2. SYNTHESIS OF ANALOG FILTERS

2.1 a) $|H_{BW}(j\omega)|^2 = \frac{1}{1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}}$ where $\varepsilon = \sqrt{10^{0.1A_{max}}-1}$

$$A_{max} = -20\log(|H(j\omega_c)|) = 10\log(1+\varepsilon^2)$$

$$A_{min} = -20\log(|H(j\omega_s)|) = 10\log\left(1+\varepsilon^2\left(\frac{\omega_s}{\omega_c}\right)^{2N}\right)$$

$$\varepsilon^2\left(\frac{\omega_s}{\omega_c}\right)^{2N} = \frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}, \quad \Rightarrow N \geq \frac{\log\left(\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}\right)}{2\log\left(\frac{\omega_s}{\omega_c}\right)}$$

b)

$$|H_{BW}(j\omega)|^2 = \frac{1}{1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}} = H(s)H(-s)|_{s=j\omega} = \frac{1}{1+\varepsilon^2\left(\frac{s}{j\omega_c}\right)^{2N}}$$

$$D(s)D(-s) = 1+\varepsilon^2\left(\frac{s}{j\omega_c}\right)^{2N} = 0 \qquad \varepsilon^2\left(\frac{s}{j\omega_c}\right)^{2N} = -1 = e^{j(2k+1)\pi}$$

$$\varepsilon^{1/N}\left(\frac{s}{j\omega_c}\right) = e^{j(2k+1)\pi/2N} \qquad \Rightarrow s_k = j\omega_c\varepsilon^{-1/N}\, e^{j(2k+1)\pi/2N}$$

2.2 $A_{max} = 10\log(1+\varepsilon^2) = -10\log(1-\rho^2)$. For example, $\rho = 5\% \Leftrightarrow A_{max} = 0.01087096$ dB

2.3 $A(\omega) = 10\log\left(1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}\right)$ and $r_{p0} = \omega_c\varepsilon^{-\frac{1}{N}}$ we get

$$A(r_{0p}) = 10\log\left(1+\varepsilon^2\left(\frac{r_{0p}}{\omega_c}\right)^{2N}\right) = 10\log\left(1+\varepsilon^2\left(\frac{\omega_c\varepsilon^{-\frac{1}{N}}}{\omega_c}\right)^{2N}\right) =$$

$$= 10\log\left(1+\varepsilon^2\left(\varepsilon^{-\frac{1}{N}}\right)^{2N}\right) = 10\log(1+\varepsilon^2\varepsilon^{-2}) = 10\log 2 \approx 3.01\text{dB}$$

2.4 $H(j\omega) = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}}} = \left(1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}\right)^{-\frac{1}{2}}, \quad \frac{1}{|H(j\omega)|^2} = |G(j\omega)|^2 = 1+\varepsilon^2\left(\frac{\omega}{\omega_c}\right)^{2N}$

which we recognize as a Taylor series. $F(x) = F(0) + \frac{F'(0)}{1!}x + \frac{F''(0)}{2!}x^2 + \ldots + \frac{F^n(0)}{n!}x^n + \ldots$ were $x = \omega/\omega_c$. Hence, the first $2N$–1 derivatives are zero for $x = 0$.

2.4 Let $h(t)$ and $s(t)$ be the impulse and step responses, respectively, of a filter $H(s)$. Scale the frequency with a factor k according to $\omega' = k\omega$. The impulse response of the frequency scaled filter is

$$h'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(jk\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(jk\omega)e^{jk\omega(t/k)}\frac{1}{k}d(k\omega) = \frac{1}{k}h\left(\frac{t}{k}\right)$$

Thus, both the time axis and the original impulse response is divided by k.

$$s'(t) = \int_0^{\infty} h'(t)dt = \int_0^{\infty} h\left(\frac{t}{k}\right)\frac{1}{k}dt = \int_0^{\infty} h\left(\frac{t}{k}\right)d\left(\frac{t}{k}\right) = s\left(\frac{t}{k}\right)$$

Thus, the time axis of the original step response is divided by k.

2.6 We get $\frac{dT}{dx} = \frac{n\sin(n\,\mathrm{acos}(x))}{\sqrt{1-x^2}}$ which is of the form 0/0 at x =1. However, the other extreme values occur for $x < 1$. We have $\sin(y) = 0 \Rightarrow y = k\pi$ for k = integer and the extreme values (maxima and minima) are obtained from $n\,\mathrm{acos}(x) = y = k\pi$. Finally, we get $x = \cos(\pi k/n)$.
For example, a filter of order $N = 5$ we get the extreme values at $x_0 = 1$, $x_1 = \cos(\pi\, 1/5) = 0.80901699$, $x_2 = \cos(\pi\, 2/5) = 0.30901699$, $x_3 = \cos(\pi\, 3/5) = -0.30901699$, $x_4 = \cos(\pi\, 4/5) = -0.80901699$, and $x_5 = -1$. The extreme values at $x = \pm1$ are neither a maxima nor a minima.

2.7 The poles are according to Eq. (2.18) where $r_{p0} = \omega_c\varepsilon^{-1/N}$ and $\varepsilon = \sqrt{10^{0.1A_{max}}-1}$.
We get $\varepsilon = 0.15262041895091921$ and $r_{p0} = 9356.1919658881052$ krad/s and the normalized poles:
$s_{p1} = -0.5 + j\,0.86602540378444$ $|s_{p1}| = 1$
$s_{p2} = -1$ $|s_{p2}| = 1$
$s_{p3} = -0.5 - j\,0.86602540378444$ $|s_{p3}| = 1$
either from the Tables or by using **[G, Z, P] = BW_POLES(Wc, Ws, Amax, Amin, N)**
We denormalize the poles by multiplying with r_{p0}which yields
$s_{p1} = -4678.0959829440526 + j\,8102.6999251429679$ krad/s $|s_{p1}| = r_{p0}$
$s_{p2} = -9356.1919658881052$ krad/s $|s_{p2}| = r_{p0}$
$s_{p3} = -4678.0959829440526 - j\,8102.6999251429679$ krad/s $|s_{p3}| = r_{p0}$

$H(s) = \frac{G}{(s-s_{p2})(s-s_{p1})(s-s_{p3})} = \frac{G}{(s+r_{p0})(s^2+r_{p0}s+r_{0p}^2)}$ We select G so that the filter get the proper gain, for example, Gain = 1 at $\omega = 0$. Hence, $G = r_{p0}{}^N$.

$$A(2\omega_c) = -10\log\left(\frac{1}{1+\varepsilon^2(2)^6}\right) = 3.8633 \text{ dB and } A(4\omega_c) = -10\log\left(\frac{1}{1+\varepsilon^2(4)^6}\right) = 19.741 \text{ dB}$$

2.8 $\varepsilon = \sqrt{10^{0.1A_{max}}-1} = 0.15262041895091921$, $x = 1/\varepsilon = 6.552203217$, $\mathrm{asinh}(x) = \ln(x+\sqrt{x^2+1}) = 2.5787215736978437$, $a = \sinh\left(\frac{\ln(x+\sqrt{x^2+1})}{N}\right) = 0.96940570903005374$, and

$$b = \cosh\left(\frac{\ln(x+\sqrt{x^2+1})}{N}\right) = 1.3927481569544657$$

Normalized poles according to Equation (2.33) are
$s_{p1} = -0.48470285451502687+1.206155284996524i$
$s_{p2} = -0.96940570903005374$
$s_{p3} = -0.48470285451502687-1.206155284996524i$
denormalize by multiplying with ω_c. We get
$s_{p1} = -2423.5142725751343 + j\,6030.7764249826196$ krad/s
$s_{p2} = -4847.0285451502687$ krad/s
$s_{p3} = -2423.5142725751343 - j\,6030.7764249826196$ krad/s
Alternatively we get the poles from the Tables or by using **[G, Z, P] = CH_I_POLES(Wc, Ws, Amax, Amin, N)**

$H(s) = \frac{G}{(s-s_{p2})(s-s_{p1})(s-s_{p3})}$ We select G so that the filter get the proper gain, for example, Gain = 1 at $\omega = 0$. Hence, $G = -2.04756350525.08664 \cdot 10^{11}$.
We can either use Eq. (2.22) or Eq (2.27), but the former is simpler to solve. We have at $\omega = 0$

$|H(0)|^2 = \frac{1}{1+\varepsilon^2 T_3^2(0)} = \frac{1}{1+\varepsilon^2}$ and $|H(j\omega)|^2 = \frac{1}{1+\varepsilon^2 T_3^2\left(\frac{\omega}{\omega_c}\right)}$. A Chebyshev polynomial can, according to standard mathematical handbooks, be computed recursively:
$T_0(x) = 1$, $T_1(x) = x$, and $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$ for $n = 2, 3, \ldots$
We get $T_2(x) = 2xT_1(x) - T_0(x) = 2x^2 - 1$ and $T_3(x) = 2xT_2(x) - T_1(x) = 2x(2x^2 - 1) - x = 4x^3 - 3x$, $T_3(1) = 4 \cdot 1^3 - 3 = 1$, $T_3(2) = 4 \cdot 2^3 - 6 = 26$ and $T_3(4) = 4 \cdot 4^3 - 12 = 244$. We get

$$|H(j2\omega_c)|^2 = \frac{1}{1+\varepsilon^2 T_3^2(2)} = \frac{1}{1+\varepsilon^2 26^2} \Rightarrow A(2\omega_c) = -10\log\left(\frac{1}{1+\varepsilon^2 26^2}\right) = 12.23913 \text{ dB}$$

and $A(4\omega_c) = -10\log\left(\frac{1}{1+\varepsilon^2(244)^2}\right) = 39.5844$ dB

2.9 We need to determine Ws. We have $x = \sqrt{\frac{10^{0.1A_{min}} - 1}{10^{0.1A_{max}} - 1}} = 655.18755984513$. Let y = Ws/Wc and with

$N = 3$ and we have $N = \frac{\ln(x + \sqrt{x^2 - 1})}{\ln(y + \sqrt{y^2 - 1})}$ Iterating we get y = 5.518 and Ws = 27.59 Mrad/s.

The following script yields

```
Amax = 0.1; Amin = 40, Wc = 5*10^6; Ws = 5.518*Wc;
N = CH_ORDER(Wc, ws, Amax, Amin)
```

N = 2.99979985985076
We select N = 3;

```
[G, Z, P] = CH_II_POLES(Wc, Ws, Amax, Amin, N)
```

yields G = 8.276526045544402e+05

Z = 1.0e+07 *	P = 1.0e+06 *
0 – 3.18524143511917i	–9.71843608351180
0 + 3.18524143511917i	–4.44539173947868 – 8.16351659857224i
∞	–4.44539173947868 + 8.16351659857224i

and with

```
W = 2*Wc;
Att = PZ_2_ATT_S(G, Z, P, W)
```

we get Att = 4.38569 dB and with W = 4*Wc; we get Att = 23.9489 dB

2.10 Trying with different Ws we get

```
Amax = 0.1;  Amin = 40;  Wc = 5*10^6;  Ws = 3.52*Wc
N = CA_ORDER(Wc, Ws, Amax, Amin)
Ws = 17600000
N = 2.99983410225726
N = 3;
[G, Z, P] = CA_POLES(Wc, Ws, Amax, Amin, N
```

yields G = 5.170164067567552e+05

Z = 1.0e+07 *	P = 1.0e+06 *
0 – 2.02146769888630i	–2.26644632155069 – 6.05802180703632i
0 + 2.02146769888630i	–2.26644632155069 + 6.05802180703632i
∞	–5.04990904985813

and

```
W = 2*Wc
A = PZ_2_ATT_S(G, Z, P, W)
W = 10000000
A = 14.281712 dB
```


and

```
W = 4*Wc
A = PZ_2_ATT_S(G, Z, P, W)
W =  20000000
A = 64.64685 dB
```

2.11 a) $N = 4.986874 \Rightarrow N = 5$. We get $\varepsilon = 0.31448545$ and $r_{p0} = 3.7809518$ Mrad/s either from the Tables or by using `[G, Z, P] = BW_POLES(Wc, Ws, Amax, Amin, N)`

$s_{p1} = -1.1683784 + j\,3.521154070589589$ Mrad/s
$s_{p2} = -3.0588543 + j\,2.2223877$ Mrad/s
$s_{p3} = -3.709518$ Mrad/s
$s_{p4} = -3.0588543 - j\,2.222387$ Mrad/s
$s_{p5} = -1.1683784 - j\,3.521154070589589$ Mrad/s

b) Solving for A_{min}: $N = \frac{\log\left(\frac{10^{0.1A_{min}} - 1}{10^{0.1A_{max}} - 1}\right)}{2\log\left(\frac{\omega_s}{\omega_c}\right)}$ for $A_{min} \Rightarrow A_{min} < 39.03$ dB

2.12 $A_{max} = 3.01$ dB yields $\varepsilon = \sqrt{10^{0.1A_{max}} - 1} = 1$ and $r_{p0} = \omega_c \varepsilon^{-1/N} = 2\pi\, 10^6$ rad/s. The poles are

$s_{pk} = r_{p0}\left(\cos\left(\frac{\pi(N+2k-1)}{2N}\right) + j\sin\left(\frac{\pi(N+2k-1)}{2N}\right)\right)$ and we get using the following program

```
N = 5;  Amax = 3.01;
epsilon = sqrt(10^(0.1*Amax)-1);
epsilon = 1;                              % We select epsilon = 1
Wc = 2*pi*10^6;
rp0 = Wc*epsilon^(-1/N);
G = 5;
for k = 1:5
      sp = rp0*(cos(pi*(N+2*k-1)/(2*N)) + i*sin(pi*(N+2*k-1)/(2*N)))
      G = G*sp;
end
G
```

sp1 = –1.941611038725466e+06 + 5.975664329483111e+06i
sp2 = –5.083203692315260e+06 + 3.693163660980913e+06i
sp3 = –6.283185307179586e+06
sp4 = –5.083203692315260e+06 – 3.693163660980913e+06i
sp5 = –1.941611038725466e+06 – 5.975664329483111e+06i
G = –4.896314956564502e+34

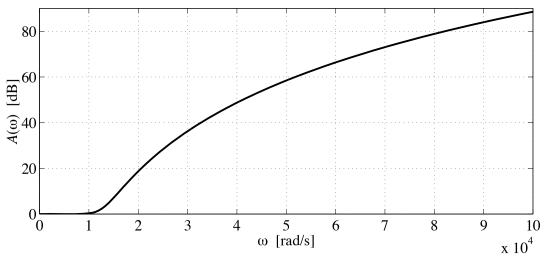
2.13 a)

```
Wc = 10e3;  Ws = 30e3; Amax = 0.3; Amin = 35;          % Specification
W = linspace(0, 10^5, 1000);                           % Frequency vector, ω-axis
t = linspace(0, 3e-3, 1000);                           % Time vector
%Butterworth
% Determine filter order
    Nbutt = buttord(Wc, Ws, Amax, Amin, 's');     %  (MATLABS Signal Processing Toolbox)
    Zbutt, Pbutt, Gbutt] = buttap(Nbutt);         % Find poles and zeros
    epsilon = sqrt(10^(0.1*Amax) - 1);            % Denormalize the poles and zeros with rp0
    rp0 = Wc*epsilon^(-1/Nbutt);
    Zbutt = Zbutt*rp0;
    Pbutt = Pbutt*rp0;
    Gbutt = Gbutt*(rp0^Nbutt);
    [Numbutt, Denbutt] = zp2tf(Zbutt, Pbutt, Gbutt);      % Compute the transfer function
    Hbutt = freqs(Numbutt,Denbutt,W);                     % Compute the frequency response
    tgbutt = groupdelay(Zbutt, Pbutt, Gbutt, W, Wc);      % Compute the group delay
    sys = zpk(Zbutt, Pbutt, Gbutt);
    hbutt = impulse(sys,t);                               % Compute the impulse response
    sbutt = step(sys,t);                                  % Compute the step response
```

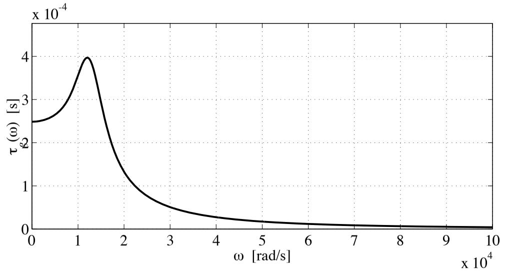
or better use our toolbox

```
Wc = 10e3;  Ws = 30e3; Amax = 0.3; Amin = 35;        % Specification
W = linspace(0, 10^5, 1000);                          % Frequency vector w-axis
t = linspace(0, 3e-3, 1000);                          % Time vector
N = BW_ORDER(Wc, Ws, Amax, Amin)
% Select an integer degree and re-run the program
        N = 5;
        [G, Z, P] = BW_POLES(Wc, Ws, Amax, Amin, N);
        Att = PZ_2_ATT_S(G, Z, P, W);
figure(1); subplot('position', [0.08 0.4 0.90 0.5]);
        PLOT_ATTENUATION_S(W, Att)

        Tg = PZ_2_TG_S(G, Z, P, W);
figure(2); subplot('position', [0.08 0.4 0.90 0.5]);
        PLOT_TG_S(W, Tg)

figure(3);   ymax = 10^5;   xmin = -4*10^4;   xmax = 10^4;
        PLOT_PZ_S(Z, P, Wc, Ws, xmin, xmax, ymax)

        [h, dirac0, t_axis] = PZ_2_IMPULSE_RESPONSE_S(G, Z, P, t);
        [s_of_t, t_axis] = PZ_2_STEP_RESPONSE_S(G, Z, P, t);

figure(4);    subplot('position', [0.08 0.4 0.90 0.5]);
        PLOT_IMPULSE_RESPONSE_S(h*10^-4, dirac0, t_axis)
        hold on
        PLOT_STEP_RESPONSE_S(s_of_t, t_axis);
```

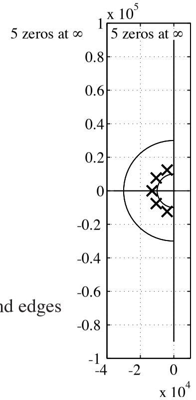
b)



c)

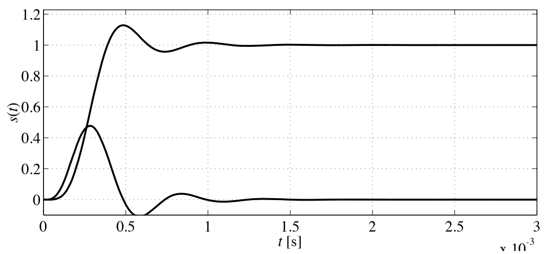


d)



The semi-circles has the same radii as the band edges

e) We have scaled the impulse response with a factor 10^{-5} in order to fit it into the same plot as $s(t)$.



2.14 a)**% Chebyshev I filter**

```
Ncheb1 = cheb1ord(Wc, Ws, Amax, Amin, 's');                   % Determine filter order
[Zcheb1, Pcheb1, Gcheb1] = cheb1ap(Ncheb1,Amax);              % Find poles and zeros
Zcheb1 = Zcheb1*Wc;                                           % Denormalize the poles and zeros
Pcheb1 = Pcheb1*Wc;
Gcheb1 = Gcheb1*(Wc^Ncheb1);
[Numcheb1, Dencheb1] = zp2tf(Zcheb1, Pcheb1, Gcheb1); % Compute the transfer function
Hcheb1 = freqs(Numcheb1,Dencheb1,W);                          % Compute the frequency response
tgcheb1 = groupdelay(Zcheb1, Pcheb1, Gcheb1, W, Wc);   % Compute the group delay
sys = zpk(Zcheb1, Pcheb1, Gcheb1);                            % Compute the impulse response
hcheb1 = impulse(sys,t);
scheb1 = step(sys,t);                                         % Compute the step response
```

or better use our toolbox

```
N = CH_ORDER(Wc, Ws, Amax, Amin)
% Select an integer degree
N = .......
[G, Z, P] = CH_I_POLES(Wc, Ws, Amax, Amin, N);
[h, dirac0, t_axis] = PZ_2_IMPULSE_RESPONSE_S(G, Z, P, t);
[s_of_t, t_axis] = PZ_2_STEP_RESPONSE_S(G, Z, P, t);
PLOT_IMPULSE_RESPONSE_S(h, dirac0, t_axis)
PLOT_STEP_RESPONSE_S(s_of_t, t_axis);
```

b)
c)
d)
e)

2.15 a)% **Chebyshev II filter**

```
Ncheb2 = cheb2ord(Wc, Ws, Amax, Amin, 's');              % Determine filter order
[Zcheb2, Pcheb2, Gcheb2] = cheb2ap(Ncheb2,Amin);         % Find poles and zeros
Zcheb2 = Zcheb2*Ws;                                      % Denormalize the poles and zeros
Pcheb2 = Pcheb2*Ws;
Gcheb2 = Gcheb2*(ws^(length(Pcheb2) - length(Zcheb2)));
[Numcheb2, Dencheb2] = zp2tf(Zcheb2, Pcheb2, Gcheb2);    % Compute the transfer function
Hcheb2 = freqs(Numcheb2,Dencheb2,W);                     % Compute the frequency response
tgcheb2 = groupdelay(Zcheb2, Pcheb2, Gcheb2, W, Wc);     %Compute the group delay
sys = zpk(Zcheb2, Pcheb2, Gcheb2);
hcheb2 = impulse(sys,t);                                 % Compute the impulse response
scheb2 = step(sys,t);                                    % Compute the step response
```

or better use our toolbox

```
N = CH_ORDER(Wc, Ws, Amax, Amin)
% Select an integer degree
N = .......
[G, Z, P] = CH_II_POLES(Wc, Ws, Amax, Amin, N);
[h, dirac0, t_axis] = PZ_2_IMPULSE_RESPONSE_S(G, Z, P, t);
[s_of_t, t_axis] = PZ_2_STEP_RESPONSE_S(G, Z, P, t);
PLOT_IMPULSE_RESPONSE_S(h, dirac0, t_axis)
PLOT_STEP_RESPONSE_S(s_of_t, t_axis);
```

b)
c)
d)
e)

2.16 a) % **Cauer filter**

```
Nca = ellipord(Wc, Ws, Amax, Amin, 's');            % Determine filter orde
[Zca, Pca, Gca] = ellipap(Nca, Amax, Amin);          %  Find poles and zeros
Zca = Zca*Wc;              % Denormalize the poles and zeros
Pca = Pca*Wc;
Gca = Gca*(Wc^(length(Pca) - length(Zca)));
[Numca, Denca] = zp2tf(Zca, Pca, Gca);               % Compute the transfer function
      Hca = freqs(Numcau,Dencau,W);                        % Compute the frequency response
tgca = groupdelay(Zca, Pca, Gca, W, Wc);             % Compute the group delay
sys = zpk(Zca, Pca, Gca);
hca = impulse(sys, t);                               % Compute the impulse response
sca = step(sys, t);                                  % Compute the step response
```

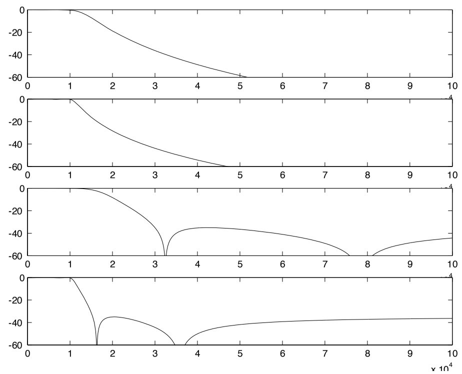
or better use our toolbox

```
N = CA_ORDER(Wc, Ws, Amax, Amin)
% Select an integer degree
N = .......
[G, Z, P] = CA_POLES(Wc, Ws, Amax, Amin, N);
[h, dirac0, t_axis] = PZ_2_IMPULSE_RESPONSE_S(G, Z, P, t);
[s_of_t, t_axis] = PZ_2_STEP_RESPONSE_S(G, Z, P, t);
PLOT_IMPULSE_RESPONSE_S(h, dirac0, t_axis)
PLOT_STEP_RESPONSE_S(s_of_t, t_axis);
```

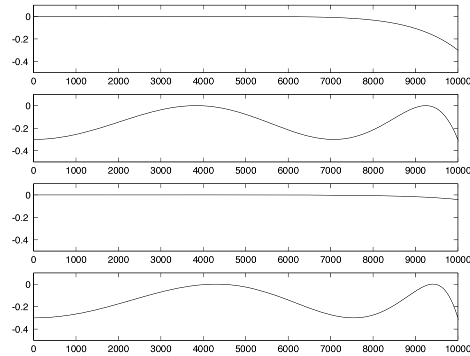
b)
c)
d)
e)

2.17

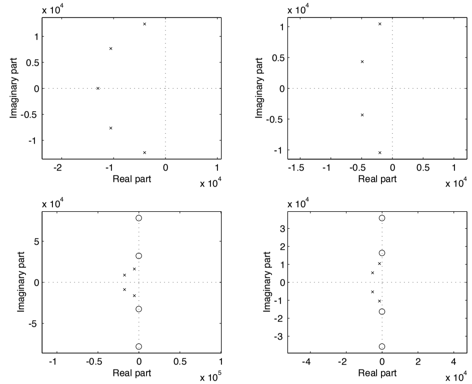
```
%Plotting
figure(1);
subplot(4, 1, 1)
plot(W, 20*log10(abs(Hbutt)))
axis([0 1e5 -60 0]);
subplot(4, 1, 2)
plot(W, 20*log10(abs(Hcheb1)))
axis([0 1e5 -60 0]);
subplot(4, 1, 3)
plot(W,20*log10(abs(Hcheb2)))
axis([0 1e5 -60 0]);
subplot(4, 1, 4)
plot(W, 20*log10(abs(Hca)))
axis([0 1e5 -60 0])
```



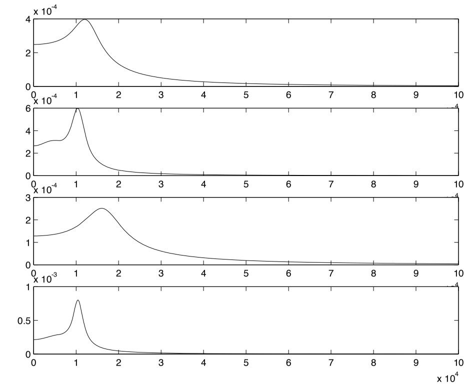

```
figure(2);
subplot(4, 1, 1)
plot(W, 20*log10(abs(Hbutt)))
axis([0 1e4 -0.5 0.1]);
subplot(4, 1, 2)
plot(W, 20*log10(abs(Hcheb1)))
axis([0 1e4 -0.5 0.1]);
subplot(4, 1, 3)
plot(W, 20*log10(abs(Hcheb2)))
axis([0 1e4 -0.5 0.1]);
subplot(4, 1, 4)
plot(W, 20*log10(abs(Hca)))
axis([0 1e4 -0.5 0.1])
```



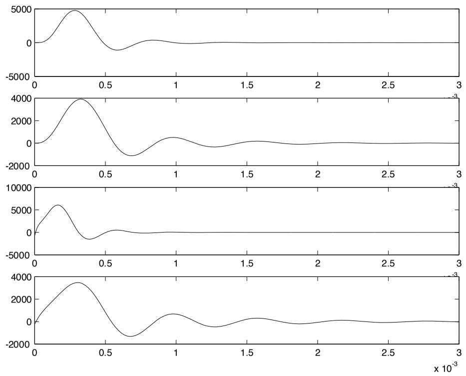
```
figure(3)
subplot(2, 2,1)
zplane(Zbutt, Pbutt)
subplot(2, 2, 2)
zplane(Zcheb1, Pcheb1)
subplot(2, 2, 3)
zplane(Zcheb2, Pcheb2)
subplot(2, 2, 4)
zplane(Zca, Pca)
```



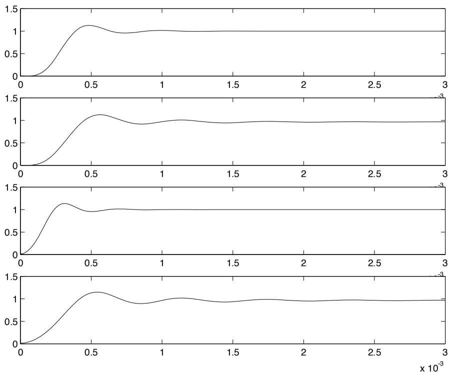

```
figure(4)
subplot(4, 1, 1)
plot(W, tgbutt)
subplot(4, 1, 2)
plot(W, tgcheb1)
subplot(4, 1, 3)
plot(W, tgcheb2)
subplot(4, 1, 4)
plot(W, tgca)
```



```
figure(5)
subplot(4, 1, 1)
plot(t, hbutt)
subplot(4, 1, 2)
plot(t, hcheb1)
subplot(4, 1, 3)
plot(t, hcheb2)
subplot(4, 1, 4)
plot(t, hca)
```



```
figure(6)
subplot(4, 1, 1)
plot(t, sbutt)
subplot(4, 1, 2)
plot(t, scheb1)
subplot(4, 1, 3)
plot(t, scheb2)
subplot(4,1, 4)
plot(t, sca)
```



2.18 $G = 1.9829574$ e+35 when $|H|_{max} = 1$ hence $G = -32 \cdot 1.9829574$ e+35 $= -6.34546368$ e+36

The difference in order of the numerator and denominator is N, hence, the rate of attenuation increase is $6N$ dB per octave (= doubling of the frequency).

2.19 a) $N = 12.6863$, select $N = 13$

$S_{p1,13} = -0.13935312021181373 \pm j\ 1.1476761992655875$
$S_{p2,12} = -0.4099606638167132 \pm j\ 1.0809774301975192$
$S_{p3,11} = -0.65674278378696671 \pm j\ 0.95145618207945792$
$S_{p4,10} = -0.8653574003264114 \pm j\ 0.76663975906144355$
$S_{p5,9} = -1.0236805902050516 \pm j\ 0.53726901986892006$
$S_{p6,8} = -1.1225111851085223 \pm j\ 0.27667415813504104$
$S_{p7} = -1.1561055100956283$

and

$s_{p1,13} = -521304.84286573727 \pm j\ 4293331.6441677595$
$s_{p2,12} = -1533618.1859958617 \pm j\ 4043818.8146342896$
$s_{p3,11} = -2456803.2146311956 \pm j\ 3559293.9342776011$
$s_{p4,10} = -3237207.7705485001 \pm j\ 2867915.8279677443$
$s_{p5,9} = -3829477.5776130003 \pm j\ 2009864.8781862874$
$s_{p6,8} = -4199192.0674512647 \pm j\ 1035007.8872462189$
$s_{p7} = -4324864.7777713826$

b) $N = 7.2035$, $N = 8$

$s_{p1,8} = -299677.12331403373 \pm j\ 4824385.7094520265$

$s_{p2,7} = -1142287.7541313658 \pm j\ 5474357.4519172823$
$s_{p3,6} = -3279485.7354346053 \pm j\ 7016952.7240264211$
$s_{p4,5} = -11032314.398128768 \pm j\ 7027140.4456866905$

c) $N = 7.2035$, $N = 8$

$s_{z1,8} = \pm j\ 2451963.2010080758$ $\quad s_{p1,8} = -36277.1442561826 \pm j\ 4030424.8910224694$
$s_{z2,7} = \pm j\ 2078674.0307563632$ $\quad s_{p2,7} = -3811964.0054222203 \pm j\ 3416829.2372569158$
$s_{z3,6} = \pm j\ 1388925.5825490057$ $\quad s_{p3,6} = -3711795.7722573373 \pm j\ 2283052.3057531635$
$s_{z4,5} = \pm j\ 487725.80504032073$ $\quad s_{p4,5} = -3198781.2152561643 \pm j\ 801701.35661917937$

d) $N = 5.0659$, $N = 6$ which yields $A_{min} \approx 76$ dB

$s_{z1,6} = \pm j\ 691008.33742590621$ $\quad s_{p1,6} = -434803.44804969523 \pm j\ 4716842.7330782395$
$s_{z2,5} = \pm j\ 1830127.0189221932$ $\quad s_{p2,5} = -1991633.9707195507 \pm j\ 5433593.8896988165$
$s_{z3,4} = \pm j\ 2425882.8186241528$ $\quad s_{p3,4} = -7416883.3161327159 \pm j\ 5088834.5452645291$

2.20 a)
```
Amax = 1; Amin = 50; Wc = 1; Ws = 2;
NBW = BW_ORDER(Wc, Ws, Amax, Amin)
NCH = CH_ORDER(Wc, Ws, Amax, Amin)
NCA = CA_ORDER(Wc, Ws, Amax, Amin)
```
yields

NBW = 9.27950878963198
NCH = 5.41035662900978
NCA = 3.89077736917283

b) The following program yields

```
Amax = 1;  Amin = 50;   Wc = 1;
Ws = 1.01;  NBW = 10;
while NBW > 4
NBW = BW_ORDER(Wc, Ws, Amax, Amin);
Ws = Ws*1.001;
end                                          % Butterworth
Ws
```
Ws = 4.99856496273532

```
Ws = 1.01; NCH = 10;
while NCH > 4
NCH = CH_ORDER(Wc, Ws, Amax, Amin);
Ws = Ws*1.001;
end                                          % Chebyshev I and II
Ws
```
Ws = 3.05688584458323

```
Ws = 1.01; NCA = 10;
while NCA > 4
NCA = CA_ORDER(Wc, Ws, Amax, Amin);
Ws = Ws*1.001;
end                                          % Cauer
Ws
```
Ws = 1.91100932207615

2.21 We get using the programs below

Butterworth

```
Wc = 10^7; Ws = 25*10^6;
Amax = -10*log10(1 - 0.15^2); Amin = 60;
N = BW_ORDER(Wc, Ws, Amax, Amin)
N = 10;                  % Re-run the program after
                         % selecting an integer order
[G, Z, P] = BW_POLES(Wc, Ws, Amax, Amin, N)
xmax = 2*10^6; xmin = -3*10^7; ymax = 4*10^7;
PLOT_PZ_S(Z, P, Wc, Ws, xmin, xmax, ymax)
```

N = 9.59684078908066
G = 6.591239977761730e+70
P = 1.0e+07 *
–1.19265996029086 – 0.18889878030815i
–1.19265996029086 + 0.18889878030815i
–1.07591409362702 – 0.54820561217270i
–1.07591409362702 + 0.54820561217270i
–0.85385025914469 – 0.85385025914469i
–0.85385025914469 + 0.85385025914469i
–0.54820561217270 – 1.07591409362702i
–0.54820561217270 + 1.07591409362702i
–0.18889878030815 – 1.19265996029086i
–0.18889878030815 + 1.19265996029086i
All 10 zeros at $s = \infty$

Chebyshev I

```
Wc = 10^7;  Ws = 25*10^6;
Amax = -10*log10(1 - 0.15^2);  Amin = 60;
N = CH_ORDER(Wc, Ws, Amax, Amin)
N = 10;                  % Re-run the program after
                         % selecting an integer order
[G, Z, P] = CH_I_POLES(Wc, Ws, Amax, Amin, N)
xmax = 2*10^6; xmin = -3*10^7; ymax = 4*10^7;
PLOT_PZ_S(Z, P, Wc, Ws, xmin, xmax, ymax)
```

N = 6.05479197414172
G = 1.029881246525271e+48
P = 1.0e+07 *
–0.37767451592427
–0.34027298104788 – 0.46379676305258i
–0.34027298104788 + 0.46379676305258i
–0.23547620910072 – 0.83573288908858i
–0.23547620910072 + 0.83573288908858i
–0.08404048601497 - –1.04214186684933i
–0.08404048601497 + 1.04214186684933i

All 7 zeros at $s = \infty$

Chebyshev II

```
Wc = 10^7;  Ws = 25*10^6;
Amax = -10*log10(1 - 0.15^2);  Amin = 60;
N = CH_ORDER(Wc, Ws, Amax, Amin)
N = 7;                   % Re-run the program after
                         % selecting an integer order
[G, Z, P] = CH_II_POLES(Wc, Ws, Amax, Amin, N)
xmax = 2*10^6; xmin = -3*10^7; ymax = 4*10^7;
PLOT_PZ_S(Z, P, Wc, Ws, xmin, xmax, ymax)
```

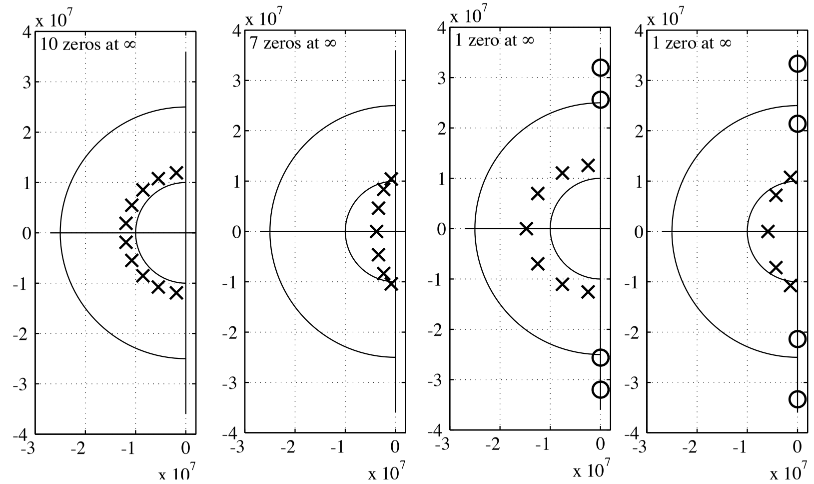
N = 6.05479197414172
G = 3.979874048506181e+04
Z = 1.0e+07 *
0 – 5.76191217740622i
0 + 5.76191217740622i
0 – 3.19762001922483i
0 + 3.19762001922483i
0 – 2.56429215818138i
0 + 2.56429215818138i
∞
P = 1.0e+07 *
–1.47642555719924
–1.24825511517649 – 0.69813016902340i
–1.24825511517649 + 0.69813016902340i
–0.75877259180592 – 1.10500666421834i
–0.75877259180592 + 1.10500666421834i
–0.24674031820480 – 1.25548408818160i
–0.24674031820480 + 1.25548408818160i

Cauer

```
Wc = 10^7;  Ws = 25*10^6;
Amax = -10*log10(1 - 0.15^2);  Amin = 60;
N = CA_ORDER(Wc, Ws, Amax, Amin)
N = 5;                   % Re-run the program after
                         % selecting an integer order
[G, Z, P] = CA_POLES(Wc, Ws, Amax, Amin, N)
xmax = 2*10^6; xmin = -3*10^7; ymax = 4*10^7;
PLOT_PZ_S(Z, P, Wc, Ws, xmin, xmax, ymax)
```

N = 4.50491210883645
G = 9.568928390354895e+04
Z = 1.0e+07 *
0 – 3.33398489680040i
0 + 3.33398489680040i
0 – 2.13849524487512i
0 + 2.13849524487512i
∞
P = 1.0e+07 *
–0.43060072446038 – 0.71909125474291i
–0.43060072446038 + 0.71909125474291i
–0.14055425087900 – 1.07446335622271i
–0.14055425087900 + 1.07446335622271i
–0.58966187555311

Note that zeros at s = ∞ ar not printed. The poles and zeros are shown below with Butterworth, Chebyshev I, Chebyshev II, and Cauer filters left to the right.



2.22

2.23 We have $N = -10\log(1+\omega^2\tau_1^2) + 10\log(1+\omega^2\tau_2^2) - 10\log(1+\omega^2\tau_3^2)$ which can be written $N = 20\log\left(\sqrt{\frac{1+\omega^2\tau_2^2}{(1+\omega^2\tau_1^2)(1+\omega^2\tau_3^2)}}\right) = 20\log\left(\frac{\sqrt{1+\omega^2\tau_2^2}}{\sqrt{1+\omega^2\tau_1^2}\sqrt{1+\omega^2\tau_3^2}}\right)$ which obviously equals $N = 20\log\left(\frac{|1+j\omega\tau_2|}{|1+j\omega\tau_1||1+j\omega\tau_3|}\right) \Rightarrow H(s) = \frac{G(1+s\tau_2)}{(1+s\tau_1)(1+s\tau_3)}$ where the real zero is at $s = -1/\tau_2$ and the two real poles are at $s = -1/\tau_1$ and $s = -1/\tau_3$, and the gain constant is $G =$ 97.9719702. Note that if the poles and zeros alternate on the real axis, as in this case, the corresponding transfer function can be realized by using a network consisting of only resistors and capacitors.

2.24

2.25

2.26

2.27 Transformation of the HP specification to the corresponding LP specification using $S = \omega_I^2/s$ and $\Omega = -\omega_I^2/\omega$ where we neglect the negative sign since the specification of the magnitude function is symmetric around $\omega = 0$. We get

$A_{max} = 1$ dB, $\Omega_c = \omega_I^2/70\ 10^6$

$A_{min} = 25$ dB, $\Omega_s = \omega_I^2/20\ 10^6$

We select $\omega_I^2 = 70$ Mrad/s to get a normalized LP specification in order to allow the use of standard tables which are normalized to $\Omega_c = 1$ and $\Omega_s = 70/20 = 3.5$. Necessary order according the Eq.(2.7) is

$$N \geq \frac{\log\left(\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}\right)}{2\log\left(\frac{\Omega_s}{\Omega_c}\right)} = \frac{\log\left(\frac{10^{2.5}-1}{10^{0.1}-1}\right)}{2\log(3.5)} = 2.836$$. We select here $N = 3$. The poles are obtained

from $S_k = R_{p0}\left(-\sin\left(\frac{\pi(2k-1)}{2N}\right) + j\cos\left(\frac{\pi(2k-1)}{2N}\right)\right)$ for $k = 1, ..N$

where $R_{p0} = \omega_c \varepsilon^{-1/N} = 1.25257639$ and $\varepsilon = \sqrt{10^{0.1A_{max}} - 1} = 0.50884714$ and all zeros are at $S = \infty$. We get

$S_{p1} = -0.62628819409051295 + j\ 1.0847629723453271$
$S_{p2} = -1.2525763881810263$
$S_{p3} = -0.62628819409051273 - j\ 1.0847629723453274$
$S_{p4} = -0.62628819409051228 - j\ 1.0847629723453276$
$S_{p5} = -1.2525763881810263$
$S_{p6} = -0.62628819409051384 + j\ 1.0847629723453267$

We select only the poles in the left-hand half of the *s*-plane

$S_{p1} = -0.62628819409051295 + j\ 1.0847629723453271$
$S_{p2} = -1.2525763881810263$
$S_{p3} = -0.62628819409051273 - j\ 1.0847629723453274$

We map the LP poles to the corresponding HP poles using

$s = \frac{\omega_I^2}{S} = \omega_I^2 \frac{(a - jb)}{a^2 + b^2}$ where $S = a + jb$ yields

$s_{p1} = -27.942407609029331 - j\ 48.397669664638016$ Mrad/s
$s_{p2} = -55.884815218058684$ Mrad/s
$s_{p3} = -27.942407609029367 + j\ 48.397669664637995$ Mrad/s

Mapping the LP zeros to the corresponding HP zeros using $s_z = \omega_I^2/S_z$ yields $s_z = 0$, $n = 1, 2, 3$ since there are 3 zeros at $S = \infty$ in the LP filter. The transfer function of the LP is

$$H_{LP}(S) = \frac{G}{(S - \sigma_0)(S^2 - 2\sigma_1 S + r_1^2)} = \frac{3911.937}{(S + 1.252576)(S^2 + 1.252576\ S + 3123.113)}$$ where $G = -\sigma_0 r_1^2$ Inserting $s = \omega_I^2/S$ gives

$$H_{HP}(s) = \frac{G}{\left(\frac{\omega_I^2}{s} - \sigma_0\right)\left(\left(\frac{\omega_I^2}{s}\right)^2 - 2\sigma_1\frac{\omega_I^2}{s} + r_1^2\right)} = \frac{Gs^3}{(\omega_I^2 - \sigma_0 s)(\omega_I^4 - 2\sigma_1\omega_I^2 s + r_1^2 s^2)}$$

$$= \frac{s^3}{(s + 55.88482)(s^2 + 0.0280746\ s + 1.568948)}$$

2.28

2.29 The zeros are $s_{z1,2} = \pm 5$ rad/s, $s_{z3,4} = \pm 2 \pm j$ rad/s, which is OK, but the poles are: $s_{p1,2,3,4} = \pm 1 \pm 2j$ and $s_{p5} = 5$ rad/s. Hence, there are three poles in the right-hand half of the *s*-plane, which makes the filter unstable. Furthermore, there are six zeros and only five poles. Hence the filter is non-causal. The filter has complex coefficients and can therefore not be realized using standard methods.

2.30

2.31 We select $\omega_I = \omega_c = 5.5$ Mrad/s and get $\Omega_c = \omega_c$ and $\Omega_s = \omega_I^2/\omega_s = 8.6428 \cdot 10^6$ and $\Omega_s/\Omega_c = 1.57142857$. The required order for the Cauer filter is $N = 4.9293$ and we select $N = 5$. We modify the

program in Example 2.7 as shown below.

```
Wc = 5.5*10^6;  Ws = 3.5*10^6;                    % Requirements for the highpass filter
Amax = -10*log10(1-0.15^2);   Amin = 45;
WI = Wc;                         % We select the transformation angular frequency WI = Wc
Omegac = Wc;                                               % Omegac = WI^2/Wc = Wc
Omegas = WI^2/Ws;
NLP = CA_ORDER(Omegac, Omegas, Amax, Amin)          % Synthesis of lowpass filter (Cauer)
NLP = 5;
[GLP, ZLP, PLP] = CA_POLES(Omegac, Omegas, Amax, Amin, NLP)
[Ghp, Zhp, Php] = PZ_2_HP_S(GLP, ZLP, PLP, WI^2)       % Transform LP to HP filter
N = NLP;                                             % LP and HP filter has the same order
subplot('position', [0.08 0.4 0.90 0.5]);
W = linspace(10, 10^7, 1000);
H = PZ_2_FREQ_S(Ghp, Zhp, Php, W);
Att = MAG_2_ATT(H);
PLOT_ATTENUATION_S(W, Att)
axis([10 10^7 0 60]);
```

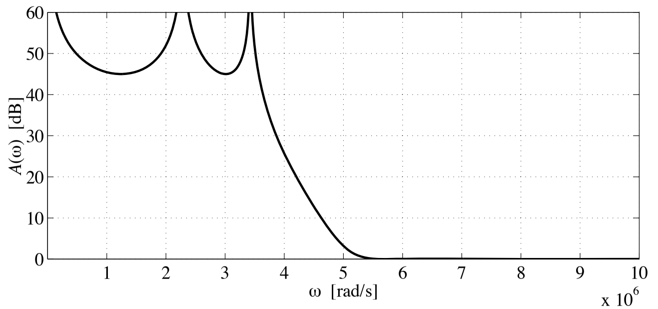
The normalized poles and zeros of the LP filter are
GLP = 2.091604896881714e+05

ZLP = 1.0e+07 *	PLP = 1.0e+06 *
0 – 1.32724942706404i	–2.31018843511470 – 4.22823690060639i
0 + 1.32724942706404i	–2.31018843511470 + 4.22823690060639i
0 – 0.88172205815098i	–0.64710842103623 – 5.87225193677382i
0 + 0.88172205815098i	–0.64710842103623 + 5.87225193677382i
∞	–3.53532051784512

The poles and zeros of the HP filter are
Ghp = –1

Zhp = 1.0e+06 *	Php = 1.0e+06 *
0 – 2.27914959940235i	–3.01026607427644 - 5.50955839897416i
0 + 2.27914959940235i	–3.01026607427644 + 5.50955839897416i
0 – 3.43078634818730i	–0.56085548311644 – 5.08954077850938i
0 + 3.43078634818730i	–0.56085548311644 + 5.08954077850938i
0	–8.55650848269855

The attenuation for the HP filter is shown below.



2.32

```
Wc = 10e3;  Ws = 6e3; Amax = 0.3; Amin = 35;
% HP filter! Transform
      Oc = Wc^2/Wc; Os = Wc^2/Ws;
      Ncheb = cheb1ord(Oc, Os, Amax, Amin, 's')      %Order Chebyshev I
      Ncauer = ellipord(Oc, Os, Amax, Amin, 's')     %Order Cauer
% Poles and zeros Chebyshev I
      [ZchebLP, PchebLP, GchebLP]= cheb1ap(Ncheb, Amax);
```

```
        ZchebLP = Oc*ZchebLP    % Denormalize Chebyshev I
        PchebLP = Oc*PchebLP
        [Zcheb, Pcheb] = zp2hp(ZchebLP, PchebLP, wc^2)       % Transform to HP Chebyshev I
%Poles and zeros Cauer
        [ZcauerLP, PcauerLP, GcauerLP]= ellipap(Ncauer, Amax, Amin);
        ZcauerLP = Oc*ZcauerLP    % Denormalize Cauer
        PcauerLP = Oc*PcauerLP
        [Zcauer, Pcauer] = zp2hp(ZcauerLP, PcauerLP, wc^2)   % Transform to HP Cauer

%Plot poles and zeros
        figure; subplot(1,2,1)
        hold on
        plot(Zcheb, 'o')
        plot(Pcheb,'x')
        axis image
        hold off
        title('Chebyshev I')
        subplot(1,2,2)
        hold on
        plot(Zcauer, 'o')
        plot(Pcauer, 'x')
axis image
        hold off
        title('Cauer')

% Determine the transfer function Chebyshev I
        [NumCheb, DenCheb] = zp2tf(Zcheb, Pcheb, 1)       % POOR ACCURACY RUTINES
% Determine the transfer function Cauer
        [NumCauer, DenCauer] = zp2tf(Zcauer, Pcauer,1)
```

Rewrite the program above by instead using the toolbox!

2.33 We have for a Butterworth filter $|H_{LP}(j\omega)|^2 = \dfrac{1}{1+\varepsilon^2\left(\dfrac{\omega}{\omega_c}\right)^{2N}}$. The corresponding HP filter is obtained by the transformation $S = \dfrac{\omega_I^2}{s}$, i.e., $\Omega = -\dfrac{\omega_I^2}{\omega}$ and by selecting $\omega_I^2 = \omega_c^2$ we obtain $|H_{HP}(j\omega)|^2 = \dfrac{1}{1+\varepsilon^2\left(\dfrac{-\omega_c}{\omega}\right)^{2N}}$. The two filters have the same gain at the crossover frequency, ω_c, i.e., $|H_{LP}(j\omega_c)|^2 = |H_{HP}(j\omega_c)|^2 = 0.5$ Hence, we must have $\varepsilon = 1$, which yields $|H_{LP}(j\omega)|^2 + |H_{HP}(j\omega)|^2 = \dfrac{1}{1+1\left(\dfrac{\omega}{\omega_c}\right)^{2N}} + \dfrac{1}{1+1\left(\dfrac{-\omega_c}{\omega}\right)^{2N}} =$

$$= \frac{2+\left(\frac{\omega_c}{\omega}\right)^{2N}+\left(\frac{\omega}{\omega_c}\right)^{2N}}{\left(1+\left(\frac{\omega}{\omega_c}\right)^{2N}\right)\left(1+\left(\frac{\omega_c}{\omega}\right)^{2N}\right)} = \frac{2+\left(\frac{\omega_c}{\omega}\right)^{2N}+\left(\frac{\omega}{\omega_c}\right)^{2N}}{2+\left(\frac{\omega_c}{\omega}\right)^{2N}+\left(\frac{\omega}{\omega_c}\right)^{2N}} = 1$$

. The two filters, which are power complementary, can be used as crossover filters in an audio system.

2.34 We have for the LP-BP transformation

$$S = s + \frac{\omega_I^2}{s} \qquad i\Omega = i\omega + \frac{\omega_I^2}{i\omega} \qquad \Omega = \frac{\omega^2+\omega_I^2}{\omega} \text{ and } \frac{\partial\Omega}{\partial\omega} = \frac{\omega^2+\omega_I^2}{\omega^2} = 1+\frac{\omega_I^2}{\omega^2}$$

From Eq.(3.5)we get $\tau_{gHP}(\omega) = \left(1+\dfrac{\omega_I^2}{\omega^2}\right)\tau_{gLP}(\Omega) = \tau_{gLP}(\Omega) + \dfrac{\omega_I^2}{\omega^2}\tau_{gLP}(\Omega)$.

Hence, the mapping of the group delay of the LP filter has two components. Since a the group delay of

a typical lowpass filter has a peak at the passband edge there will be two peaks in the BP filters group delay; one at each band edge. The second component is weighted with the factor ω_I^2/ω^2, which is large for low frequencies. Hence, the peak at the lower band edge is larger than that at the higher band edge.

2.35 First, we map the BP specification to the corresponding LP specification.
The geometric requirement is in this case satisfied since
$\omega_{c1}\,\omega_{c2} = 6 \cdot 8.5 = 51$ (krad/s)^2 = $\omega_{s1}\,\omega_{s2} = 2 \cdot 25.5 = 51$ (krad/s)^2
We get
$\Omega_c = \omega_{c2} - \omega_{c1} = 8.5 - 6 = 2.5$ krad/s
$\Omega_s = \omega_{s2} - \omega_{s1} = 25.5 - 2 = 23.5$ krad/s
Nomogram or MATLAB with $\omega\Omega_s/\omega\Omega_c = 9.4 \Rightarrow N = 3$
Normalized poles: (Approximately!)
$S_{pLP}1 = -0.4941$
$S_{pLP2} = -0.2470+j0.9659$
$S_{pLP3} = -0.2470-j0.9659$

Denormalization of Chebyshev I filter are done by multiplication with $\Omega_c = 2.5$ krad/s.
Denormalized poles:
$S_{pLP}1 = -1.2352$ krad/s
$S_{pLP2} = -0.6175 + j2.4148$ krad/s
$S_{pLP3} = --0.6175 - j2.4148$ krad/s

and three zeros at infinity. (There are always equal numbers of poles and zeros.)

The LP transfer function is $H_{LP}(S) = \dfrac{G}{(S-S_{pLP1})(S-S_{pLP2})(S-S_{pLP3})}$

LP toBP transformation $LP \to BP \Rightarrow S \to s + \dfrac{\omega_I^2}{s} \Rightarrow$

The BP transfer function is $H_{BP}(S) = \dfrac{G}{\left(s+\dfrac{\omega_I^2}{s}-S_{pLP1}\right)\left(s+\dfrac{\omega_I^2}{s}-S_{pLP2}\right)\left(s+\dfrac{\omega_I^2}{s}-S_{pLP3}\right)}$

The bandpass poles can be found from equation: $S_{pBP} = \dfrac{S_{pLP}}{2} \pm \dfrac{1}{2}\sqrt{S_{pLP}^2 - 4\omega_I^2}$

2.36 First, we map the BP specification to the corresponding LP specification using $S = s + \omega_I^2/s$.
We have $A_{max} = -10\log(1-0.09) = 0.4$ dB, $A_{min1} = A_{min2} = 35$ dB,
The geometric requirement: $\omega_I^2 = \omega_{c2}\,\omega_{c1} = \omega_{s2}\,\omega_{s1} \Rightarrow 15 \cdot 10 = 150 \neq 32 \cdot 5 = 160$. We select to reduce: $\omega_{s2} = 30$ krad/s.

$\Omega_c = \omega_{c2} - \omega_{c1} = 15 - 10 = 5$ krad/s
$\Omega_s = \omega_{s2} - \omega_{s1} = 30 - 5 = 25$ krad/s

We select $\omega_I^2 = 5$ krad/s to get a normalized LP specification with $\Omega_c = 1$ and $\Omega_s = 25/5 = 5$. The order for the LP filter is according to Eq.(2.16)

$$N \geq \frac{\operatorname{acosh}\left(\sqrt{\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}}\right)}{\operatorname{acosh}\left(\frac{\omega_s}{\omega_c}\right)} = \frac{\operatorname{acosh}\left(\sqrt{\frac{10^{3.5}-1}{10^{0.04}-1}}\right)}{\operatorname{acosh}(5.4)} = \frac{\operatorname{acosh}(181.01593)}{\operatorname{acosh}(5)} = 2.57$$

where $\operatorname{acosh}(x) = x + \sqrt{x^2 - 1}$. We select here $N = 3$ and get the LP poles for the Chebyshev I filter

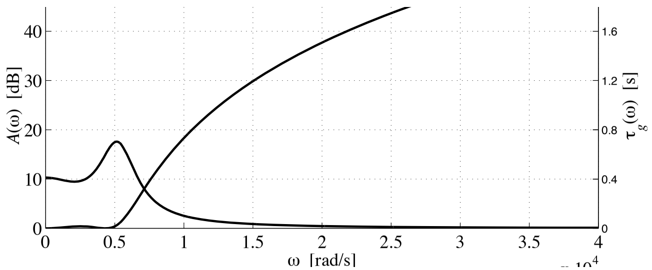
$$S_{pk} = -\omega\Omega_c a \sin\left((2k-1)\frac{\pi}{2N}\right) + j\omega\Omega_c b \cos\left((2k-1)\frac{\pi}{2N}\right) \text{ for } k = 1, 2, ..N. \text{ where}$$

$$a = \sinh\left(\frac{1}{N}\operatorname{asinh}\left(\frac{1}{\varepsilon}\right)\right),\ b = \cosh\left(\frac{1}{N}\operatorname{asinh}\left(\frac{1}{\varepsilon}\right)\right),\quad \varepsilon = \sqrt{10^{0.1A_{max}} - 1}.$$

We have $\varepsilon = 00.34931140018895$ and since $\operatorname{asinh}(x) = x + \sqrt{x^2 + 1}$ we get $a = 0.62645648634027$ and $b = 1.1800202240969$
Poles are
$S_{p1} = -1665.0493083673987+5202.6116132077605*i$
$S_{p2} = -3330.0986167347974$
$S_{p3} = -1665.0493083673987-5202.6116132077605*i$
and

$$H_{LP}(S) = \frac{G}{(S-\sigma_0)(S^2 - 2\sigma_1 S + r_1^2)} = \frac{9.9368667 \cdot 10^{10}}{(S + 3330.0986)(S^2 + 3330.0986\ S + 29839557)}$$

where G has been selected so that $H_{LP}(0) = 1$. All LP zeros are at $S = \infty$. The attenuation and group delay functions for the lowpass filter are shown below.



Mapping of LP poles and zeros are done according to $S = s + \frac{\omega_I^2}{s} = \frac{s^2 + \omega_I^2}{s}$ which can be rewritten

$s^2 - Ss + \omega_I^2 = 0$ and $s = \frac{S}{2} \pm \frac{\sqrt{(S^2 - 4\omega_I^2)}}{2}$. To compute the left hand side we proceeds as follows. For example, for a pole, $S_{p1} = -1665.0493 + j\,5202.61161$, we first compute the square root.

$$(S_{p1}^2 - 4\omega_I^2) = (-1665.0493 + j\ 5202.61161)^2 - 6 \cdot 10^8 = -6.2429487 \cdot 10^8 - j17325210$$

$$= 6.2453513 \cdot 10^8 \angle \operatorname{atan}\left(\frac{-17325210}{-624294870}\right) = 6.2453513 \cdot 10^8 \angle (0.027744528 + \pi)$$

where we must add π rad since the pole must be in the lhp. The BP poles are computed

$$s_{p1,2} = \frac{-1665.0493 + j\ 5202.61161}{2} \pm \frac{\sqrt{6.2453513 \cdot 10^8}}{2} \angle \frac{3.1693372}{2}$$

$$= -832.52465 + j\ 2601.3058 - \frac{\sqrt{623.86093}}{2} \angle \frac{3.1672499}{2}$$

$$= -832.52465 + j\ 2601.3058 - 12495.35 \angle 1.5846686$$

$$= -832.52465 + j\ 2601.3058 - 12495.35(\cos(1.5846686) + j\sin(1.5846686))$$

$$= -832.52465 + j\ 2601.3058 - (-173.33324 + j\ 12494.148)$$

$$= \begin{cases} -1005.8579 + j\ 15095.454 \\ -659.19141 - j\ 9892.8424 \end{cases}$$

Note that we get two BP poles (zeros) for every LP pole (zero). In the same way we get for $S_{p2} = -$

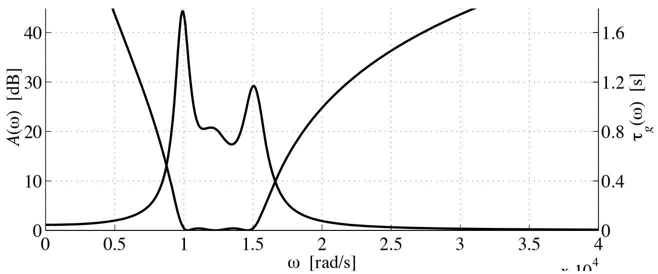
3330.0986 yields the bandpass poles $s_{p3,4} = \begin{cases} -1665.0493 + j\ 12133.739 \\ -1665.0493 + -j\ 12133.739 \end{cases}$ and

$S_{p3} = -1665.0493 + j\,5202.6161$ yields $s_{p5,6} = \begin{cases} -1005.8579 - j\ 15095.454 \\ -659.19141 - j9892.8424 \end{cases}$

An LP Chebyshev I have all zeros at $S = \infty$, each resulting in a zero at $s = \infty$ and $s = 0$. The numerator is therefore s^3 since the denominator has order 6. The transfer function is

$$H_{BP}(s) = \frac{9.9368667 \cdot 10^{10} s^3}{(s^2 + 1318.3828\ s + 98302864)(s^2 + 2011.7158\ s + 2.288845 \cdot 10^8)(s^2 + 3330.0986\ s + 1.5 \cdot 10^8)}$$

The attenuation and group delay is shown below.



2.37

2.38

2.39 We get

$\Omega_c = \omega_I^2/(\omega_{c2} - \omega_{c1}) = \omega_I^2/(51.5 - 48.5)2\pi$

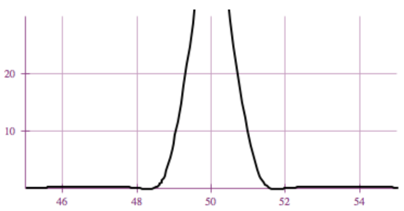
$\Omega_s = \omega_I^2/(\omega_{s2} - \omega_{s1}) = \omega_I^2/(50.5 - 49.5)2\pi$

The symmetry requirement yields: $\omega_{c2}\omega_{c1}$= 98607.22 (rad/s)^2 and $\omega_{s2}\omega_{s1}$= 98686.174 (rad/s)^2 We select to decrease ω_{s1} to $\omega_{s1} = \omega_{c2}\omega_{c1}/\omega_{s2}$ = 310.7688 rad/s and $\omega_I^2 = \omega_{c2}\omega_{c1}$= 98686.174 since we do not want to change the passband edges. We get Ω_c = 5235.464 and Ω_s = 15706.39.
Nomogram or MATLAB with Ω_s/Ω_c = 2.885714 => $N = 3$
Normalized poles for a Chebyshev I filter are:
$S'_{pLP1} = -0.45322183$
$S'_{pLP2,3} = -0.2266109 \pm j0.9508194$
Denormalizing with Ω_c = 5231.2754 yields
$S_{pLP1} = -3486.923$ rad/s
$S_{pLP2,3} = -1743.4613 \pm j5447.617$ rad/s

Transformation to a SB filter using $S = \frac{\omega_I^2 s}{s^2 + \omega_I^2}$, i.e., $s = \frac{\omega_I^2}{2S} \pm \sqrt{\left(\frac{\omega_I^2}{2S}\right)^2 - \omega_I^2}$ yields

$s_{pBS1,2} = -14.1509 \pm j313.6989$ rad/s (from the real LP pole)
$s_{pBS3,4} = -2.698297 \pm j322.3305$ rad/s
$s_{pBS5,6} = -2.560764 \pm j306.0238$ rad/s
The zeros of the SB filer are obtained from the 3 zeros of the LP filter, i.e., from $S = \infty$. Hence, we get 3 zeros at $s_{z1,2,3} = \pm j\omega_I = \pm j314.1436 = \pm j\ 49.9975 \cdot 2\pi$ rad/s
All together 6 zeros. The transfer function is

$$H_{BS}(s) = \frac{(s^2 + 98686.174)^3}{(s^2 + 5.121529s + 93657.13)(s^2 + 5.396536\ s + 103985.26)} \cdot$$
$$\cdot \frac{1}{(s^2 + 28.30180\ s + 98686.174)}$$



2.40

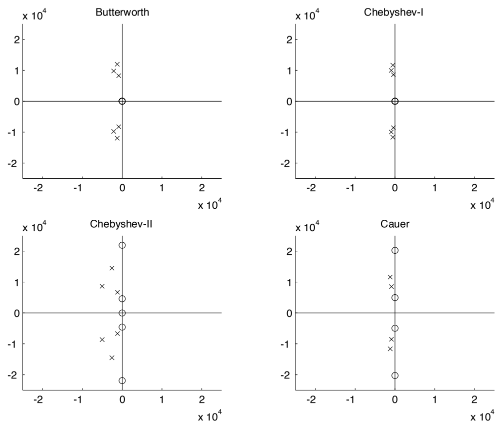
$N = 6$

$s_{z1,2,3} = 0$

a) $s_{p1,2} = -817 \pm j8270$

$s_{p3,4} = -1259 \pm j11959$

$s_{p5,6} = -2130 \pm j9771$



$N = 6$

$zs_{z1,2,3} = 0$

b) $s_{p1,2} = -399 \pm j8573$

$s_{p3,4} = -541 \pm j11639$

$s_{p5,6} = -940 \pm j9956$

$N = 6$

$s_{z1} = 0$

$s_{z2,3} = \pm j21889$

c) $s_{z4,5} = \pm j4569$

$s_{p1,2} = -1185 \pm j6678$

$s_{p3,4} = -2576 \pm j14519$

$s_{p5,6} = -5028 \pm j8644$

$N = 4$

$s_{z1,2} = \pm j4946$

d) $s_{z3,4} = \pm j20218$

$s_{p1,2} = -876 \pm j8518$

$s_{p3,4} = -1195 \pm j11616$

e)

2.41

2.42

2.43