$. Hence, a passive element.

b) We have for an inductor $v(t) = L\frac{d}{dt}i(t)$ which yields $w(t) = Re\left\{\int_{-\infty}^{t} i^*(\tau)L\frac{d}{d\tau}i(\tau)d\tau\right\} = \frac{L}{2}i^2(t) \geq 0$ if $L \geq 0$. The stored reactive energy in an inductor depends only on the current at time t.

Inductors (including time varying) are lossless since the stored reactive energy from a current that flows under a finite time period, i.e., $i(\infty) = 0$, is

$$w(t) = Re\left\{\int_{-\infty}^{\infty} i^*(\tau)L\frac{d}{d\tau}i(\tau)d\tau\right\} = \lim_{t\to\infty}\frac{L}{2}i^2(t) = 0 .$$

c) For a capacitor we have $i(t) = C\frac{d}{dt}v(t)$ and $w(t) = Re\left\{\int_{-\infty}^{t} v^*\tau C\frac{d}{d\tau}v\tau d\tau\right\} = \frac{C}{2}v^2(t) \geq 0$ if $C \geq 0$.

Capacitors are also lossless.

d) active according to b)

e) active according to c)

f) a negative resistor can according to a) generate energy, i.e., it is active.

5.3 a) For a two-port with a series impedance we have $V_1 = Z\,I_1 + V_2$ and $I_1 = -I_2$ and $V_1 = V_2 + Z\,(-I_2)$

$I_1 = 0 + 1\,(-I_2)$ which can be written: $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = K_{serie}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

b) For a two-port consisting of a shunt impedance we have $V_1 = V_2$ and $V_1 = Z\,(I_1 + I_2)$ and $I_1 = V_1/Z - I_2 = V_2/Z + 1(-I_2)$ and we get the $\mathbb{K}$ matrix, $K_{shunt} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$

5.4 $K_{VCVS} = \begin{bmatrix} \frac{1}{A} & 0 \\ 0 & 0 \end{bmatrix}$, $K_{VCCS} = \begin{bmatrix} 0 & -\frac{1}{g} \\ 0 & 0 \end{bmatrix}$, $K_{CCVS} = \begin{bmatrix} 0 & 0 \\ \frac{1}{r} & 0 \end{bmatrix}$, $K_{CCCS} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\alpha} \end{bmatrix}$

5.5 $\mathbb{K}^{-1} = \frac{1}{AD + -BC}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

5.6 For a two-port we have according to its definition of the $\mathbb{K}$ matrix

$V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

and $V_{in} = R_1I_1 + V_1$ and $V_2 = -R_2I_2$ where the currents are defined into the two-port. Elimination yields $I_2 = -V_2/R_2$ and using the equations above yields $V_1 = AV_2 + BV_2/R_2 = (A + B/R_2)V_2$ and $I_1 = CV_2 + DV_2/R_2 = (C + D/R_2)V_2$ and we get $V_{in} = R_1I_1 + V_1 = R_1(C + D/R_2)V_2 + (A + B/R_2)V_2$, and finally we get $H(s) = \frac{V_2}{V_{in}} = \frac{1}{A + \frac{B}{R_2} + CR_1 + D\frac{R_1}{R_2}}$

5.6 The $\mathbb{K}$ matrix for the LC ladder is

$$K_{LC} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & Z_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{Z_1}{Z_2}+1 & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & Z_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{Z_1}{Z_2}+1 & \left(\left(\frac{Z_1}{Z_2}+1\right)Z_3 + Z_1\right) \\ \frac{1}{Z_2} & \frac{Z_3}{Z_2}+1 \end{bmatrix} =$$

$$\begin{bmatrix} C_2L_1s^2+1 & (C_2L_3L_1s^3 + (L_1+L_3)s) \\ C_2s & C_2L_3s^2+1 \end{bmatrix}$$

Insertion into the expressions derived in Problem 5.6 yields the transfer function.

$$H(s) = \frac{R_L}{C_2L_3L_1s^3 + (L_1R_L + L_3R_s)C_2s^2 + (L_1 + L_3 + C_2R_LR_s)s + R_s + R_L} \quad \text{and}$$

$$H(s) = \frac{5.0505010^8}{s^3 + 2000s^2 + 2010101s + 1.01010110^9} =$$

$$= \frac{5.0505010^8}{(s+1000)(s+500-j871.83)(s+500+j871.83)} = \frac{5.0505010^8}{(s+1000)(s^2+1000s+1010101)}$$

The DC gain for the filter is $H(0) = \frac{R_L}{R_s + R_L}$. Note that here the transfer function is different from the normal transfer function, i.e., the ratio of the output and input voltage is maximally $R_L/(R_s + R_L)$. The normal transfer function is normalized to be maximally equal to unity.

5.7 $$\mathbb{K} = \begin{bmatrix} 1 & sL_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ sC_2 & 1 \end{bmatrix}\begin{bmatrix} 1 & sL_3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ sC_4 & 1 \end{bmatrix}\begin{bmatrix} 1 & sL_5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+s^2L_1C_2 & sL_1 \\ sC_2 & 1 \end{bmatrix}\begin{bmatrix} 1+s^2L_3C_4 & sL_3 \\ sC_4 & 1 \end{bmatrix}\begin{bmatrix} 1 & sL_5 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+s^2L_1C_2 & sL_1 \\ sC_2 & 1 \end{bmatrix}\begin{bmatrix} C_4L_3s^2+1 & C_4L_5L_1s^3 + (L_3+L_5)s \\ C_4s & C_4L_5s^2+1 \end{bmatrix} =$$

$$\begin{bmatrix} (C_4L_3s^2+1)(1+s^2L_1C_2) + C_4L_1s^2 & sL_1(C_4L_5s^2+1) + (C_4L_5L_1s^3 + (L_3+L_5)s)(1+s^2L_1C_2) \\ sC_2(C_4L_3s^2+1) + C_4s & sC_2(C_4L_5L_1s^3 + (L_3+L_5)s + C_4L_5s^2+1) \end{bmatrix}$$

$$= \begin{bmatrix} 3.2358s^4 + 5.2358s^2 + 1 & 1.9997s^5 + 5.2356s^3 + 3.236s \\ 5.2358s^3 + 3.236s & 3.2358s^4 + 5.2358s^2 + 1 \end{bmatrix}$$. Hence, we get

$$H(s) = \frac{1}{A + \frac{B}{R_L} + CR_1 + D\frac{R_s}{R_L}} = \frac{1}{1.9997s^5 + 6.4715s^4 + 10.471s^3 + 10.472s^2 + 6.472s + 2} =$$

$= \dfrac{1}{1.9997(s^2+0.61807s+1)(s^2+1.6181s+1)(s+1)}$. Note that the pole radius is equal for all the poles in a Butterworth filter.

5.8 a We have for an ideal operational amplifier

$V_1 = V_2$

$R_1 I_1 = R_3 I_2$

$V_2 = -R_2 I_2$

where I_1 and I_2 are the currents are into the two-port.

We get the chain matrix $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_3}{R_1} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$ which do not correspond to a reciprocal two-port since $AD - BC \neq 1$. Furthermore it is active since $A \neq 1/D$.

The two-port is a NIC (Negative Impedance Converter) with $n_1 = 1$ and $n_2 = R_3/R_1$. The particular circuit is denoted INIC where I refer to current since port voltages are equal and $I_1 = (R_3/R_1)I_2$. Also VNIC circuits exist where the input currents are equal and the port voltages are $V_1 = kV_2$.

b) We get from the equations above: $Z_{in} = -(R_1/R_3)R_2$

The input impedance correspond to a negative resistor.

The input impedance to a NIC is $Z_{in} = -(R_1/R_3)Z_2$ where Z_2 is the load impedance. The right port is open-circuit-stable and the left port is short-circuit-stable, i.e., the two NICs stable if they have a high or low load impedances, respectively.

5.9 a In the same ways as in Problem 5.9 a we get the $\mathbb{K}$ matrices for the two NICs

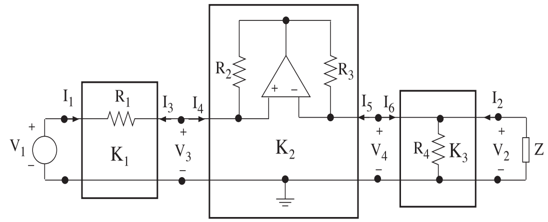
$\begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_2}{Z_1} \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_4}{Z_3} \end{bmatrix}$ and $\mathbb{K}$ matrices for the two cascade connected NICs yields

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_2}{Z_1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\frac{Z_4}{Z_3} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$. Furthermore, we have $V_2 = -Z_5 I_2$, which with the $\mathbb{K}$ matrix yields $Z_{in} = \dfrac{V_1}{I_1} = \dfrac{V_2}{-\frac{Z_2 Z_4}{Z_1 Z_3} I_2} = \dfrac{Z_1 Z_3}{Z_2 Z_4} Z_5$. For the $\mathbb{K}$ matrix we have $A = 1$, $B = C = 0$ and $D = (Z_2Z_4)/(Z_1Z_3)$. Hence, the circuit is a positive impedance converter (PIC). It is nonreciprocal since $AD - BC \neq 1$ if $D \neq 1$ and active if $A \neq 1/D$.

b) The $\mathbb{K}$ matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, i.e., $V_1 = V_2$ and $I_1 = -I_2$ which represent a direct connection of the two ports.

5.10 Consider the circuit that consist of three cascaded two-ports. Using the notation according to the figure we get the chain matrix, $\mathbb{K}_1$ for series resistor R_1: $V_1 = R_1 I_1 + V_2$ and $I_1 = -I_2$ which gives

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$. In Problem 5.8 a we derived the chain matrix for the inner two-port $\mathbb{K}_2$ and with the new notation we get $\begin{bmatrix} V_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_3}{R_2} \end{bmatrix} \begin{bmatrix} V_4 \\ -I_5 \end{bmatrix}$



For the last two-port we have $V_2 = R_4(I_6 + I_2)$ and $V_4 = V_2$. The chain matrix, $\mathbb{K}_3$ for the parallel resistor R_4 is $\begin{bmatrix} V_4 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_4} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

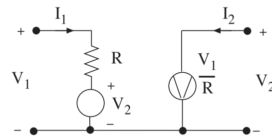
The resulting chain matrix is obtained by successively multiply the three $\mathbb{K}$ matrices, i.e.,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_3}{R_2} \end{bmatrix} \begin{bmatrix} V_4 \\ -I_5 \end{bmatrix} = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_3}{R_2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_4} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $I_3 = -I_4$ and $I_5 = -I_6$. We get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{R_3 R_1}{R_2} \\ 0 & -\frac{R_3}{R_2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_4} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} 1 - \frac{R_3 R_1}{R_2 R_4} & -\frac{R_3 R_1}{R_2} \\ -\frac{R_3}{R_2 R_4} & -\frac{R_3}{R_2} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$ and if $R_1 = R_2 = R_3 = R_4 = R$ we get

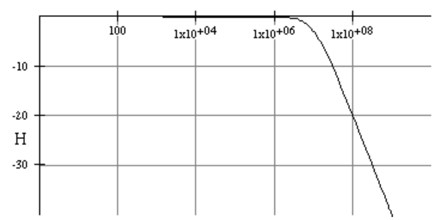
$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & -R \\ -\frac{1}{R} & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$, i.e., $V_1 = R I_2 \Rightarrow I_2 = V_1/R$ and $I_1 = -V_2/R + I_2 \Rightarrow V_1 = R I_1 + V_2$ The circuit is a voltage controlled (V_1) current source (I_2) where the current is independent of the impedance Z. The circuit can therefore be used as a voltage to current converter.



5.11 For the inverting amplifier we get $\begin{cases} Z_1^{-1}(V_1 - V_-) + Z_2^{-1}(V_2 - V_-) = 0 \\ A(V_+ - V_-) = V_2 \\ V_+ = 0 \end{cases}$ and $\dfrac{V_2}{V_1} = -\dfrac{Z_2}{Z_1 + \frac{Z_1 + Z_2}{A}}$

and $H(s) = -\dfrac{R_2}{R_1 + \frac{R_1 + R_2}{A}} = -\dfrac{10^4}{10^4 + \frac{2 10^4}{\frac{2 10^7}{s+100}}} = -\dfrac{10^7}{s + 1.00001 \cdot 10^7}$ where

$A(s) = \dfrac{A_0 \omega_{3dB}}{s + \omega_{3dB}} = \dfrac{2 \cdot 10^7}{s + 100}$. We get a real pole at $s_p = -10.0001$ Mrad/s. The bandwidth (3 dB) = 10.0001 Mrad/s. Note it is not always that we define the band edge at the 3-dB edge.



We get for the non-inverting amplifier

$$\begin{cases} V_- = \dfrac{Z_1}{Z_1 + Z_2} V_2 \\ V_+ = V_1 \\ A(V_+ - V_-) = V_2 \end{cases}$$ which yields $H(s) = \dfrac{Z_1 + Z_2}{Z_1 + \dfrac{Z_1 + Z_2}{A}} = \dfrac{2 \cdot 10^7}{s + 1.00001 \cdot 10^7}$. Hence, the same frequency response as for the inverting amplifier but with the DC gain, $H(0) \approx +2$. Also in this case, we get a real pole at $s_p = -10.0001$ Mrad/s. The bandwidth (3-dB) = 10.0001 Mrad/s is in this case the same for the inverting and non-inverting amplifier.

b) For the inverting amplifier we have $H(s) = -\dfrac{10^5}{10^4 + \dfrac{11 \cdot 10^4}{\dfrac{210^7}{s + 100}}} = -\dfrac{1.8182 \cdot 10^7}{s + 1.81828 \cdot 10^6}$. We get a real pole at $s_p = -1.81828$ Mrad/s and bandwidth (3 dB) = 1.81828 Mrad/s. The DC-gain is $H(0) = -9.99945$

For the non-inverting amplifier we get $H(s) = \dfrac{2 \cdot 10^7}{s + 1.81828 \cdot 10^6}$. We get the same frequency response as for the inverting amplifier but with the DC-gain, $H(0) \approx +10.9994$.

c) For the inverting amplifier is the case with $R_1 = \infty$ not relevant since it leads to a non-working amplifier. For the non-inverting amplifier we get $H(s) = \dfrac{A}{A+1} = \dfrac{2 \cdot 10^7}{s + 2.00001 \cdot 10^7}$. The DC gain becomes, $H(0) \approx +0.999995$ and bandwidth 20.0001 Mrad/s.

A comparison yields

Inverting amplifier

|H(0)|	Bandwidth, Mrad/s
–1	10.0001
–10	1.81828

Non-inverting amplifier

|H(0)|	Bandwidth, Mrad/s
1	20.0001
2	10.0001
11	1.81828

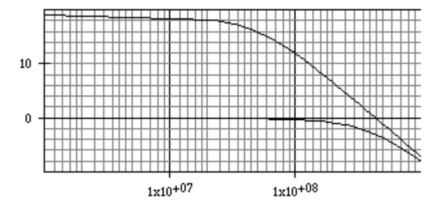
Hence, the bandwidth reduces rapidly with increasing gain and the non-inverting amplifier have a slightly larger bandwidth at the same gain compared to the inverting amplifier.

5.12 The resistor r is used to compensate for the offset-error that occur at the input of the amplifier due to the bias currents goes through different DC-impedances. Note that there are several cause for the off-set voltage. If both inputs has the same bias-currents and sees the same DC-impedance, then the voltage at the amplifiers input will be zero. Typically, the resulting offset-error will in practice be reduced with a factor of 4 if the DC impedance are the same. For both the inverting amplifier and for the non-inverting amplifier we shall select $r \approx R_1 // R_2$. If Z_2 is a capacitance we select $r \approx R_1$.

5.13 The operational amplifier is used in the circuit for the non-inverting amplifier. We have

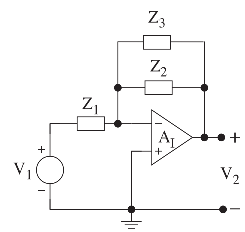
$$H(s) = \frac{Z_1 + Z_2}{Z_1 + \dfrac{Z_1 + Z_2}{A}} = \begin{cases} \dfrac{1}{\dfrac{1}{A} + 1} & \text{f□r Gain = 1} \\ 10\dfrac{A}{A + 10} & \text{f□r Gain = 10 med } Z_2 = 9Z_1 \end{cases}$$ and for operational amplifier

We get $A(s) = \dfrac{A_0 \omega_{3dB}}{s + \omega_{3dB}}$. Let $A(s)$ in $H(s)$ and equate the numerators real and imaginary parts, which yields the amplifiers 3-dB-band edge. We get A_0 = 80 and ω_{3db} = 5.5555 Mrad/s, i.e., f_{3db} = 884.2 kHz.



5.14 According to Problem 5.11 we have $\dfrac{V_2}{V_1} = -\dfrac{R_2}{R_1 + \dfrac{R_1 + R_2}{A}}$. For high frequencies, i.e., $\omega >> \omega_{3dB}$.

We have $A(s) = \dfrac{A_0 \omega_{3dB}}{s + \omega_{3dB}} \approx \dfrac{A_0 \omega_{3dB}}{s} = \dfrac{\omega_t}{s}$. Inserting $A(s)$ into the above gives $H(s) = -\dfrac{R_2}{R_1 + \dfrac{(R_1 + R_2)s}{\omega_t}}$. We replace the original resistor R_2 with an impedance Z_3 so that $H(s) = -\dfrac{R_2}{R_1 + \dfrac{(R_1 + R_2)s}{\omega_t}} = -\dfrac{Z_3}{R_1}$. This correspond to an inverting amplifier with an ideal operational amplifier with gain $A_I = \infty$. A resistor R_2 parallel with a capacitor C_3 has the impedance $\dfrac{R_2}{C_2 R_2 s + 1}$. We get $Z_3 = \dfrac{\omega_t R_2 R_1}{\omega_t R_1 + (R_1 + R_2)s} = \dfrac{R_2}{C_3 R_2 s + 1}$ and



$C_3 = \dfrac{(R_1 + R_2)}{\omega_t R_2 R_1}$. For example, $Z_1 = R_1 = 10$ kΩ and $Z_2 = R_2 = 100$ kΩ and operational amplifier with $A_0 = 2\ 10^5$, $\omega_{3dB} = 200$ rad/s, $\omega_t = 40$ Mrad/s yields $C_3 = \dfrac{10 + 100}{4 \cdot 10^7 \cdot 10 \cdot 100} = 2.75\ 10^{-12}$.

The parallel capacitor shall have the capacitance 2.75 pF.

Note that the correction admittance should be equal to the sum of all admittances that are connected to the (–)-input, divided by ω_t /s.

5.15 According to the example above, the correction admittance Y_3 equals the sum of all admittances connected to the (–)-input divided by GB/s. In this case,

$$Y_3 = \frac{(Y_1 + Y_2)}{\dfrac{GB}{s}} = \frac{\left(\dfrac{1}{R} + sC\right)s}{GB} = \frac{s}{RGB} + \frac{s^2 C}{GB}$$ Identification yields $C_1 = \dfrac{1}{RGB}$ and $D = \dfrac{C}{GB}$.

In order to correct for the finite GB, an extra correction capacitor and en supercapacitor is required. For example, $R = 10$ kΩ and $C = 100$ nF and operational amplifier have $A_0 = 2\ 10^5$, $\omega_{3dB} = 200$ rad/s, $GB = 40$ Mrad/s we get $C_1 = 2.5\ 10^{-15} = 2.5$ fF (this is a very low value) and $D = 100$ nFs. Of couse, this solution is not efficient in practice.

5.15 For the circuit we have

$$\begin{cases}\dfrac{(V_1 - V_-)}{R} + sC(V_2 - V_-) = 0 \\ A(V_+ - V_-) = V_2 \\ V_+ = 0\end{cases}$$ Elimination gives $H(s) = -\dfrac{\frac{1}{sC}}{R + \dfrac{R + \frac{1}{sC}}{A}} = -\dfrac{1}{sRC + \dfrac{1 + sRC}{A}}$. With

$A(s) = \dfrac{A_0\omega_{3dB}}{s + \omega_{3dB}} \approx \dfrac{GB}{s}$ we get $H(s) = -\dfrac{GB}{\left(s + GB + \frac{1}{RC}\right)sRC}$

We get, if the operational amplifier gain is large, $H(s) = -\dfrac{1}{sRC}$.

We get, with the values above, $H(s) = -\dfrac{10^{14}}{(s + 20 \cdot 10^6)s}$. The pole $s_p = -20$ Mrad/s is fare from the $j\omega$-axis and do not effect integrator significantly. With $A(s) = \dfrac{A_0\omega_{3dB}}{s + \omega_{3dB}}$ without any approximation we get $H(s) = -\dfrac{10^{14}}{(s + 20 \cdot 10^6)(s + 50)}$, i.e., the pole at $s = 0$ have moved into the LHP. A resistor, $r = R$, should be used between ground and (+)-input in order to minimize the effect of the bias currents.

5.16 We get for the node currents

$$\begin{cases}\dfrac{(V_1 - V_+)}{R_1} - sCV_+ + \dfrac{(V_2 - V_+)}{R_2} = 0 \\ \dfrac{(V_2 - V_-)}{R_3} - \dfrac{V_-}{R_4} = 0 \\ A(V_+ - V_-) = V_2\end{cases}$$ and

$\dfrac{V_2}{V_1} = \dfrac{(R_3 + R_4)R_2}{sCR_1R_2R_4 - R_1R_3 + R_2R_4 + \dfrac{(R_3 + R_4)((sCR_1 + 1)R_2 + R_1)}{A}}$ and with $R_1 = R_2 = R_3 = R_4 = R$

we get $H(s) = \dfrac{2}{sRC + \dfrac{2(sRC + 2)}{A}}$ and with an ideal operational amplifier we get $H(s) = \dfrac{2}{sRC}$. The circuit is called **Deboo's integrator**. With the element values used above we get $H(s) = \dfrac{10^{14}}{(s + 80)(s + 2.5\ 10^7)}$. The pole $s_p = -25$ Mrad/s is fare from the $j\omega$-axis and do not effect integrator significantly while the pole $s_p = -80$ rad/s is more important and it lies further away compared to the real pole in the Miller-integrator in Problem 5.15. The most important difference is that Deboo's integrator is positive.

With an ideal operational amplifier we get $H(s) = \dfrac{(R_3 + R_4)R_2}{sCR_1R_2R_4 - R_1R_3 + R_2R_4}$ which can be written

$H(s) = \dfrac{(R_3 + R_4)R_2}{CR_1R_2R_4} \dfrac{1}{s + \dfrac{-R_1R_3 + R_2R_4}{CR_1R_2R_4}}$. The pole is $s_p = \dfrac{1}{CR_1}\left(\dfrac{R_3}{R_4}\right)\left(\dfrac{R_1}{R_2} - \dfrac{R_4}{R_3}\right)$. We must therefore select $\dfrac{R_4}{R_3} \gg \dfrac{R_1}{R_2}$ (pole in the LHP). Unfortunately we get a large deviation in the phase response. The input impedance is frequency independent. The circuit is a NIC that is connected to the R_1C section, compared with Problem 5.8 a, and realize a en negative resistor R_0 in parallel with C where $R_0 = -R_2R_4/R_3 = -R$. The voltage over the capacitor is $V_C = \dfrac{R_1V_1}{R_1 + Z}$ where $Z = 1/sC//(-R_0)$ =>

$Z = \dfrac{-R_0}{1 - sCR_0}$ which yields $H_C = \dfrac{1}{sCR}$ and the operational amplifier gives a gain with a factor 2.

5.17 a) $H(s) = \dfrac{-1}{sRC}$ The integrator is less sensitive for the operational amplifiers finite GB.

b) $H(s) = \dfrac{+1}{sRC}\dfrac{R_2}{R_1}$. Positive integrator.

c) $H(s) = \left(\dfrac{1 + R_1C_1}{1 + R_2C_2}\right)\dfrac{+1}{sR_1C_1} = \dfrac{+1}{sRC}$ if $R_1 = R_2 = R$ and $C_1 = C_2 = C$. The input impedance to the integrator is $Z_{in} = R + 1/sC$ and the circuit must have a low impedance signal source.

d) $H(s) = \dfrac{+2}{sRC}$ The integrator is less sensitive for the operational amplifiers finite GB.

5.18

5.19

5.20 For a transconductor we have: $I = g_m(V_+ - V_-)$ For this circuit we have

$I_1 + I_2 + I_3 = 0$, $I_1 = g_{m_1}(V_1 - 0)$, $I_2 = g_{m_2}(0 - V_2)$, $I_3 = g_{m_3}(0 - V_3)$ which yields

$g_{m_1}V_1 - g_{m_2}V_2 - g_{m_3}V_3 = 0$ and $V_3 = \dfrac{g_{m_1}}{g_{m_3}}V_1 - \dfrac{g_{m_2}}{g_{m_3}}V_2$

5.21 We have $V_3 = \dfrac{1}{sC}(I_1 + I_2)$, $I_1 = g_{m_1}(V_1 - 0)$, $I_2 = g_{m_2}(0 - V_2)$ which yields

$V_3 = \dfrac{1}{sC}(g_{m_1}V_1 - g_{m_2}V_2)$

5.22 The circuit realizes a NIC.

5.23 The transfer function for an inverting amplifier has been derive before

$$H(s) = -\dfrac{Z_2}{Z_1 + \dfrac{Z_1 + Z_2}{A}} = \dfrac{-\left(r_x + \frac{1}{sC}\right)}{R - r_x + \dfrac{R - r_x + r_x + \frac{1}{sC}}{A}} = \dfrac{-(sr_xC + 1)}{(R - r_x)sC + \dfrac{sRC + 1}{A}}$$

For reasonable high frequencies we have $A(s) \approx \omega_t/s$ where $\omega_t = A_0\omega_{3dB}$. We get

$$H(s) = \dfrac{-(sr_xC + 1)}{(R - r_x)sC + \dfrac{(sRC + 1)s}{\omega_t}} = \dfrac{-(sr_xC + 1)\omega_t}{(sRC - r_xC\omega_t + \omega_tRC + 1)s}.$$

We obtain, if we select $r_x = \dfrac{1}{C\omega_t} = \dfrac{1}{10 \cdot 10^{-9} \cdot 10^5} = 1000\ \Omega$,

$H(s) = \dfrac{-\left(\left(\frac{s}{\omega_t} + 1\right)\right)\omega_t}{(sRC - 1 + \omega_tRC + 1)s} = \dfrac{-(s + \omega_t))}{(s + \omega_t)RCs} = \dfrac{-1}{RCs}$ With this value of r_x, the real pole in the amplifier is cancelled. However, the pole must be known and remain constant, which is not the case in practice when, for example, the temperature varies. However, a significant improvement is obtained in practice.

5.24 The input impedance to the PII (gyrator) is found from $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & r \\ \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$ and with

$Z_2 = 1/sC_2$ we get $Z_{in} = \frac{r^2}{Z_2} = r^2 sC_2$, i.e., the input impedance correspond to an inductor, with

inductance $L = r^2 C_2 = 6.75$ mH. Hence, the transfer function is $H(s) = \frac{Z}{R+Z}$ where

$$Z = \frac{sL\left(\frac{1}{sC_1}\right)}{sL + \frac{1}{sC_1}} = \frac{sL}{LC_1 s^2 + 1} \text{ and } H(s) = \frac{\frac{s}{RC_1}}{s^2 + \frac{s}{RC_1} + \frac{1}{LC_1}}$$ i.e., a BP section. We get

$$H(s) = \frac{\frac{s}{RC_1}}{s^2 + \frac{r_p s}{Q} + r_p^2} = \frac{10^4 s}{s^2 + 10^4 s + 1.481510^{10}}$$ Hence, resonance frequency (pole radius) is $r_p =$ 1.2171 10^5 rad/s and $Q = 12.1716$.

5.25 a) With an ideal operational amplifier is the voltage between the inputs zero, i.e., $V_1 = V_2$.
Denote the currents through the impedances form left to the right, I_1, I_2, I_3, I_4 and $-I_5$.

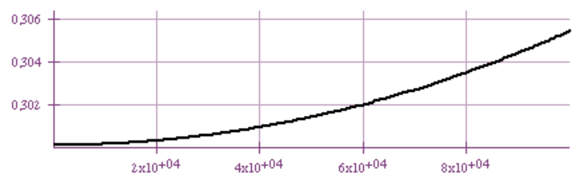
$Z_1 I_1 + Z_2 I_2 = 0 \Rightarrow$ $I_2 = (-Z_1/Z_2)I_1 \Rightarrow I_3 = (-Z_1/Z_2)I_1$
$Z_3 I_3 + Z_4 I_4 = 0 \Rightarrow$ $I_4 = (-Z_3/Z_4)I_3 \Rightarrow I_5 = (Z_3/Z_4)I_3$
$V_2 = -Z_5 I_5 \Rightarrow$ $V_2 = -Z_5 I_5 = -Z_5 (Z_3/Z_4)I_3 = -Z_5 (Z_3/Z_4)(-Z_1/Z_2)I_1$
$I_2 = I_3$
$I_4 = -I_5$

$V_1 = V_2$ and finally $Z_{in} = \frac{Z_1 Z_3}{Z_2 Z_4} Z_5$

b) We have from above $V_1 = V_2$ and $I_1 = (-Z_2/Z_1)I_2 = (-Z_2/Z_1)I_3 = (-Z_2/Z_1)I_5/(Z_3/Z_4)$ and finally $I_1 = (Z_2 Z_4/Z_1)/Z_3)(-I_5)$ where I_5 is the current into port 2. Comparing with the definition for the $\mathbb{K}$ matrix yields

$$K = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix}$$

5.26 a) Select relatively small resistors, e.g., $R_2 = R_3 = 10$ kΩ in order to obtain low sensitivity for parasitic stray capacitances. Select, e.g., $C_4 = 10$ nF which yields $sL = Z_1 s C_4 Z_5$ and select, e.g., $R_1 = R_5 = 5.48$ kΩ. With an operational amplifier with $\omega_t = 20\pi$ Mrad/s we get the inductance (magnitude of $Z_{in}/j\omega$) according to the figure below.



b) Select, e.g., $R_4 = R_5 = 10$ kΩ and $C_1 = C_3 = 330$ nF which yields $R_2 = \frac{D}{C_1 C_3} = 91.83$ kΩ.

5.27 We have $Z_1 = \frac{1}{sC_1}$ $Z_2 = R_2$ $Z_3 = \frac{R_3}{1 + sR_3 C_2}$ $Z_4 = R_4$ $Z_5 = R_5$

We have for an ideal operational amplifier, i.e., $V_+ = V_-$

$V_{in} = R_1 I_{in} - R_5 I_5 \Rightarrow I_5 = (R_1 I_{in} - V_{in})/R_5$
$Z_1 I_{in} + Z_2 I_2 = 0 \Rightarrow I_{in} = -Z_2 I_2/Z_1 \Rightarrow I_{in} = (-Z_2/Z_1)(Z_4/Z_3)I_5$
$Z_3 I_2 - Z_4 I_5 = 0 \Rightarrow I_2 = Z_4 I_5/Z_3$
$V_2 = -(R_4 + R_5)I_5 \Rightarrow V_2 = -(R_4 + R_5)I_5$

which after simplification we get $\frac{V_2}{V_{in}} = \frac{(Z_4 + Z_5)Z_3 Z_1}{R_1 Z_2 Z_4 + Z_1 Z_3 Z_5}$

With $R_1 = R_2 = R_4 = R_5 = R$, $R_3 = Q\,R$ and $C_1 = C_2 = C$ and $r_p = 1/RC$ we get the transfer function according Eq.(9.4), i.e., an LP section. Select, e.g., $C = 10$ nF and $R = 1/(r_p C) = 5.0$ kΩ and $R_3 = Q\,R = 1/\sqrt{2} = 3.5255$ kΩ. The section DC gain is 2. The gain can be reduced by replacing R_1 with a voltage divider. If R_1 is split into two equal size resistors with the resistance $2R_1$, the section DC gain = 1.