RGB Filter design using the properties of the Weibull Manifold

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Abstract

Combining the channels of a multi-band image with the help of a pixelwise weighted sum is one of the basic operations in color and multispectral image processing. A typical example is the conversion of RGB- to intensity images. Usually the weights are given by some standard values or chosen heuristically. This does not take into account neither the statistical nature of the image source nor the intended further processing of the scalar image. In this paper we will present a framework in which we specify the statistical properties of the input data with the help of a representative collection of image patches. On the output side we specify the intended processing of the scalar image with the help of a filter kernel with zero-mean filter coefficients. Given the image patches and the filter kernel we use the Fisher information of the manifold of two-parameter Weibull distributions to introduce the trace of the Fisher information matrix as a cost function on the space of weight vectors of unit length. We will illustrate the properties of the method with the help of a database of scanned leaves and some color images from the internet. For the green leaves we find that the result of the mapping is similar to standard mappings like Matlab's RGB2Gray *weights. We then change the colour of the leaf using a global shift in the HSV representation of the original image and show how the proposed mapping adapts to this color change. This is also confirmed with other natural images where the new mapping reveals much more subtle details in the processed image. In the last experiment we show that the mapping emphasizes visually salient points in the image whereas the standard mapping only emphasizes global intensity changes. The proposed approach to RGB filter design provides thus a new methodology based only on the properties of the image statistics and the intended post-processing. It adapts to color changes of the input images and, due to its foundation in the statistics of extreme-value distributions, it is suitable for detecting salient regions in an image.*

Introduction

Channel combination of a multi-band image with the help of a pixelwise weighted sum, is one of the basic operations in colour and multispectral image processing. A typical example is the conversion from RGB to intensity images. Usually, the weights are given by some standard values or are chosen heuristically. This does not take into account the statistical nature of the image source, nor the intended further processing of the scalar image.

The standard selection of weights might not be optimal when the statistics of the input images deviate significantly from common situations. An example where this might be the case is automated inspection of input samples with very special visual properties, such as images of plants and especially of their leaves. This is of particular interest in applications where the growth of individual plants is monitored by a robotic system. Such a system has to locate the plant in a scene and extract relevant features from it. Apart from the boundary of the leaf, its texture and the structure of its veins give significant information about the conditions of the plant. It is therefore important to design optimized methods to extract such information. This can be difficult to achieve in practice since the colour properties of the leaves can vary significantly between different plants and can also be highly specific for some type of species.

The contributions in this paper are the following:

- We will present a framework, which combines the principles of group-theoretically designed filter systems, the statistical models of the extreme-value distributions (especially the two-parameter Weibull distributions) and the tools from information geometry to define a cost function on the space of filter design parameters, that allows us to design filter functions by an optimization or selection process.
- We demonstrate the properties of the optimization criterion in experiments where the only free parameters are the weight coefficients for the R,G and B combination.
- We show that the selection process leads to weight vectors that are useful in detecting salient regions in the image and that provide a more detailed description of the structure in the case of objects with a very narrow range of colours.

Filters and Weibull Distributions

In the following we will use filter functions based on the representation theory of the dihedral groups which are the symmetry groups of the square and the hexagonal grids. Specific details of the construction are described extensively in [6, 7]. One property that is important here, is the fact that these filter systems consist of orthonormal vectors and one of the filter vectors consists of constant coefficients only. From the orthogonality property, it follows that the sum of the filter coefficients of the non-constant filter functions is always zero. We will use only the simplest filter functions defined over a 3×3 neighbourhood, where it can be shown that in that case all the filter coefficients have either the value one, minus one or zero. They are therefore computed by additions and subtractions only.

Since the filter kernels consist of an equal number of ones and minus ones we can expect that a large proportion of the filter results will have a very small magnitude. Intuitively it is also clear that the large magnitude filter results indicate visually important events and that the distribution of these non-zero filter results should characterize the visual appearance. A important class of statistical distributions that describe non-negative valued stochastic variables are the extreme-value distributions. For these filter systems previous work [11, 8, 5, 2, 4] has shown that for a vast majority of images the distributions of the magnitude of these filter responses follow these extreme value distributions. In these studies the authors argued that without further a-priori knowledge it seems reasonable to assume that the R, G and B channels in colour images should be treated equally. Therefore the permutation invariant combination R+G+B was used there. For many image sources the three channels are obviously of different statistical nature and in the following we will thus use the construction of the weighted sum of the R, G and B channels as an example demonstrating an application of the theoretical framework to be described.

In this work, we follow the above mentioned research on derivative filters and sums of correlated variables, and choose to describe the statistical distributions of the filtered images by the 2-parameter Weibull distribution. The probability density function (pdf) of the (2-parameter) Weibull distribution is given by:

$$p(x,\{k,\lambda\}) = \frac{e^{-\left(\frac{x}{\lambda}\right)^k} k \left(\frac{x}{\lambda}\right)^{-1+k}}{\lambda} \quad (1)$$

where k is the shape and λ is the scale parameter. It is defined for positive values of x. The Weibull distribution and especially its 3-parameter variant [11, 10], have shown very good fitting performance with similar type of filtered data such as ours. The 3-parameter version has an extra degree of freedom (location parameter), which gives the flexibility of fitting to a larger range of filtered images. The disadvantage however is that the geometry of the 3-parameter Weibull becomes very complicated (e.g. the metric tensor vanishes) for any practical work to be carried out. As such, we have opted to first fit a 3-parameter Weibull using MLE [11], extract the location parameter and then subtract it from the data; effectively removing that extra degree of freedom since it is of little interest. This gives us a 2-parameter Weibull, with the same properties (scale and shape parameters) as in the 3-parameter case, but now we have closed form expressions for all the relevant components of the geometry of the Weibull manifold. We will briefly describe these components in the next section.

The Geometry of the Weibull Distribution

From equation (1) we observe that the Weibull distribution depends on two parameters, and we may consider every realisation from the same family as a point in the 2-dimensional Weibull space. These two parameters act as the coordinate vector of that point. In the framework of information geometry [1, 9], it is possible to consider the space of Weibull distributions as a manifold with a Riemmanian geometry, in which properties such as distances, angles and geodesics may be defined. We give here only an intuitive description of the necessary, basic concepts.

In Riemann geometry a manifold is a geometric object that looks locally like a flat Euclidean space. In the case of the Weibull distributions the manifold looks locally like a plane since it depends on two variables and has thus two dimensions. On the manifold one can define directional derivatives, which form the tangent space at this point. The geometry is defined by a metric on the tangent space at each point. This metric is given by a symmetric positive matrix whose elements are traditionally denoted by g_{ij}. In information geometry these are the elements of the Fisher information matrix and are computed as

$$g_{ij} = \int \frac{\partial \log p(x,\theta)}{\partial \theta_i} \frac{\partial \log p(x,\theta)}{\partial \theta_j} p(x,\theta)\, dx \quad (2)$$

where θ is the parameter vector of the distributions, $p(x,\theta)$ is the the pdf and the integral is computed over the range of the distribution. The terms $\frac{\partial \log p(x,\theta}{\partial \theta_i}$ measure how the pdf varies as a function of the parameters and the integral is the expectation of the product of these two partial derivatives. An equivalent expression is:

$$g_{ij} = \int \frac{-\partial^2 \log p(x,\theta)}{\partial \theta_i \partial \theta_j}) p(x,\theta)\, dx \quad (3)$$

which is the expectation of the second order partial derivative of the log-likelihood function $-\log p(x,\theta)$.

For the two-parameter Weibull distribution the parameter vector θ is given by the shape-scale pair $\{k,\lambda\}$ and the three elements in the matrix defining the metric are given in [3] as:

$$g_{11} = \frac{k^2}{\lambda^2} \quad (4)$$

$$g_{12} = \frac{\gamma - 1}{\lambda} \quad (5)$$

$$g_{22} = \frac{1 - 2\gamma + \gamma^2 + \pi^2/6}{k^2}, \quad (6)$$

where $\gamma \approx 0.577216$ is Euler's constant.

For the specific case of 2×2 metric tensors, their eigenvalues and eigenvectors as well as their combinations (e.g. trace) can be computed analytically. In addition, these entities can be computed with systems like Mathematica and we find for the trace the following expression:

$$\mathrm{tr}(g_{ij}) = \frac{6 + 6(-2+\gamma)\gamma + \pi^2 + \frac{6k^4}{\lambda^2}}{6k^2}. \quad (7)$$

In Figures 1 and 2 we show a few typical example pdfs for different parameter pairs of $\{k,\lambda\}$. We see that for lower values of shape and scale the mode of the distribution is near the origin whereas for combinations of high shape and scale the contributions are more concentrated away from the origin. Distributions with high scale and high shape values are therefore visually more detailed and interesting. If we consider a filter operation as a transformation from the original image to a scalar valued image, it seems reasonable to favor transformations that lead to high-scale/high-shape parameter pairs of the distributions of the absolute filter results.

The metric tensor of a 2-parameter Weibull distribution describing the local geometric properties around a point in the manifold, is a symmetric 2×2 matrix and therefore given by three elements. Geometrically, it is easier to describe its properties by the trace, its eigenvalues and the orientation of the first eigenvector. Here we choose as descriptor the trace (7) of the matrix. The

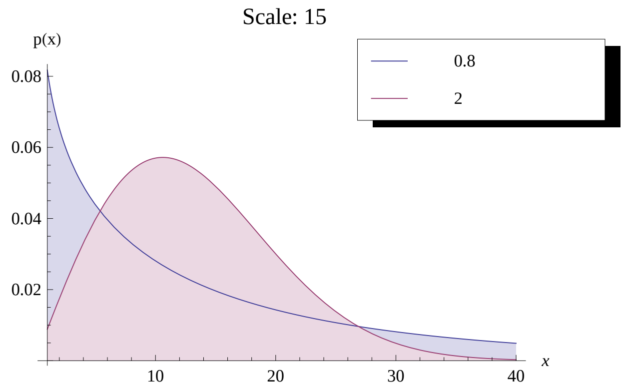

Figure 1. *Weibull Distributions with Scale = 1.5, Shape = 0.8 and 2.0*

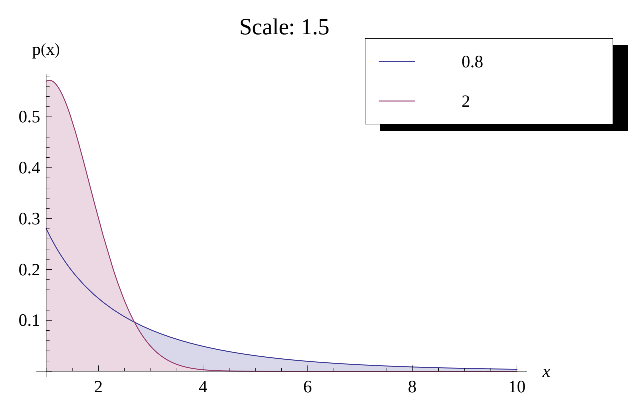

Figure 2. *Weibull Distributions with Scale = 15, Shape = 0.8 and 2.0*

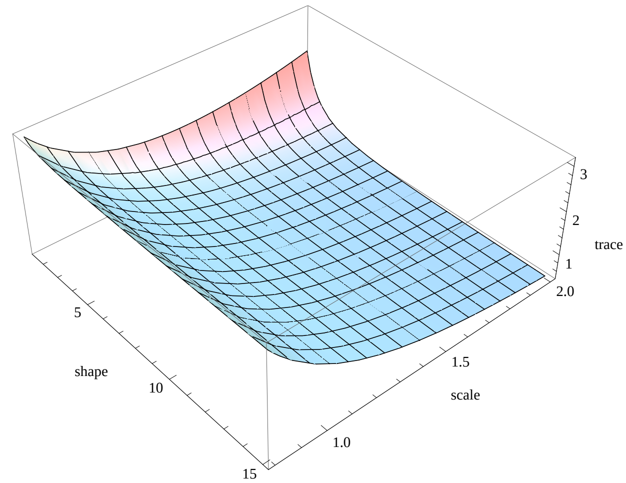

Figure 3. *Weibull distributions trace as a function of the two parameters, scale and shape. Small trace values imply large scale and shape combinations.*

plot of the trace in the region specified by the distributions in Figures 1 and 2, is illustrated in Figure 3. We see that low values of the trace imply high-scale/high-shape parameter pairs and as a result, we propose to select RGB weight vectors for which the fitted Weibull distribution has minimum trace.

Experiments

In this section we illustrate the properties of our approach by selecting RGB weight vectors that map RGB images to scalar valued images. We have generated a collection of 81 different unit vectors specifying the weight vectors. They represent 81 points on the upper half sphere. For a collection of 15 different types of leaves and all directions, we compute the filtered image, estimate the Weibull parameters and compute the corresponding trace. We consider each trace value as a vote for the corresponding weight vector. The accumulated votes (trace values) measure how good this weight vector performs for the whole class of leaves.

In Figure 5 we mark the positions on the upper half sphere with “×” and for selected points we show the accumulated votes. We also mark the position of the equal weight vector by “Identity” and the weight vector of the Matlab function RGB2Gray by “Rgb2gray”. We see that for the class of green leaves both weight vectors lie in a region of low votes which might explain their good performance for the leaf-images. After applying the RGB mapping we use the following pairs of edge filters and compute the magnitude of the resulting feature vector.

$$F1 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & -1 \end{pmatrix} \qquad F2 = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix} \quad (8)$$

Following that, we estimate the Weibull parameters and compute the trace. We then select the weight vector with the lowest trace.

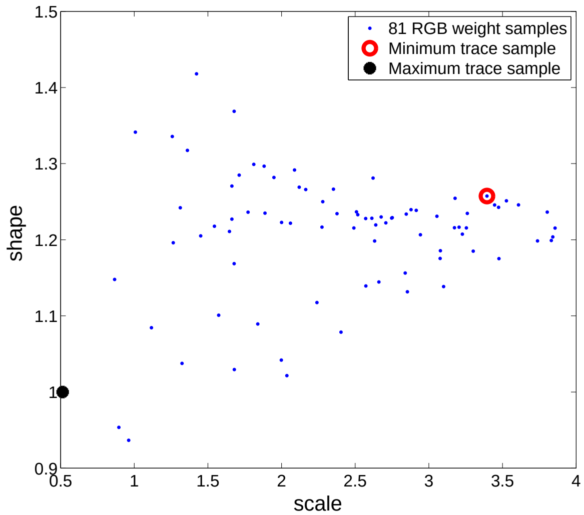

Figure 4. *The scatter plot of the 81 RGB weight samples in the Weibull (scale, shape) space. What is interesting to show is that the sample with the minimum trace has a large scale and shape parameters and the sample with the highest trace has the lower scale and shape parameters. Note that this plot is only a Euclidean approximation of the manifold used for illustration purposes only.*

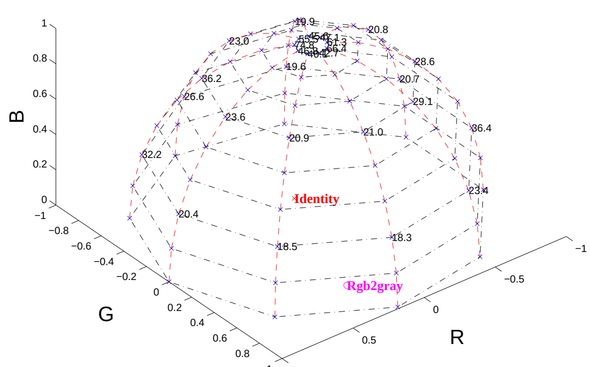

Figure 5. *81 weight RGB samples as points in the upper half sphere. This technique is used to determine the optimal mapping.*

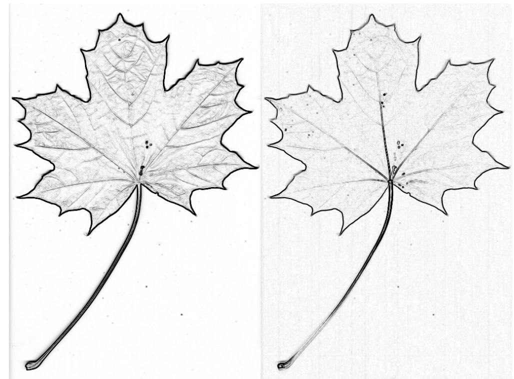

Figure 6. *Lowest (left) and highest (right) trace value mapping results.*

For large images (like the leaves) we optimized the mapping using a small patch of 64×64 pixels from the interior of the leaf. For the other, smaller images we used the full image. We present the results as images of the magnitude of the resulting filter vectors. These raw result images are then normalized to values between zero and one and shown as black-and-white images.

In Figure 6 we see the results where we used the weight vector with the lowest trace value to obtain the left image and the weight vector with the highest trace value to obtain the image on the right. Just as expected, we see that the lowest trace choice leads to a result with much more detailed information preserved, especially in the vein structure of the leaf.

The adaptivity of the selection process is illustrated in the next two Figures 7 and 8. Here we started with the original image of the leaf. We then apply a RGB2HSV transformation in Matlab and change the hue values by a common shift. The resulting image is transformed back to RGB via HSV2RGB. We then applied the weight vectors with the lowest trace to obtain the image on the left and a constant weight vector resulting in the image on the right. Again we see that the lowest-trace solution results in a much more detailed image.

A similar result, demonstrating the difference between adaptive weight selection and a fixed transformation, is shown in Figure 9. In these experiments we compare again the results of using the trace-based weight vector with the identity vector. We see the original image (left) together with the two filter results, one using the trace-vector (centre) and the other the identity vector (left), i.e. averaging of the RGB channels. The results are practically identical.

In Figure 10 we see the same type of experiments now applied to a part of a purple leaf. We see that the trace based mapping brings out much finer details than the identity mapping. We also include a scatter plot in the Weibull scale, shape space (Euclidean approximation) in Figure 4 of the 81 weight samples

Figure 7. *An example of an adjusted HSV-blue leaf that we have used of the comparison between the two mapping approaches.*

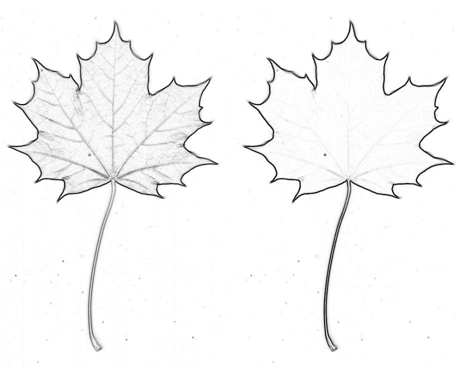

Figure 8. *RGB mappings of the HSV-blue leaf. The left is for our approach with a low trace where important vein information is preserved. On the right we show results from generic, constant grayscale conversion with loss of detail.*

taken. We see that, just like in Figure 3, low trace weights give higher scale and are as such more informative and preserve more of the finer details in the image.

Finally we show an example where the trace based weighting leads to a selection of salient parts of the image that are visually much more important than the mere intensity based differences. The original image is shown in Figure 11 (a), the trace-based result in Figure 11 (b) and the solution using Matlab's *RGB2Gray* weights in Figure 11 (c). Note that the *RGB2Gray* function is using the CCIR 601 luma weights, with the formula $Y' = 0.299R + 0.587G + 0.114B$. We see that the trace based image brings out the red details in the original image while the standard map concentrates on the global intensity differences.

Conclusions

We showed that the Fisher information matrix of the Weibull distribution provides a natural cost function which can be used to map RGB images to scalar valued images with great richness in detail. This approach has the advantage that on the input side it is driven by the image statistics and therefore adaptive and tuned to the input images under investigation. On the output side we assume that the result of the processing follows the two-parameter Weibull distribution which is often the case when the processing consists of a linear filtering with filters of zero-mean coefficients. In the current illustration we only selected the R, G, and B weight coefficients from a table of pre-defined unit vectors. Since the quality of the processing is defined in terms of the statistical properties of the processing results it is possible to generalize the procedure to an optimization process where the general form of the filter kernels can be learned from examples. The proposed optimality criterion can therefore be combined with the group theoretical filter design method which allows various combinations of the group theoretically defined filter systems that are all equally good regarding the group theoretical properties. Apart from the technological advantage of defining a cost function that can be used to derive filter functions from examples the proposed framework should also be helpful in analyzing properties of other vision systems, like those found in animals and humans, since it provides a statistically motivated characterization of the usefulness of low-level vision processes.

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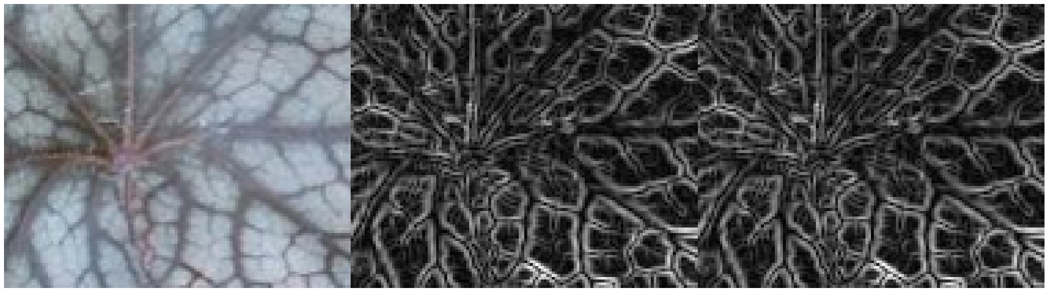


Figure 9. *SilverPatch. No significant differences between the two mappings and the same amount of details are preserved. Original image (left), trace-based mapping (middle) and identity mapping (right).*

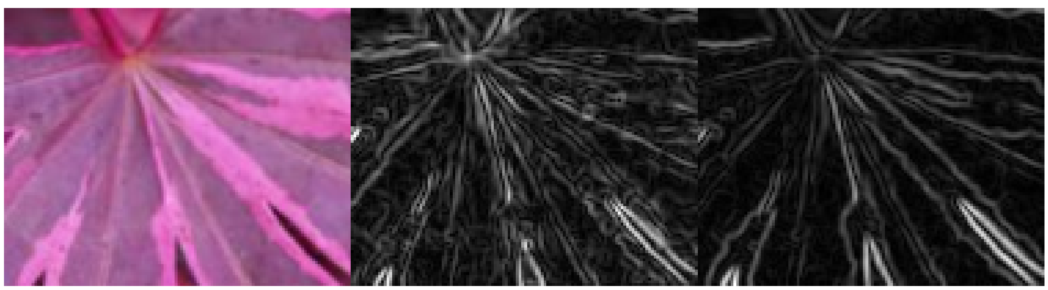
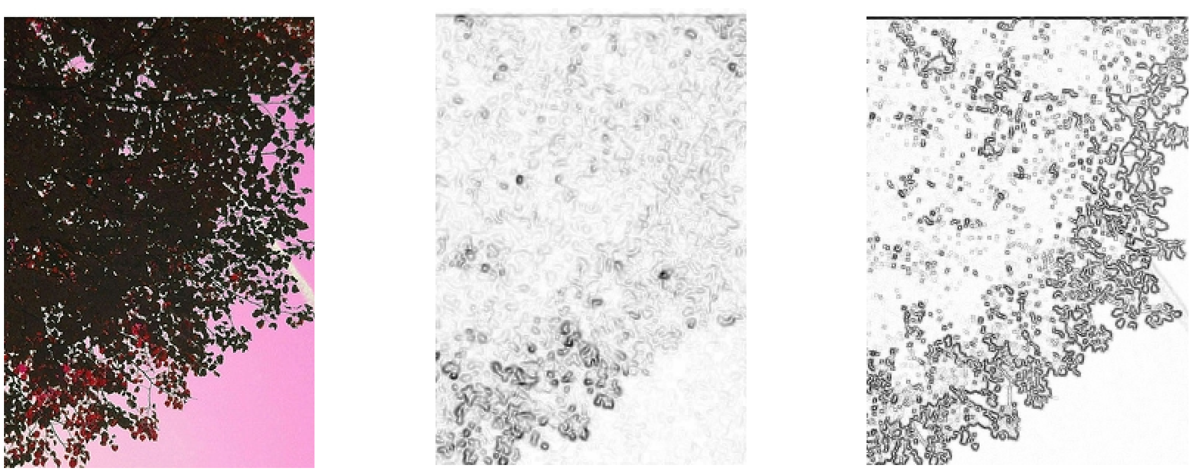


Figure 10. *Purple Patch. We can see that the Weibull trace mapping preserves much more details than the identity mapping. Original image (left), trace-based mapping (middle) and identity mapping (right).*



(a) (b) (c)

Figure 11. *An example where the trace-based mapping can be used for saliency enhancement. If we compare this with Matlab's* RGB2Gray *function we can see that while our method still retains all the main edges and outlines in the image, similar to* RGB2Gray*, we can also highlight colour-salient regions in the image (red structures in the tree). Note that the image colourmaps have been automatically scaled.*