Fast and accurate motion segmentation using Linear Combination of Views

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What is motion segmentation

"The task of separating a sequence of images into different regions, each corresponding to a distinct, consistent 3d motion"

More specifically

- 3d **rigid** motions
- **Sparse** features in **correspondence** across **all** frames

- Simplifications:
  - Weak perspective effects
  - Number of motions assumed known

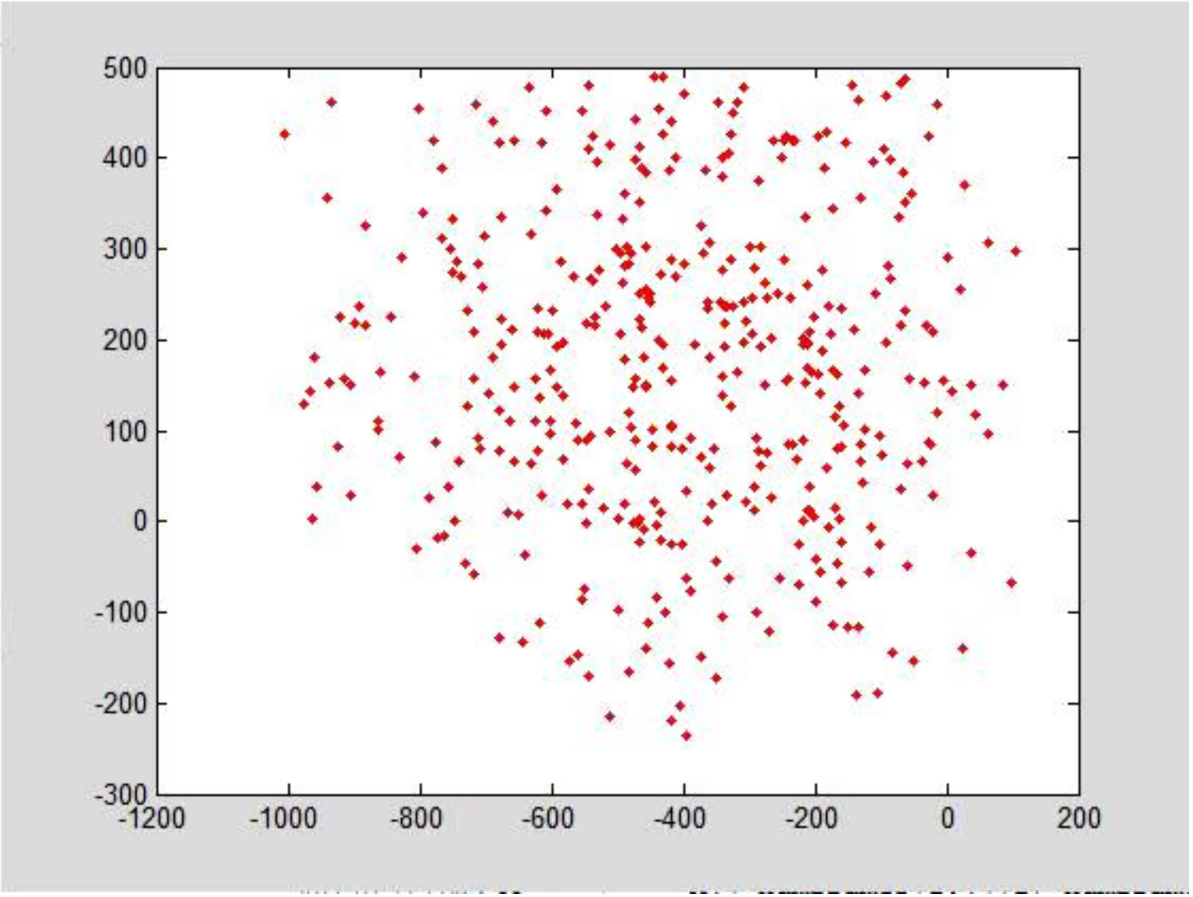
Applications of motion segmentation

- Tracking
- Multi-body structure from motion
- Navigation
- Activity / Anomaly detection
- Image semantic analysis

Why is it a difficult problem?

- Relying only on geometry (motion trajectories) can be difficult
- Humans rely also on secondary features (spatial, colour, texture)
- Such secondary features can aid segmentation
- But the core problem has to be solved in geometry

Example (motion trajectories only)



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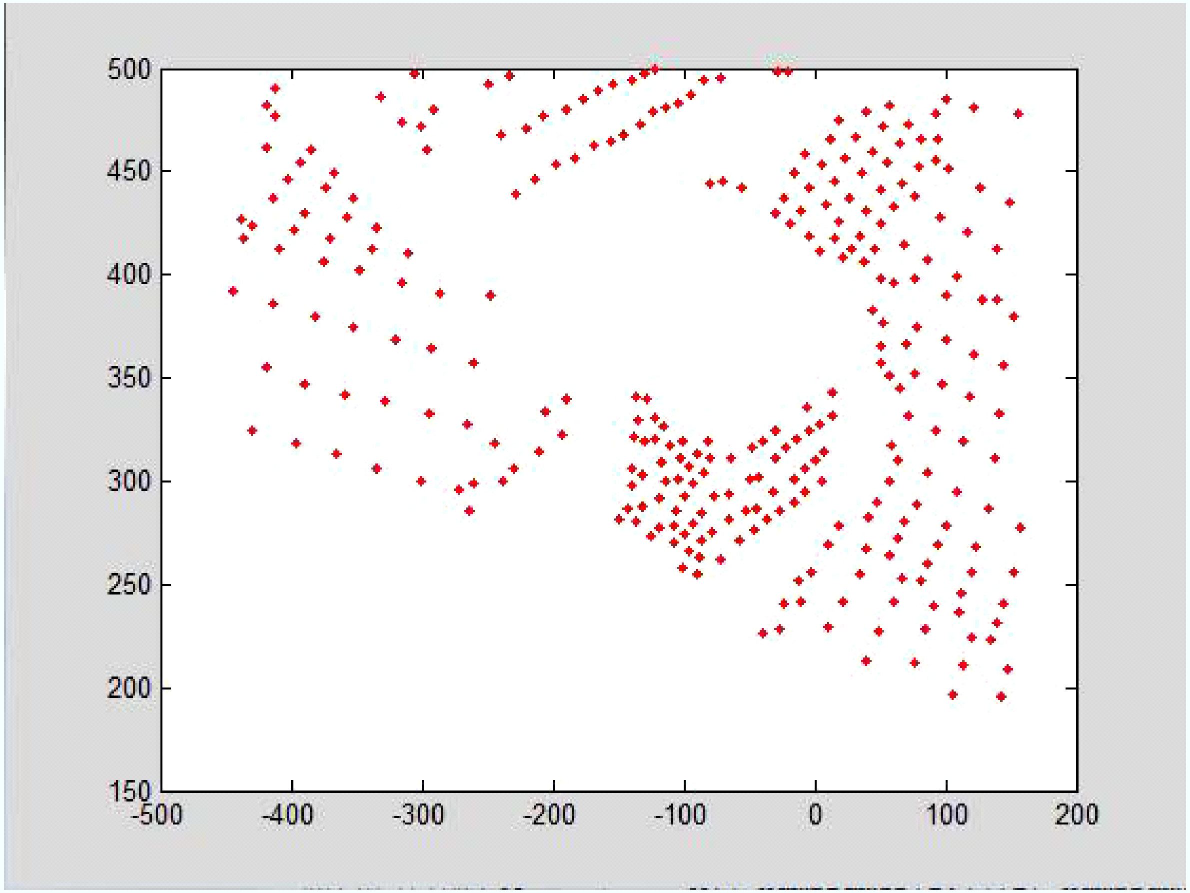
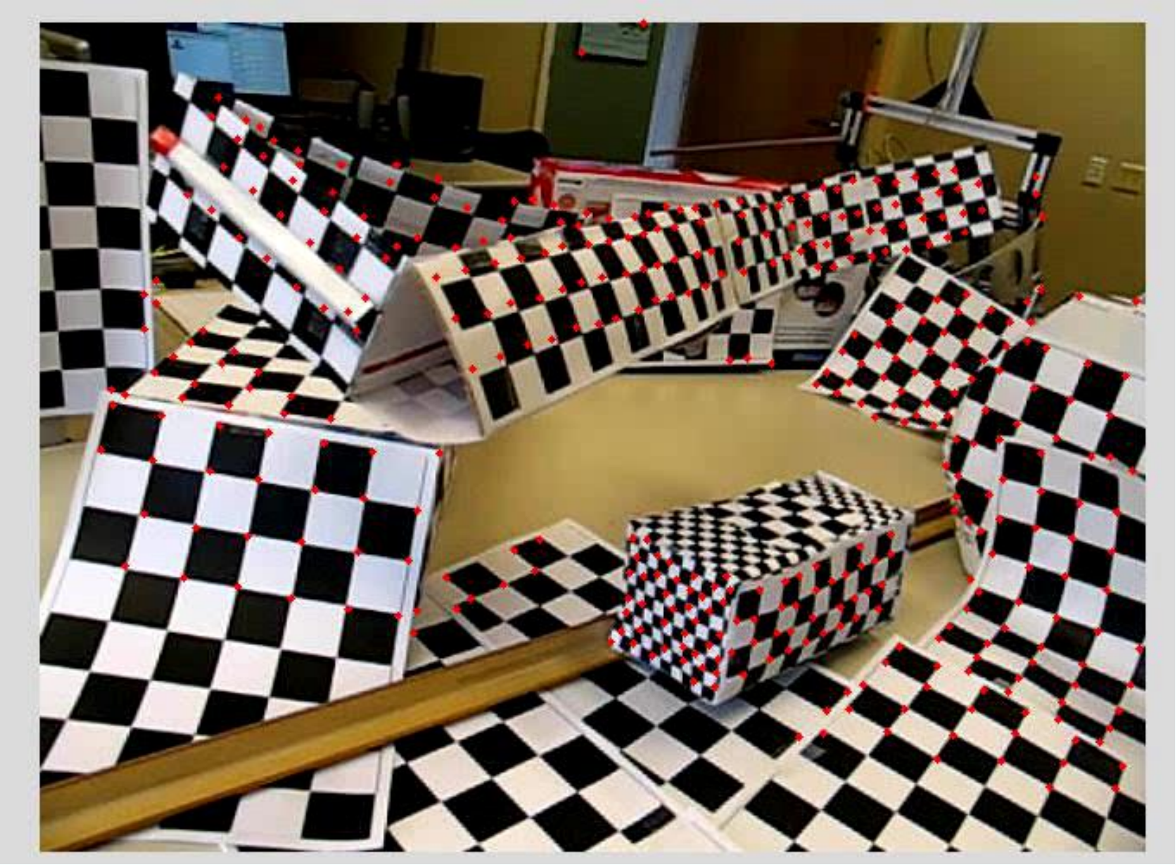
Example (including spatial configuration)



Example (All features)



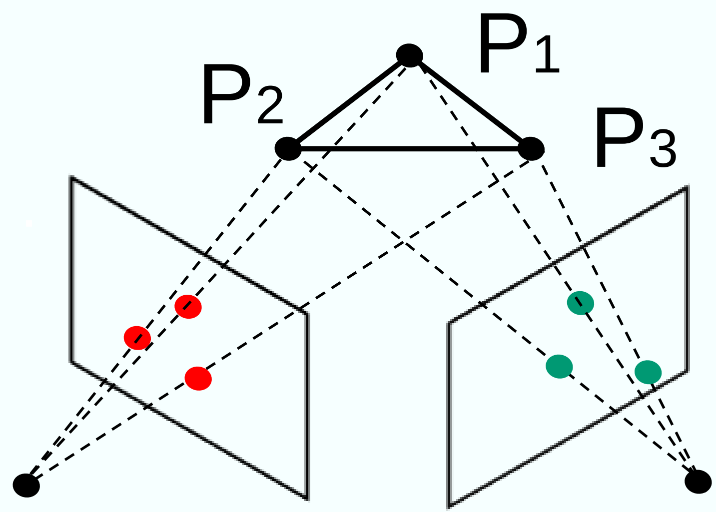
Current methods

- Most are subspace clustering methods that work in trajectory space
- Some employ spatial relations
- Differ on their definition of affinity
- Very slow or too complex

Multi-view geometry Affine camera

- 3d scene with N points on a single object

$$P_j = [X_j, Y_j, Z_j, 1]^T \quad j = 1, ..., N$$



- F images $i = 1, ..., F$
- Projected to the i[th] image $p_{ij} = m_i \; P_j$
- m_i is the 2x4 affine camera projection matrix

Multi-view geometry Affine camera: **The View**

- The set of N 2d points, in image i, define a ***view*** of the object:

$$v_i = m_i \ [P_1 \ P_2 \ ... \ P_N] = m_i S, \quad i = 1, ..., F$$

- The 4xN matrix S, is the 3d ***shape*** matrix of the object.

Multi-view geometry Affine camera: **The trajectory**

- Alternatively, a single 3d point projected onto all the images defines a trajectory:

$$t_j = \begin{bmatrix} m_1 \\ \vdots \\ m_F \end{bmatrix} P_j = M P_j, \quad j = 1, ..., N$$

- The 2Fx4 matrix M is the ***motion*** matrix

Multi-view geometry
Affine camera

- Combine everything into the 2FxN ***measurement*** matrix W:

$$W = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ y_1^1 & y_2^1 & \dots & y_N^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^F & x_2^F & \dots & x_N^F \\ y_1^F & y_2^F & \dots & y_N^F \end{bmatrix} = M\,S$$

- Also $\mathrm{rank}(W) \leq \min\,(\mathrm{rank}(M), \mathrm{rank}(S)) \leq 4$

The Tomasi-Kanade factorisation

- Use SVD to obtain a similar factorisation of W into **shape** x **motion**
- Provides basis vectors that span the R^{2F} space of W called the ***Joint Image Space*** (JIS)
- The JIS represents a connection between 2d and 3d where the camera parameters have been eliminated

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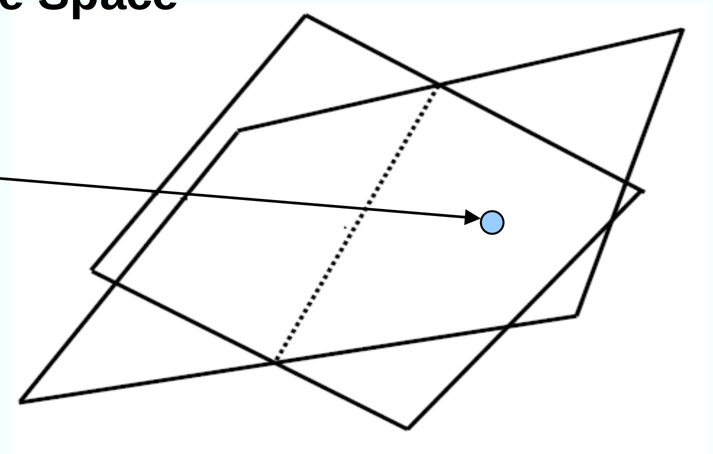
The T-K factorisation for motion segmentation

$$\begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_N^1 \\ y_1^1 & y_2^1 & \cdots & y_N^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^F & x_2^F & \cdots & x_N^F \\ y_1^F & y_2^F & \cdots & y_N^F \end{bmatrix}$$

Joint Image Space



- Rank 4 means a 4d linear subspace
- For k rigidly moving objects, their trajectories will lie in a union of k linear subspaces in R^{2F}
- Motion segmentation equates to subspace clustering

Ullman-Basri
Linear combination of views

- We can instead look at the row-space of W.
- Each row is a vector in R^N space called the ***Joint-Point-Space*** (JPS)

$$\begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ y_1^1 & y_2^1 & \dots & y_N^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^F & x_2^F & \dots & x_N^F \\ y_1^F & y_2^F & \dots & y_N^F \end{bmatrix}$$

Half-view

The JPS represents a connection between 2d and camera parameters where the 3d shape has been eliminated

Linear combination of views

- Given any 2 basis views v_1, v_2 we can reconstruct the 3d shape S by:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} S = M_{12}\, S \;\Rightarrow\; S = M_{12}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

- We are not interested in S explicitly but a third view can be synthesised by:

$$v_3 = m_3\, S = m_3\, M_{12}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Linear combination of views

- The inversion of M_{12} is valid if it has full rank. For the general case:

$$v_3 = Q \begin{bmatrix} 1 \\ v_1 \\ v_2 \end{bmatrix}, \quad Q = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 \end{bmatrix}$$

- This leads to the familiar equations:

$$x_j^3 = a_0 + a_1 x_j^1 + a_2 y_j^1 + a_3 x_j^2 + a_4 y_j^2$$
$$y_j^3 = b_0 + b_1 x_j^1 + b_2 y_j^1 + b_3 x_j^2 + b_4 y_j^2,$$

Linear combination of views

- This expression is valid for all the points on an object in the third view
- Q is not known but can be found from 5 or more corresponding points, visible in 3 views

$$\begin{bmatrix} x_1^3, \ldots, x_r^3 \\ y_1^3, \ldots, y_r^3 \end{bmatrix} = \underbrace{\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 \end{bmatrix}}_{Q} \begin{bmatrix} 1, \ldots, 1 \\ x_1^1, \ldots, x_r^1 \\ y_1^1, \ldots, y_r^1 \\ x_1^2, \ldots, x_r^2 \\ x_1^2, \ldots, x_r^2 \end{bmatrix}$$

LCV in summary

- Given **2 basis views** of an object
- And a set of LCV **coefficients** Q
- We can synthesise a **novel view** of the object
- Applications: Object recognition and view synthesis

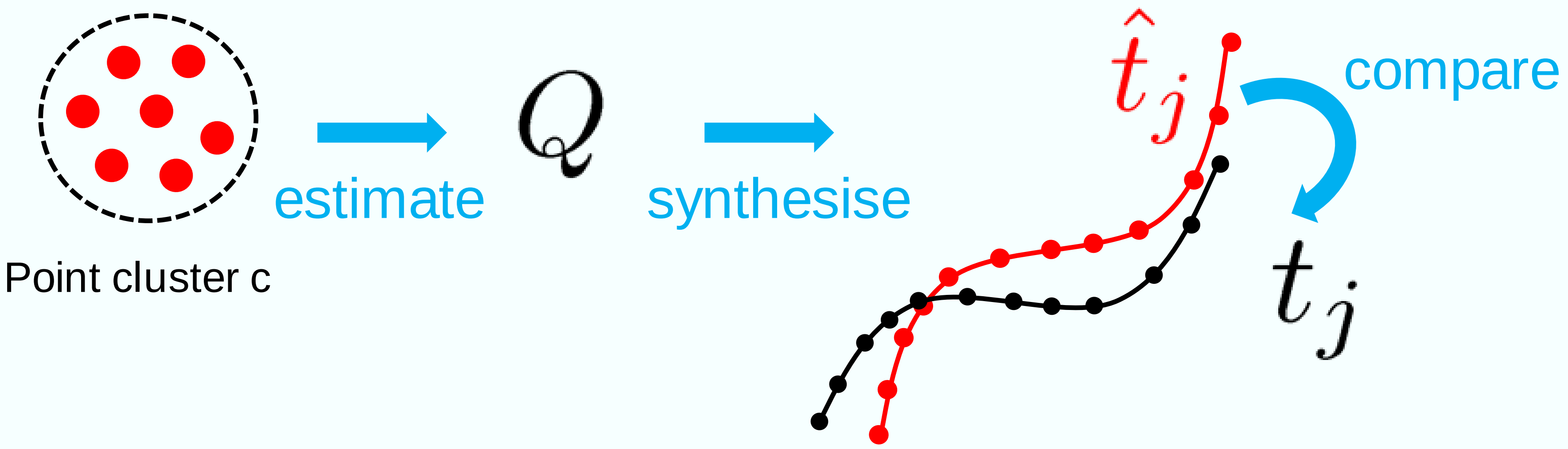
LCV for motion segmentation: What is required?

- Motion-based affinity (n-tuple or pairwise)
  - If pairwise then we can use standard clustering approaches (e.g. spectral clustering)
- Some clustering algorithm

LCV for motion segmentation: Simple concept

- Given a set Q of LCV coefficients we can synthesise a trajectory $\hat{t}_j$ of a point p_j and compare it with its real trajectory t_j
- We may then say something about p_j and the points used to estimate Q.

Illustration of concept

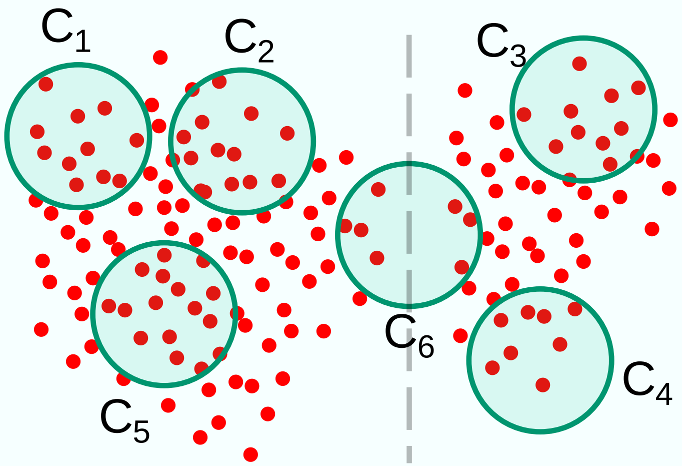


- If then $\hat{t}_j \approx t_j$ then Q describes well the motion of the point p_j
- Also if Q was estimated from points c, where $p_j \notin c$ then p_j and c probably lie on the same object

LCV for motion segmentation: 3-step algorithm outline

- Step 1: Motion hypotheses sampling
- Step 2: Trajectory synthesis + affinity generation
- Step 3: Clustering

Algorithm: Step 1 – Motion hypotheses sampling



Sample in image space. Any single frame will do.

Estimate by inverting a 5xn matrix for each

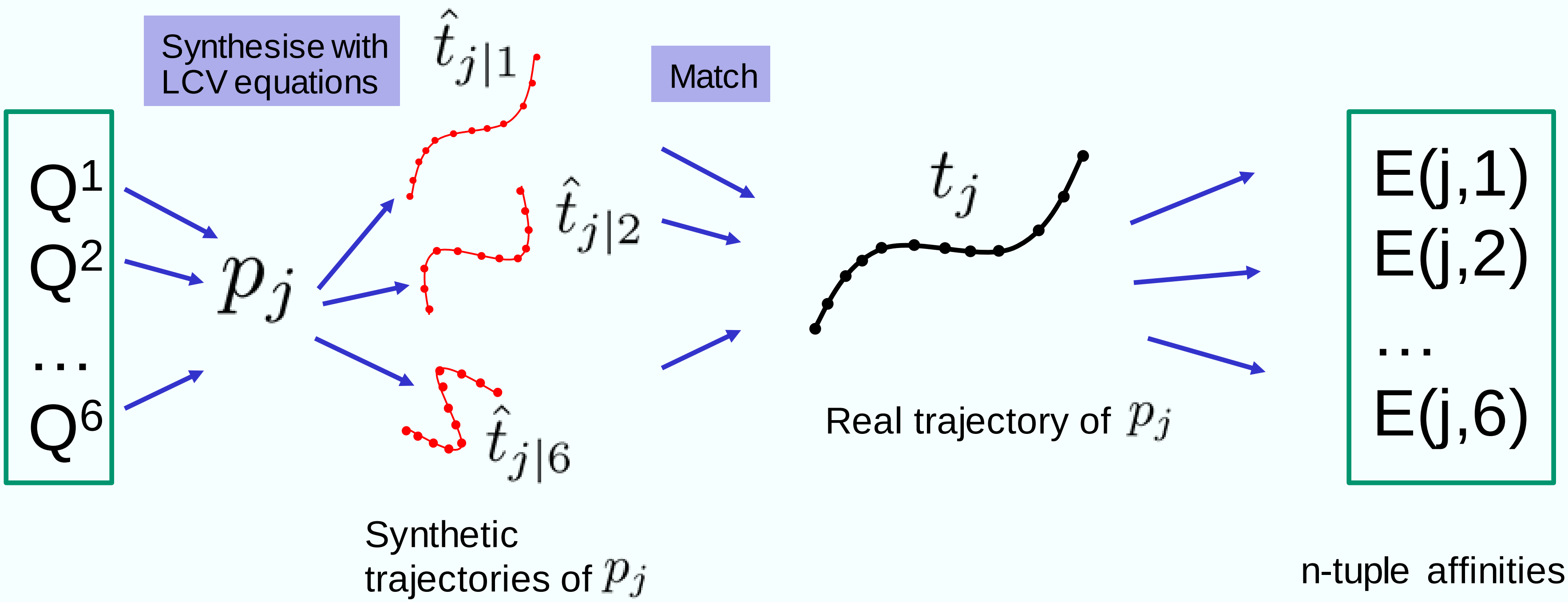
Q^1
Q^2
Q^3
$\dots$
Q^6

Each LCV coefficient matrix represents a "*motion hypothesis*".

They are not necessarily unique

- Using as basis views first and last frame of the sequence
- Sample C n-point clusters in any frame (e.g. n=7)
- n-closest point in Euclidean distance (spatial configuration implicit)
- Estimate the LCV coefficients Q^c

Algorithm: Step 2 – Trajectory synthesis and affinity matrix



Algorithm: Step 2 – Trajectory synthesis and affinity matrix

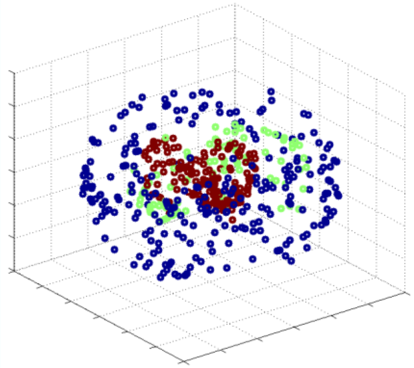
- The n-tuple affinity between the point p_j and the n-points c is defined as:

$$E(j,c) = K\left(\left\|t_j - \hat{t}_{j|c}\right\|_H / F\right)$$

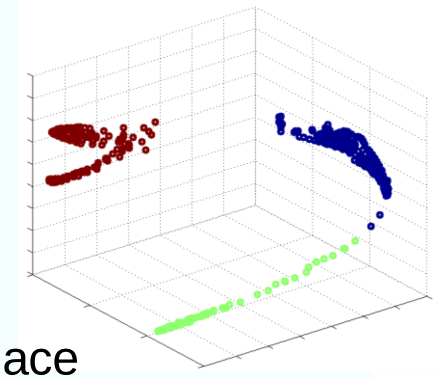
- K is a kernel function $K(x,\sigma) = \left(x^2 + \sigma^2\right)^{-1/2}$
- $\left\|.\right\|_H$ is the robust Huber norm
- Affinity is defined in image space, in pixels
- Pairwise affinity is then A $\approx$ EE^T

Algorithm: Step 3 - Spectral clustering

- Uses the eigen-structure of the affinity matrix A for dimensionality reduction and clustering
- Usually one parameter (kernel width)
- We search for the optimal kernel width



Data space



Kernel eigen-space

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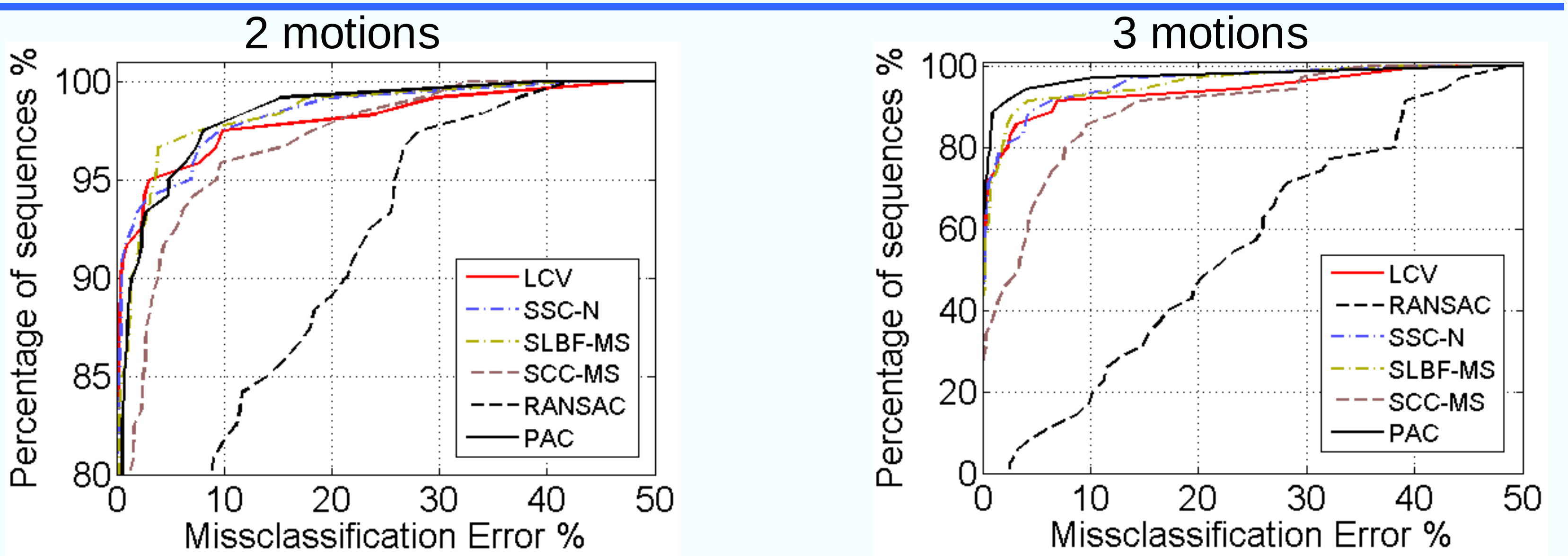
Experiments and comparison to other methods

- State-of-the-art methods of the last few years (SSC, SCC, SLBF, PAC). All of them use the T-K factorisation

- Hopkins155
  - Standard dataset for sparse motion segmentation methods
  - Manually refined, complete trajectories (20-30 frames, 100-500 points)
  - 155 sequences of 2 and 3 motions.
  - Varying difficulty (general, degenerate, articulated)
- No per sequence tuning. Either fixed parameters or determined automatically.
- The number of motions are assumed known

Accuracy and speed



Method	RANSAC	SCC-MS	SLBF-MS	SSC-N	PAC	LCV
Average time (sec)	0.387	1.264	10.83	165	952.25	0.93
Total time (sec)	60	196	1680	25620	147600	145
Average error (%)	9.48	2.70	1.35	1.36	1.24	1.86

The average and total runtime on the full Hokpins155 dataset

Algorithm complexity overview

N: number of scene points, F number of frames, C: number of 7-point clusters

- Step1 (motion sampling):
  - K-means
  - C*F matrix inversions of size 5x7
- Step2 (synthesis+affinity):
  - N*C*F LCV equations
  - N*C*Affinity
- Step3 (spectral clustering with kernel width search):
  - 10*eigen-decompositions of a NxN matrix
  - 10*K-means

Conclusion

- Motion segmentation method based on LCV theory for affine camera model
- Good combined accuracy and speed.
- Easy to implement and can be used in practice (fast and parameters are auto tuned)
- Outstanding issues: missing trajectories, number of objects, degeneracies