Linköping University

xsens

UPPSALA UNIVERSITET

Calibration of a magnetometer in combination with inertial sensors

Manon Kok, Linköping University, Sweden

Joint work with:
Thomas Schön, Uppsala University, Sweden
Jeroen Hol, Xsens Technologies, the Netherlands
Fredrik Gustafsson, Linköping University, Sweden
Henk Luinge, Xsens Technologies, the Netherlands

March 11th 2015

Summary of our work on magnetometer calibration

- Manon Kok, Jeroen Hol, Thomas Schön, Fredrik Gustafsson and Henk Luinge, **Calibration of a magnetometer in combination with inertial sensors.** *The 15th International Conference on Information Fusion*, Singapore, July 2012.
- Manon Kok and Thomas Schön, **Maximum likelihood calibration of a magnetometer using inertial sensors.** *Proceedings of the 19th World Congress of the International Federation of Automatic Control (IFAC)*, Cape Town, South Africa, August 2014.
- Manon Kok and Thomas Schön, **Magnetometer calibration using inertial sensors**, 2014. Paper A in: Manon Kok, **Probabilistic modeling for positioning applications using inertial sensors**, 2014 (Licentiate thesis).

http://users.isy.liu.se/en/rt/manko/

Introduction: sensors

- Magnetometers

Introduction: sensors

- Magnetometers
- Accelerometers } Inertial sensors
- Gyroscopes

Introduction: sensors

- Magnetometers
- Accelerometers
- Gyroscopes

} Inertial sensors (accelerometers and gyroscopes)

Inertial sensors and magnetometers are widely used for orientation estimation.

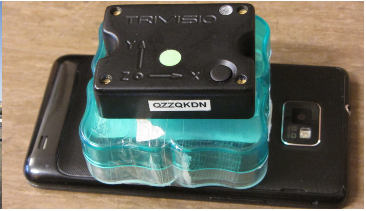
Magnetometers can be used in combination with inertial sensors to estimate the sensor's orientation provided that the magnetometer is properly calibrated.

Introduction: magnetometer calibration

Magnetometers can be used in combination with inertial sensors to estimate the sensor's orientation provided that the magnetometer is properly calibrated.

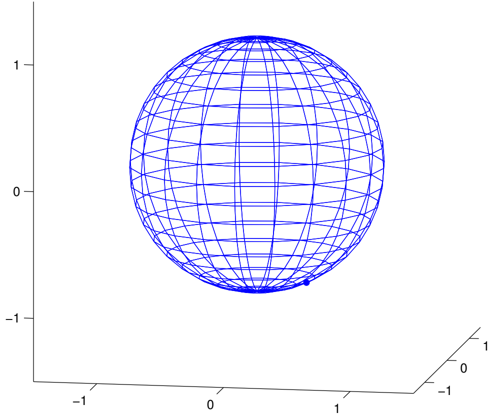
The magnetometer needs to be calibrated for

1. magnetometer sensor errors,
2. presence of magnetic material rigidly attached to the sensor,
3. misalignment between magnetometer and inertial sensor axes.

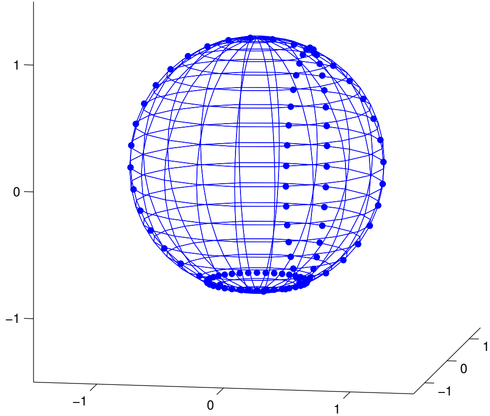
The magnetometer needs to be calibrated for

1. magnetometer sensor errors,
2. presence of magnetic material rigidly attached to the sensor,
3. misalignment between magnetometer and inertial sensor axes.



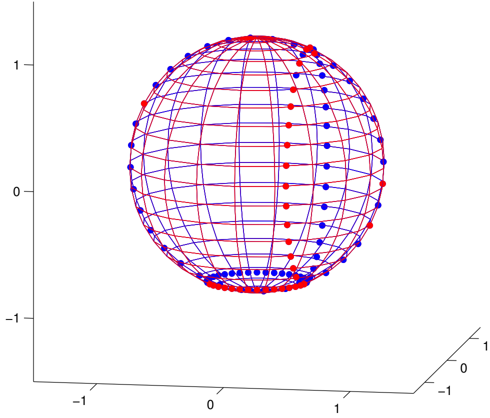
Rotate the IMU obtain a sphere of magnetometer data

$$y_{\mathrm{m},t} = R_t^{\mathrm{bn}} m^{\mathrm{n}}$$



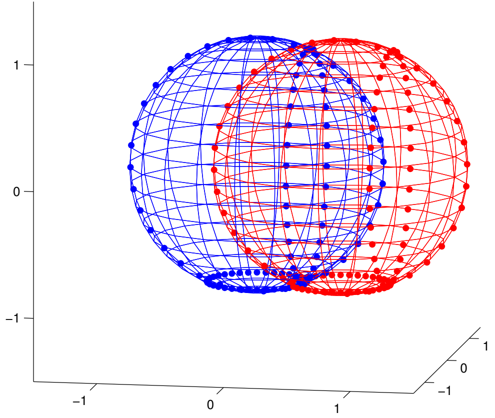
Rotate the IMU obtain a sphere of magnetometer data

$$y_{\mathrm{m},t} = R_t^{\mathrm{bn}} m^{\mathrm{n}}$$



Uncalibrated magnetometer data will have an offset (o)

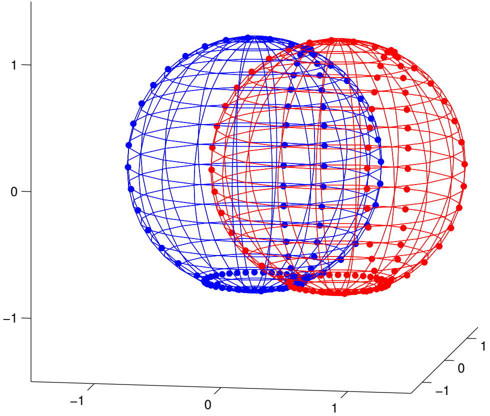
$$y_{\mathrm{m},t} = R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$



Uncalibrated magnetometer data will have an offset (o)

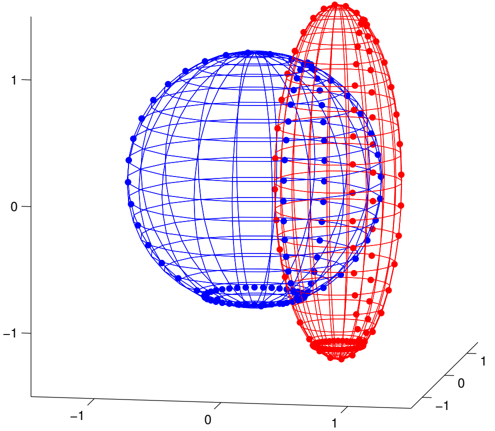
$$y_{\mathrm{m},t} = R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$

Introduction: magnetometer calibration data



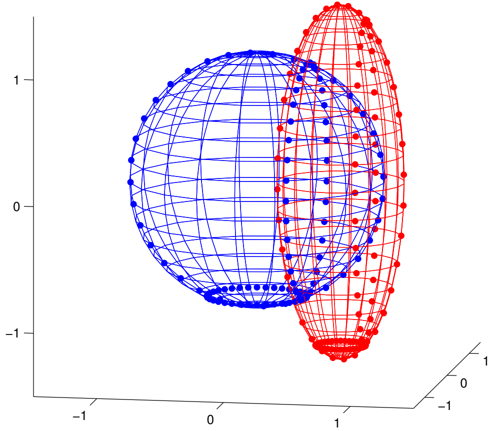
Uncalibrated magnetometer data will have an offset (o) and it will be scaled (D)

$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$



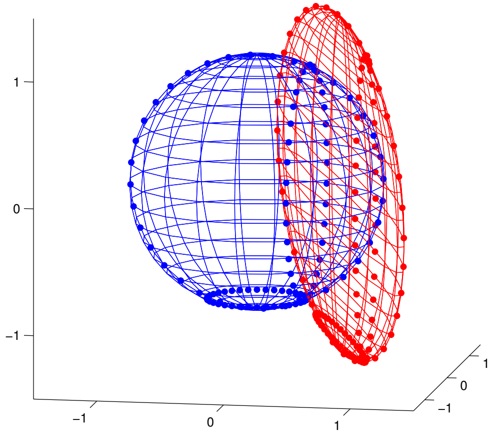
Uncalibrated magnetometer data will have an offset (o) and it will be scaled (D)

$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$



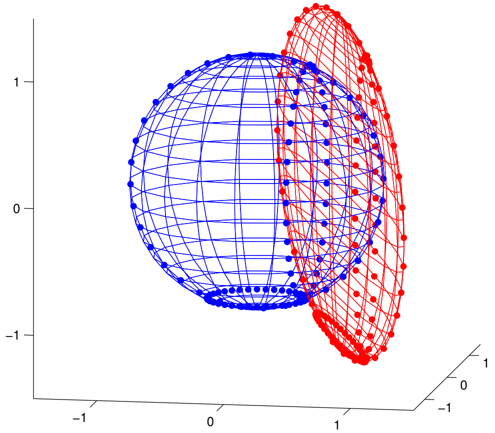
Uncalibrated magnetometer data will have an offset (o) and it will be scaled, skewed (D)

$$y_{m,t} = D R_t^{bn} m^n + o$$



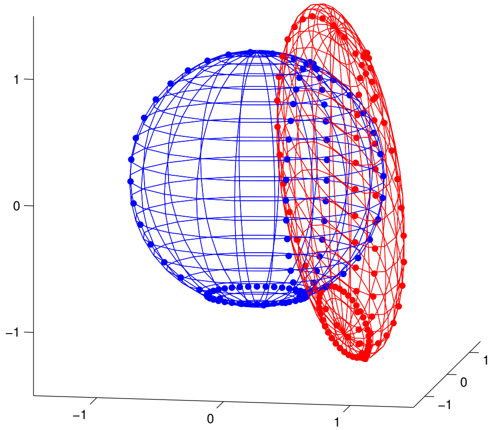
Uncalibrated magnetometer data will have an offset (o) and it will be scaled, skewed (D)

$$y_{m,t} = D R_t^{bn} m^n + o$$



Uncalibrated magnetometer data will have an offset (o) and it will be scaled, skewed and rotated (D).

$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$



Uncalibrated magnetometer data will have an offset (o) and it will be scaled, skewed and rotated (D).

$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$

Magnetometer calibration algorithm

1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.

Magnetometer calibration algorithm

Magnetometer calibration algorithm

1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.
   - ⇒ Uncalibrated magnetometer measurements
   - ⇒ Wrong orientation estimates

Magnetometer calibration algorithm

Magnetometer calibration algorithm

1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.
   - ⇒ Uncalibrated magnetometer measurements
   - ⇒ Wrong orientation estimates
3. Estimate the calibration parameters
   1. Obtain an initial estimate of the parameters.
   2. Obtain a maximum likelihood estimate of the parameters.

Magnetometer calibration algorithm

Magnetometer calibration algorithm

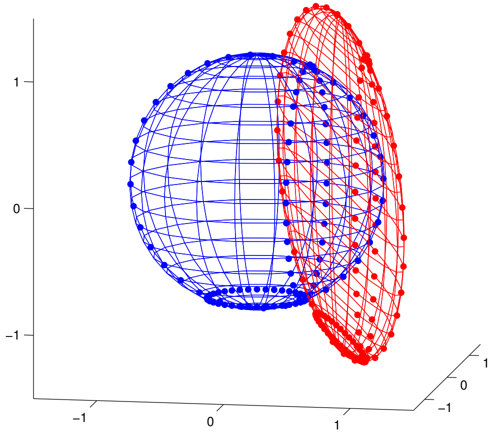
1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.
   - $\Rightarrow$ Uncalibrated magnetometer measurements
   - $\Rightarrow$ Wrong orientation estimates
3. Estimate the calibration parameters
   1. Obtain an initial estimate of the parameters.
   2. Obtain a maximum likelihood estimate of the parameters.
4. Use the estimated calibration parameters together with the magnetometer measurements.

Magnetometer calibration algorithm

Magnetometer calibration algorithm

1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.
   - $\Rightarrow$ Uncalibrated magnetometer measurements
   - $\Rightarrow$ Wrong orientation estimates
3. Estimate the calibration parameters
   1. Obtain an initial estimate of the parameters.
   2. Obtain a maximum likelihood estimate of the parameters.
4. Use the estimated calibration parameters together with the magnetometer measurements.
   - $\Rightarrow$ Calibrated magnetometer measurements
   - $\Rightarrow$ Correct orientation estimates

Introduction: magnetometer calibration data



Uncalibrated magnetometer data will have an offset (o) and it will be scaled, skewed and rotated (D).

$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o$$

Summary of our work on magnetometer calibration

- Manon Kok, Jeroen Hol, Thomas Schön, Fredrik Gustafsson and Henk Luinge, **Calibration of a magnetometer in combination with inertial sensors.** *The 15th International Conference on Information Fusion*, Singapore, July 2012.
- Manon Kok and Thomas Schön, **Maximum likelihood calibration of a magnetometer using inertial sensors.** *Proceedings of the 19th World Congress of the International Federation of Automatic Control (IFAC)*, Cape Town, South Africa, August 2014.
- Manon Kok and Thomas Schön, **Magnetometer calibration using inertial sensors**, 2014. Paper A in: Manon Kok, **Probabilistic modeling for positioning applications using inertial sensors**, 2014 (Licentiate thesis).

http://users.isy.liu.se/en/rt/manko/

Contributions

- Derived a magnetometer calibration algorithm by solving a maximum likelihood problem.
- Validated the algorithm using experimental data showing that it works and leads to improved heading estimates even in the presence of large disturbances.
- Performed an identifiability analysis to quantify how much rotation that is needed to be able to solve the calibration problem.

Contributions

- Derived a magnetometer calibration algorithm by solving a maximum likelihood problem.
- Validated the algorithm using experimental data showing that it works and leads to improved heading estimates even in the presence of large disturbances.
- Performed an identifiability analysis to quantify how much rotation that is needed to be able to solve the calibration problem.

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$
\begin{aligned}
x_{t+1} &= f_t(x_t, u_t, \theta) + B(x_t) v_t(\theta) \\
y_t &= h_t(x_t, \theta) + e_t(\theta)
\end{aligned}
$$

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$\begin{aligned} x_{t+1} &= f_t(x_t, u_t, \theta) + B(x_t) v_t(\theta) \\ y_t &= h_t(x_t, \theta) + e_t(\theta) \end{aligned}$$

Dynamic model:

- the state x_t represents the sensor's orientation,

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$
\begin{aligned}
x_{t+1} &= f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta) \\
y_t &= h_t(x_t, \theta) + e_t(\theta)
\end{aligned}
$$

Dynamic model:

- the state x_t represents the sensor's orientation,
- the gyroscope measurements ($y_{\omega,t}$) are used as input to the dynamic model.

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$x_{t+1} = f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta)$$
$$y_t = h_t(x_t, \theta) + e_t(\theta)$$

Measurement model:

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$x_{t+1} = f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta)$$
$$y_{\mathrm{a},t} = R_t^{\mathrm{bn}} \left(a_t^{\mathrm{n}} - g^{\mathrm{n}} \right) + e_{\mathrm{a},t} \approx -R_t^{\mathrm{bn}} g^{\mathrm{n}} + e_{\mathrm{a},t}$$

Measurement model:

- accelerometer measurement model

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$x_{t+1} = f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta)$$
$$y_{\mathrm{a},t} = R_t^{\mathrm{bn}} (a_t^{\mathrm{n}} - g^{\mathrm{n}}) + e_{\mathrm{a},t} \approx -R_t^{\mathrm{bn}} g^{\mathrm{n}} + e_{\mathrm{a},t}$$

Measurement model:

- accelerometer measurement model
  - assumes that the sensor's acceleration is approximately zero.

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$
\begin{aligned}
x_{t+1} &= f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta) \\
y_{\mathrm{a},t} &= R_t^{\mathrm{bn}} \left(a_t^{\mathrm{n}} - g^{\mathrm{n}} \right) + e_{\mathrm{a},t} \approx -R_t^{\mathrm{bn}} g^{\mathrm{n}} + e_{\mathrm{a},t} \\
y_{\mathrm{m},t} &= D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o + e_{\mathrm{m},t}
\end{aligned}
$$

Measurement model:

- accelerometer measurement model
  - assumes that the sensor's acceleration is approximately zero.
- magnetometer measurement model

Problem formulation

State-space model with unknown states (x_t) and parameters (θ)

$$
\begin{aligned}
x_{t+1} &= f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta) \\
y_{\mathrm{a},t} &= R_t^{\mathrm{bn}} \left(a_t^{\mathrm{n}} - g^{\mathrm{n}} \right) + e_{\mathrm{a},t} \approx -R_t^{\mathrm{bn}} g^{\mathrm{n}} + e_{\mathrm{a},t} \\
y_{\mathrm{m},t} &= D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o + e_{\mathrm{m},t}
\end{aligned}
$$

Measurement model:

- accelerometer measurement model
  - assumes that the sensor's acceleration is approximately zero.
- magnetometer measurement model
  - assumes that the magnetometer measurements can be calibrated using a calibration matrix D and offset vector o

State-space model with unknown states (x_t) and parameters (θ)

$$\begin{aligned}
x_{t+1} &= f_t(x_t, y_{\omega,t}, \theta) + B(x_t) v_{\omega,t}(\theta) \\
y_{\mathrm{a},t} &= R_t^{\mathrm{bn}} \left(a_t^{\mathrm{n}} - g^{\mathrm{n}} \right) + e_{\mathrm{a},t} \approx -R_t^{\mathrm{bn}} g^{\mathrm{n}} + e_{\mathrm{a},t} \\
y_{\mathrm{m},t} &= D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o + e_{\mathrm{m},t}
\end{aligned}$$

where

$$\begin{aligned}
y_{\omega,t} &= \omega_t + b_\omega + e_{\omega,t} \\
e_{\omega,t} &\sim \mathcal{N}(0, \Sigma_\omega) \\
e_{\mathrm{a},t} &\sim \mathcal{N}(0, \Sigma_{\mathrm{a}}) \\
e_{\mathrm{m},t} &\sim \mathcal{N}(0, \Sigma_{\mathrm{m}})
\end{aligned}$$

Maximum likelihood estimate

Compute a maximum likelihood estimate of the unknown parameters,

$$\widehat{\theta}^{\mathrm{ML}} = \arg\max_{\theta \in \Theta} \; p_\theta(y_{1:N})$$

Compute a maximum likelihood estimate of the unknown parameters,

$$\widehat{\theta}^{\text{ML}} = \arg\max_{\theta \in \Theta} \; p_\theta(y_{1:N})$$
$$\approx \arg\min_{\theta \in \Theta} \; \frac{1}{2} \sum_{t=1}^{N} \|y_t - \widehat{y}_{t|t-1}(\theta)\|^2_{S_t^{-1}(\theta)} + \log\det S_t(\theta)$$

Maximum likelihood estimate

Compute a maximum likelihood estimate of the unknown parameters,

$$\begin{aligned}\widehat{\theta}^{\mathrm{ML}} &= \arg\max_{\theta \in \Theta} \; p_\theta(y_{1:N}) \\ &\approx \arg\min_{\theta \in \Theta} \; \frac{1}{2} \sum_{t=1}^{N} \|y_t - \widehat{y}_{t|t-1}(\theta)\|^2_{S_t^{-1}(\theta)} + \log\det S_t(\theta)\end{aligned}$$

where $\widehat{y}_{t|t-1}$ and S_t are the predicted measurement and the residual covariance from an extended Kalman filter.

Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathsf{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathsf{a}}, \Sigma_{\mathsf{m}}\}$$

Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathrm{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathrm{a}}, \Sigma_{\mathrm{m}}\}$$

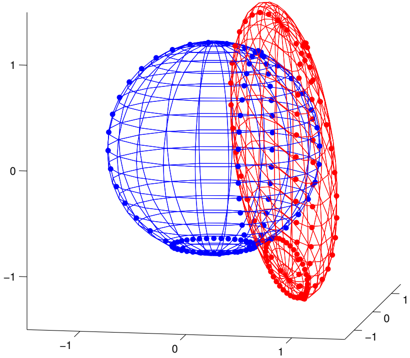
1. Obtain initial sensor characteristics.

Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathrm{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathrm{a}}, \Sigma_{\mathrm{m}}\}$$

1. Obtain initial sensor characteristics.
2. Ellipse fitting (to a unit sphere).

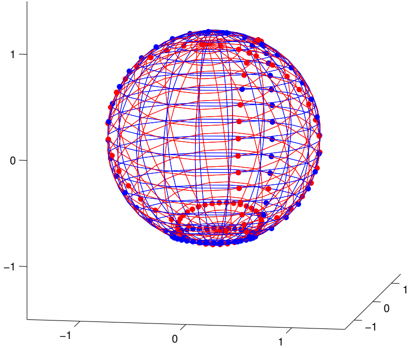


Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathrm{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathrm{a}}, \Sigma_{\mathrm{m}}\}$$

1. Obtain initial sensor characteristics.
2. Ellipse fitting (to a unit sphere).

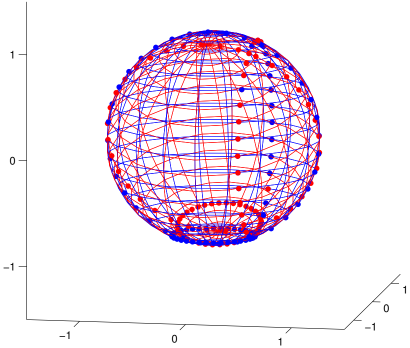


Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathrm{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathrm{a}}, \Sigma_{\mathrm{m}}\}$$

1. Obtain initial sensor characteristics.
2. Ellipse fitting (to a unit sphere).
3. Determine the rotation of the ellipse to align the magnetometer and inertial sensor axes.

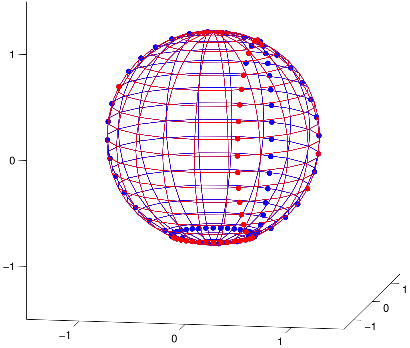


Initial estimate

The maximum likelihood problem is non-convex and needs proper initialization.

$$\theta = \{D, o, m^{\mathrm{n}}, \delta_\omega, \Sigma_\omega, \Sigma_{\mathrm{a}}, \Sigma_{\mathrm{m}}\}$$

1. Obtain initial sensor characteristics.
2. Ellipse fitting (to a unit sphere).
3. Determine the rotation of the ellipse to align the magnetometer and inertial sensor axes.

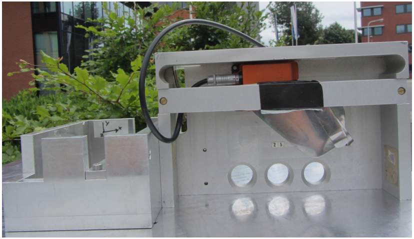


Magnetometer calibration algorithm

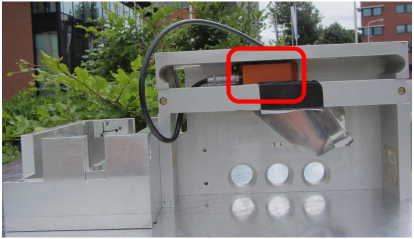
Magnetometer calibration algorithm

1. Rigidly mount the magnetometer and the inertial sensors on the platform to be used.
2. Rotate the assembly to collect inertial and magnetometer measurements.
   - ⇒ Uncalibrated magnetometer measurements
   - ⇒ Wrong orientation estimates
3. Estimate the calibration parameters
   1. Obtain an initial estimate of the parameters.
   2. Obtain a maximum likelihood estimate of the parameters.
4. Use the estimated calibration parameters together with the magnetometer measurements.
   - ⇒ Calibrated magnetometer measurements
   - ⇒ Correct orientation estimates

Experimental setup

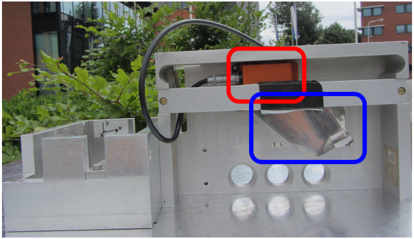


Experimental setup



Inertial measurement unit (inertial sensors + magnetometer)

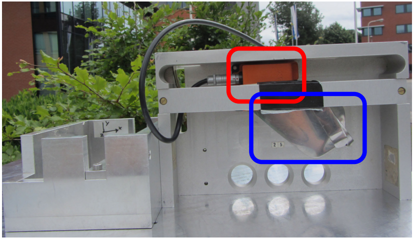
Experimental setup



Inertial measurement unit (inertial sensors + magnetometer)

Magnetic disturbance

Experimental setup

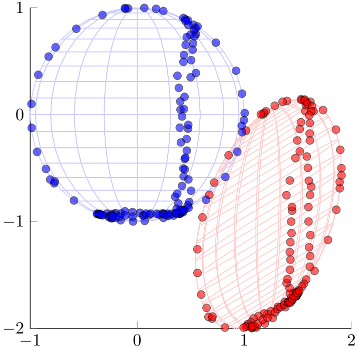


Inertial measurement unit (inertial sensors + magnetometer)

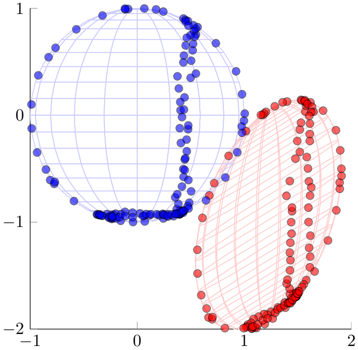
Magnetic disturbance

The IMU is placed in a block that can be put in orientations differing 90 degrees from each other.

Experimental results



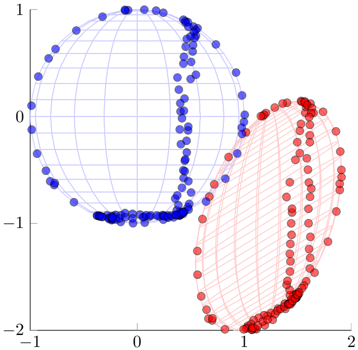
Experimental results

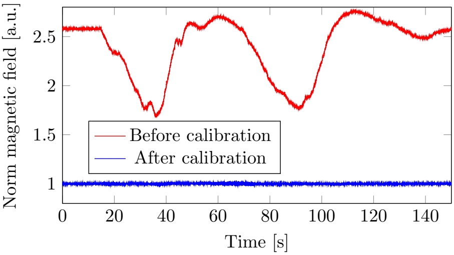


$$y_{\mathrm{m},t} = D R_t^{\mathrm{bn}} m^{\mathrm{n}} + o + e_{\mathrm{m},t}$$

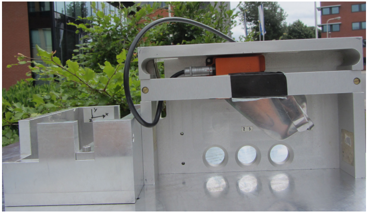
$$\widehat{D} = \begin{pmatrix} 0.74 & -0.14 & 0.02 \\ -0.12 & 0.68 & 0.01 \\ -0.04 & 0.43 & 1.00 \end{pmatrix}, \quad \widehat{o} = \begin{pmatrix} 1.37 \\ 1.22 \\ -0.94 \end{pmatrix}$$

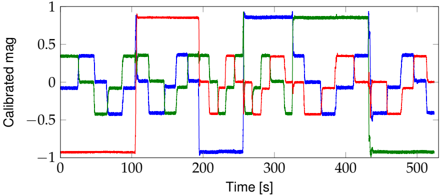
Experimental results





The norm of the calibrated magnetometer measurements is around 1, i.e. the data is properly mapped to a unit sphere.





Heading error in degrees after calibration for 90° turns

z up	z down	x up	x down	y down
-0.23	-0.84	0.08	0.98	-0.31
0.21	-2.70	-0.02	1.66	0.35
-0.44	1.81	-0.82	-0.71	-0.07
0.42	2.00	0.36	-1.89	0.45

Conclusions

- Developed a magnetometer calibration algorithm which calibrates a magnetometer in combination with inertial sensors to obtain better orientation estimates.
- It corrects for
  - magnetometer sensor errors,
  - presence of magnetic material rigidly attached to the sensor,
  - misalignment between magnetometer and inertial sensor axes.
- We applied the algorithm to real data, showing that improved heading estimates are obtained.

An optimization-based approach to human body motion capture using inertial sensors

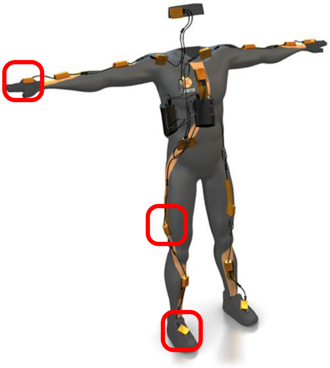
Manon Kok[1], Jeroen D. Hol[2] and Thomas B. Schön[3]

[1]Linköping University, Sweden
[2]Xsens Technologies, the Netherlands
[3]Uppsala University, Sweden

March 13th 2015

Inertial motion capture

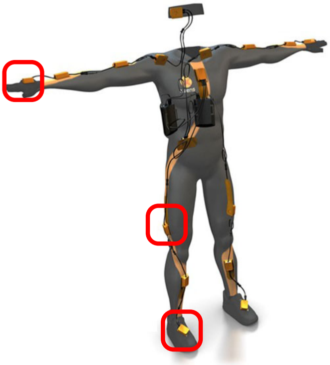
Inertial motion capture



17 inertial measurement units containing:

- Accelerometers } Inertial sensors
- Gyroscopes } Inertial sensors
- Magnetometers

Inertial motion capture



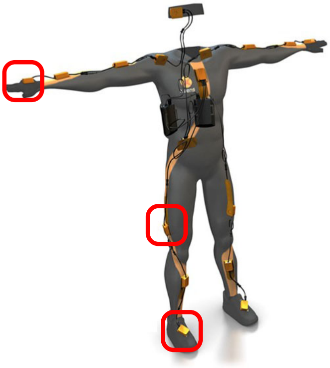
17 inertial measurement units containing:

- Accelerometers
- Gyroscopes
- Magnetometers

} Inertial sensors

The magnetic field at the different sensor locations is typically different.

Inertial motion capture



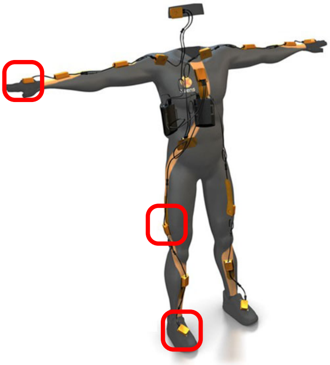
17 inertial measurement units containing:

- Accelerometers } Inertial sensors
- Gyroscopes }
- Magnetometers

\+ Biomechanical model

The magnetic field at the different sensor locations is typically different.

Inertial motion capture



17 inertial measurement units containing:

- Accelerometers
- Gyroscopes

} Inertial sensors

- Magnetometers

+ Biomechanical model

Assuming that the body segments are connected to each other, the *relative* position and orientation of the body is observable (if the subject is not standing completely still).

We solve the motion capture problem by solving a *maximum a posteriori* (MAP) problem

$$\min_{z=\{x_{1:N},\theta\}} \underbrace{-\log p(x_1 \mid y_1) - \log p(\theta)}_{\text{initial state + prior}}$$

$$\underbrace{- \sum_{t=2}^{N} \log p(x_t \mid x_{t-1}, \theta)}_{\text{dynamic model}} \underbrace{- \sum_{t=1}^{N} \log p(y_t \mid x_t, \theta)}_{\text{biomechanical/sensor model}}$$

$$\text{s.t.} \quad \underbrace{c_{\text{bio}}(z) = 0}_{\text{biomechanical model}}$$

Problem formulation

We solve the motion capture problem by solving a *maximum a posteriori* (MAP) problem

$$\min_{z=\{x_{1:N},\theta\}} \underbrace{-\log p(x_1 \mid y_1) - \log p(\theta)}_{\text{initial state + prior}} \underbrace{- \sum_{t=2}^{N} \log p(x_t \mid x_{t-1}, \theta)}_{\text{dynamic model}} \underbrace{- \sum_{t=1}^{N} \log p(y_t \mid x_t, \theta)}_{\text{biomechanical/sensor model}}$$

$$\text{s.t.} \quad \underbrace{c_{\text{bio}}(z) = 0}_{\text{biomechanical model}}$$

$x_{1:N}$: time-varying states such as the sensor positions, velocities and orientations, the body segment positions and orientations.
θ: constant model parameters such as sensor biases.
$y_{1:N}$: inertial measurements.

Problem formulation

We solve the motion capture problem by solving a *maximum a posteriori* (MAP) problem

$$\min_{z=\{x_{1:N},\theta\}} \underbrace{-\log p(x_1 \mid y_1) - \log p(\theta)}_{\text{initial state + prior}}$$

$$\underbrace{-\sum_{t=2}^{N} \log p(x_t \mid x_{t-1}, \theta)}_{\text{dynamic model}} \underbrace{-\sum_{t=1}^{N} \log p(y_t \mid x_t, \theta)}_{\text{biomechanical/sensor model}}$$

$$\text{s.t.} \quad \underbrace{c_{\text{bio}}(z) = 0}_{\text{biomechanical model}}$$

$c_{\text{bio}}(z)$: constraints imposed by the biomechanical model.

Problem formulation

We solve the motion capture problem by solving a *maximum a posteriori* (MAP) problem

$$\min_{z=\{x_{1:N},\theta\}} \underbrace{-\log p(x_1 \mid y_1) - \log p(\theta)}_{\text{initial state + prior}}$$

$$\underbrace{-\sum_{t=2}^{N} \log p(x_t \mid x_{t-1}, \theta)}_{\text{dynamic model}} \underbrace{-\sum_{t=1}^{N} \log p(y_t \mid x_t, \theta)}_{\text{biomechanical/sensor model}}$$

$$\text{s.t.} \quad \underbrace{c_{\text{bio}}(z) = 0}_{\text{biomechanical model}}$$

$c_{\text{bio}}(z)$: constraints imposed by the biomechanical model.

$\Rightarrow$ A constrained nonlinear least-squares problem.

Problem formulation

We solve the motion capture problem by solving a *maximum a posteriori* (MAP) problem

$$\min_{z=\{x_{1:N},\theta\}} \underbrace{-\log p(x_1 \mid y_1) - \log p(\theta)}_{\text{initial state + prior}}$$

$$\underbrace{-\sum_{t=2}^{N} \log p(x_t \mid x_{t-1}, \theta)}_{\text{dynamic model}} \underbrace{-\sum_{t=1}^{N} \log p(y_t \mid x_t, \theta)}_{\text{biomechanical/sensor model}}$$

$$\text{s.t.} \quad \underbrace{c_{\text{bio}}(z) = 0}_{\text{biomechanical model}}$$

$c_{\text{bio}}(z)$: constraints imposed by the biomechanical model.

$\Rightarrow$ A constrained nonlinear least-squares problem.

Solve this as a batch problem using standard solvers.

Biomechanical model - strategy

The body segments are connected at the joints.

Biomechanical model - strategy

The body segments are connected at the joints.

⇒ constraint

Biomechanical model - strategy

The body segments are connected at the joints.
⇒ constraint

The position and orientation of the sensors on the body is approximately constant.

Biomechanical model - strategy

The body segments are connected at the joints.
⇒ constraint

The position and orientation of the sensors on the body is approximately constant.
⇒ objective function

Biomechanical model - strategy

The body segments are connected at the joints.
⇒ constraint

The position and orientation of the sensors on the body is approximately constant.
⇒ objective function

Some joints are restricted in their rotational freedom (optional).

The body segments are connected at the joints.
$\Rightarrow$ constraint

The position and orientation of the sensors on the body is approximately constant.
$\Rightarrow$ objective function

Some joints are restricted in their rotational freedom (optional).
$\Rightarrow$ objective function

Biomechanical model - strategy

The body segments are connected at the joints.
⇒ constraint

The position and orientation of the sensors on the body is approximately constant.
⇒ objective function

Some joints are restricted in their rotational freedom (optional).
⇒ objective function

The positions and orientations of the sensors on the body are assumed to be known but it is possible to extend the algorithm to estimate these as well.

LINKÖPINGS UNIVERSITET

LINKÖPINGS UNIVERSITET