A Track Before Detect Algorithm for Tracking Extended Targets

Yvo Boers and Hans Driessen
THALES Nederland B.V., Hengelo, The Netherlands
Johan Torstensson, Mikael Trieb, Rickard Karlsson and Fredrik Gustafsson
Linköping University, Linköping, Sweden

***Abstract*— For certain types of sensor-target configurations a point target model or approach is not suitable and the physical extent of the target has to be accounted for in the processing. An extended target track before detect algorithm is presented and the performance is compared to an algorithm based on the point target assumption. Simulations illustrate the gain in performance obtained by using the extended target model where a particle filter is used for the track before detect implementation.**

***Index Terms*— Track Before Detect, Particle Filters, Nonlinear Filtering, Target Tracking, Radar, Extended Targets.**

I. Introduction

In this paper an extended target tracking radar application is studied. The application under consideration deals with tracking on the basis of unthresholded measurements, as opposed to tracking on the basis of thresholded measurements, or radar plots. This method is sometimes also referred to as *track before detect* (TBD), see [1, Chapter 11] for an overview. An efficient method to implement such a TBD processing is provided by the *particle filter* (PF), see the seminal papers on PF TBD [2], [3]. More work in this area has been performed recently, once more [1] contains an overview and many additional references. For PF TBD related issues also see [4].

Mostly, in tracking and detection, targets are assumed to be point targets. Often this is quite a valid assumption. However, there also exist situations in which this is not the case. If in such situations the target is still treated as a point target performance degradation or worse failure or divergence of the algorithm might occur.

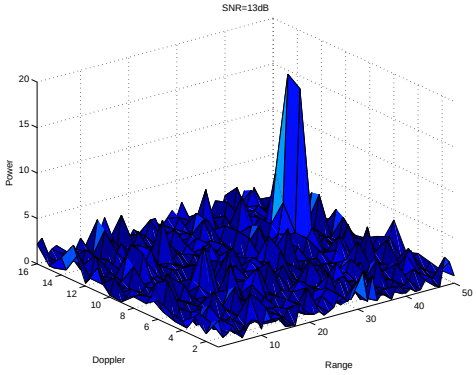
Summarizing, the contributions of this paper are: incorporation of an extended target model in a PF TBD algorithm, illustration of the working of the new algorithm, a comparison of the extended target model with the point target model and showing the superior performance and robustness of the extended target algorithm in an example.

The paper is organized as follows: In Section II the TBD problem is introduced. In Section III the nonlinear Bayesian particle filtering technique is described. Section IV presents a PF TBD simulation study. In Section V concluding remarks are given.

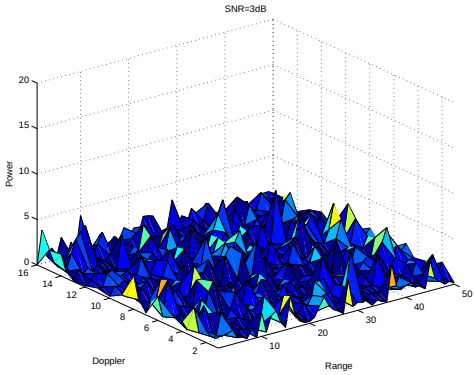
II. Track Before Detect

In Fig 1 typical illustrations of radar measurements are given, where the power is plotted as a function of range and Doppler cells for one fixed bearing angle. In Fig 1 (a), the target is a point target with a relatively high *signal to noise ratio* (SNR), hence easily detected and spotted if the received energy is above the detection threshold. This is the normal procedure for traditional radar receivers, which only delivers a single measurement from the radar video signal. This extracted data is often referred to as a radar plot, consisting of for instance a single value (r_t, d_t, b_t), for measured range, Doppler, and bearing. Traditionally radar tracking systems are based on plots from thresholded data, [5], [6]. In Fig 1 (b) the target is present as well, though much weaker, and it is not obvious that by means of classical extraction the target would be detected.

Corresponding Author: Yvo Boers, THALES Nederland B.V., Hengelo, The Netherlands, E-mail:yvo.boers@nl.thalesgroup.com



(a) SNR=13 dB. A high SNR makes it easy to detect the point target.



(b) SNR=3 dB. A low SNR makes the target hard to detect in a cluttered environment.

Fig. 1. Examples of simulated measurement data for a point target in noise with different SNR.

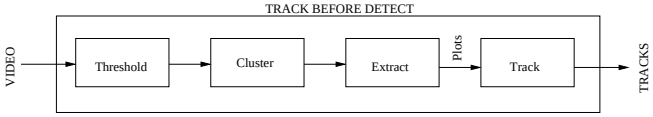


Fig. 2. Radar signal processing : Classical methods based on thresholded data and the TBD method where tracking and detection is done simultaneously.

For *track before detect* (TBD) problems, [1], [2], [3], [7], much lower SNR values are possible, where the target is not easily distinguished from the cluttered noisy background for any given frame. The TBD, despite of its name, consists of a simultaneous detection and tracking part. Instead of thresholding, the entire radar video signal is used as the measurement, i.e., the received power, $P(r_j, d_k, b_l), \forall j, k, l$. In the application under consideration the measurements consist of the power levels in $N_r \times N_d \times N_b$ sensor cells, where N_r, N_d, and N_b are the number of range, Doppler, and bearing cells, respectively.

A schematic overview of a TBD processing chain in comparison with a more classical one is given in Fig 2. The smaller boxes represent the steps that are normally distinguished in a classical setup, i.e., thresholding, clustering, and feature extraction to obtain a radar plot from the analog video signal. These plots are filtered and connected over time to form trajectories, or tracks, in the tracking algorithm.

A. Measurement Model

First, the measurement model under the assumption of a point target is briefly discussed, as given in [8]. Thus one measurement consists of a frame of reflected power levels over a three dimensional array. The measurement model describes how these measurements are related to the target state. For each range-Doppler-bearing cell the received power in the measurement relation is given as

$$y_t^{jkl} = \begin{cases} e_t^{jkl}, & \text{if } \ m_t = 0, \\ h_P^{jkl}(x_t^{jkl}, e_t^{jkl}), & \text{if } \ m_t = 1, \end{cases} \tag{1}$$

where $j = 1, \ldots, N_r,\ k = 1, \ldots, N_d,\ l = 1, \ldots, N_b$. The modal state, m_t, denotes if the target is present or not, and e_t^{jkl} denotes the measurement noise in a cell, and the function $h_P^{jkl}(\cdot)$ represents the received radar power in a cell. The power measurement per range-Doppler-bearing cell is related to the signal amplitude by

$$y_t^{jkl} = y_{P,t}^{jkl} = |y_{A,t}^{jkl}|^2, \tag{2}$$

where $y_{A,t}^{jkl}$ is the complex amplitude of the target

$$y_{A,t}^{jkl} = A_t^{jkl} \cdot h_A^{jkl}(x_t) + e_t^{jkl}, \tag{3}$$

with

$$A_t^{jkl} = \tilde{A}_t^{jkl} \mathrm{e}^{\imath \phi_t}, \quad \phi_t \in [0, 2\pi[\ , \tag{4}$$

and where $h_A^{jkl}(x_t)$ is the reflection form that is defined for every range-Doppler-bearing cell by

$$h_A^{jkl}(x_t) = \mathrm{e}^{-\frac{(r_j - r_t)^2}{2R}\lambda_r - \frac{(d_k - d_t)^2}{2D}\lambda_d - \frac{(b_l - b_t)^2}{2B}\lambda_b}. \tag{5}$$

The constants R, D, and B are related to the size of the range cell, the Doppler cell, and the bearing cell. Losses are represented by λ_r, λ_d, and λ_b.

The noise is defined by

$$e_t = e_{I,t} + \imath \cdot e_{Q,t}, \tag{6}$$

which is complex Gaussian, where $e_{I,t}$ and $e_{Q,t}$ are independent, zero-mean white Gaussian with variance σ_e^2, for the in-phase and quadrature-phase, respectively. In this way the power measurement in a specific range-Doppler-bearing cell is defined by

$$y_t^{jkl} = \left| y_{A,t}^{jkl} \right|^2 = \left| A_t^{jkl} \cdot h_A^{jkl}(x_t) + e_{I,t}^{jkl} + \imath \cdot e_{Q,t}^{jkl} \right|^2. \tag{7}$$

These measurements conditioned on the system state $\{x_t, m_t\}$ are now exponentially distributed, and the likelihood function is given as

$$p(y_t^{jkl}|x_t, m_t) = \frac{1}{\mu_0^{jkl}} \mathrm{e}^{-\frac{1}{\mu_0^{jkl}} y_t^{jkl}}, \tag{8}$$

where

$$\mu_0^{jkl} = \mathbb{E}(y_t^{jkl}|x_t, m_t), \tag{9}$$

and

$$\begin{aligned} \mu_0^{jkl} &= \mathbb{E}_{e_I, e_Q}(y_t^{jkl}|x_t, m_t) \\ &= \mathbb{E}_{e_I, e_Q}(|\tilde{A}_t \mathrm{e}^{\imath\phi_t} h_A^{jkl}(x_t) + e_{I,t}^{jkl}) + \imath e_{Q,t}^{jkl}|x_t, m_t)|^2) \\ &= \mathbb{E}_{e_I, e_Q}((\tilde{A}_t h_A^{jkl}(x_t)\cos(\phi_t) + e_{I,t}^{jkl}|x_t, m_t)^2 \\ &\quad +(\tilde{A} h_A^{jkl}(x_t)\sin(\phi_t) + e_{Q,t}^{jkl})^2) \\ &= \tilde{A}^2 \cdot (h_A^{jkl}(x_t))^2 + 2\sigma_e^2 = \tilde{P} \cdot h_P^{jkl}(x_t) + 2\sigma_e^2, \end{aligned}$$

with

$$\begin{aligned} h_P^{jkl}(x_t) &= (h_A^{jkl}(x_t))^2 \\ &= \mathrm{e}^{-\frac{(r_j - r_t)^2}{R}\lambda_r - \frac{(d_k - d_t)^2}{D}\lambda_d - \frac{(b_l - b_t)^2}{B}\lambda_b}. \end{aligned} \tag{10}$$

Observe that the likelihood under the noise only assumption is readily obtained from the above formulations as well. The above model has been taken from [8].

B. Extended Target Model

A target is denoted extended whenever the target extent is larger than the sensor resolution. Thus, whether or not a target is considered to be extended does not only depend on the physical size of the target, but also on the physical size relative to the sensor resolution. Typically for point targets the underlying assumption is that the target occupies one resolution cell. Very recent work on extended target tracking on a plot basis has been done in [9], where a diffuse spatial distribution over the target extent has been assumed. Here a similar approach is followed, but unlike in [9] no data association has to be considered. This is because unthresholded data are considered here, so no hypothesizing over clutter target hypotheses have to be performed, which is a well established additional advantage of TBD, [1]. Another difference with the approach of [9] is that no a priori knowledge of the target extent is assumed. The target extent and orientation are to be inferred from the data. This has been mentioned in [9] under "further developments".

In this section a specific type of extended target model is introduced by partially following the discussion in [9, Section 2.3], where a spatial distribution model for extended objects is assumed. The spatial extension is modeled by the distribution $p(\tilde{x}|x)$, which can be interpreted as a generator of a point source $\tilde{x}$ from an extended target with its center and orientation given by the state vector x. Receiving a measurement from a source $\tilde{x}$ somewhere on the target leads to a likelihood conditioned on a specific source $\tilde{x}$, $\Lambda(x) = p(y|\tilde{x})$. Using this model the total likelihood, conditioned only on x, is obtained as the convolution between $p(y|x)$ and $p(\tilde{x}|x)$, i.e.,

$$p(y|x) = \int p(y|\tilde{x})p(\tilde{x}|x)d\tilde{x}. \tag{11}$$

Point Target: The model (11) also covers the point target case, which is retrieved for:

$$p(\tilde{x}|x) = \delta(\tilde{x} - x), \tag{12}$$

where δ denotes the delta-Dirac function.

Point Sources: Also the case of a finite number of point target sources over the target extent is covered by the above model. For example if there are M sources at location $x^{(i)}$, $i = 1, \ldots, M$, the pdf $p(\tilde{x}|x)$ is defined as:

$$p(\tilde{x}|x) = \sum_{i=1}^{M} \Lambda(x^{(i)})\delta(\tilde{x} - x^{(i)}). \tag{13}$$

Extended Target: In its general form the pdf $p(\tilde{x}|x)$ could or should reflect (prior) knowledge on the position of possible scatterers on the target extent. The general expression for the likelihood of an extended target, as represented by (11), might not be available analytically. In this case this expression can always be approximated numerically through an importance sampling approach. This approximation is readily calculated by:

$$p(y|x) \approx \frac{1}{\tilde{M}} \sum_{i=1}^{\tilde{M}} p(y|\tilde{x}^{(i)}), \tag{14}$$

with $\tilde{x}^{(i)}$, independently drawn according to $p(\tilde{x}|x)$ for $i = 1, \ldots, \tilde{M}$.

III. Nonlinear State Estimation

Consider a general nonlinear and non-Gaussian system evolving according to

$$\begin{aligned} x_{t+1} &= f(x_t, m_t, w_t), && \text{(15a)} \\ y_t &= h(x_t, m_t, e_t), && \text{(15b)} \\ \Pi_{ij} &= \text{Prob}\left(m_{t+1} = i | m_t = j\right),\ i, j \in \{0, 1\} && \text{(15c)} \end{aligned}$$

where $x_t \in \mathbb{R}^n$ is the kinematic state of the system, $m_t \in \mathbb{N}$ is the modal state of the system, $y_t \in \mathbb{R}^p$ is the measurement, w_t is the process noise, e_t is the measurement noise, f is the system dynamic function, h is the measurement function, and Π is the Markov matrix, including all modal state transitions. It is assumed that the sample period is T. The system defined by (15) is referred to as a jump Markov system, see e.g. [7] and [10]. The key feature of such a system is that it involves both continuous as well as discrete state variables. The continuous state variables represent kinematic information, e.g. position, orientation, and velocity. The discrete state variable describes the modal state. In this application the modal state is an indicator of target absence/presence, see (1) and (15).

A. Bayesian Estimation

The recursive Bayesian estimation problem can be formulated as a time-update and a measurement-update for the posterior pdf, [11]. By extending the state space with the modal state, i.e., $\chi_t = (x_t^T,\ m_t)^T$, it can be expressed as

$$p(\chi_{t+1}|\mathbb{Y}_t) = \int_{\mathbb{R}^n} p(\chi_{t+1}|\chi_t)p(\chi_t|\mathbb{Y}_t)\, d\chi_t, \tag{16a}$$

$$p(\chi_t|\mathbb{Y}_t) = \frac{p(y_t|\chi_t)p(\chi_t|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})}, \tag{16b}$$

where the integration of the modal-space should be interpreted as a summation over the possible combinations and where $p(\chi_{t+1}|\mathbb{Y}_t)$ is the prediction density and $p(\chi_t|\mathbb{Y}_t)$ the filtering density.

The solution to the problem can in general not be represented in a finite dimension. There are two fundamentally different ways to approximately solve the problem:

- The *extended Kalman filter* (EKF), [12], [13], that is the sub-optimal filter for an approximate linear Gaussian model, or the optimal *linear* filter for linear non-Gaussian systems.
- Numerical approaches, such as the *particle filter* (PF) [14], [15], [1], that give an arbitrary good approximation of the optimal solution to the Bayesian filtering problem.

As the TBD problem is highly nonlinear and non-Gaussian, the PF is preferred in this application.

B. The Particle Filter

In this section the *particle filter* (PF) theory is presented according to [16], [15], [14], [17], [1]. The particle filter provides an approximative solution to the discrete time Bayesian estimation problem formulated in (16) by updating an approximate description of the posterior filtering density. The particle filter approximates the density $p(x_t|\mathbb{Y}_t)$ by a large set of N samples (particles), $\{x_t^{(i)}\}_{i=1}^N$, where each particle has an assigned relative weight, $\tilde{\gamma}_t^{(i)}$, chosen so that all weights sum to unity. The location and weight of each particle reflect the value of the density in that region of state space. The particle filter updates the particle location and the corresponding weights recursively with each new observed measurement. If the measurement noise is assumed additive, the unnormalized weights are given by

$$\gamma_t^{(i)} = p_e(y_t - h(x_t^{(i)})), \quad i = 1, \ldots, N. \tag{17}$$

Using the samples (particles) and the corresponding weights the Bayesian equations can be approximately solved. To avoid divergence a resampling step is introduced. This is referred to as the *sampling importance resampling* (SIR), [14].

As the estimate for each time, choose the minimum mean square estimate, i.e.,

$$\hat{x}_t = \mathbb{E}(x_t|\mathbb{Y}_t) = \int_{\mathbb{R}^n} x_t\, p(x_t|\mathbb{Y}_t)\, dx_t \approx \sum_{i=1}^{N} \tilde{\gamma}_t^{(i)} x_t^{(i)}. \quad (18)$$

The PF approximates the posterior pdf, $p(x_t|\mathbb{Y}_t)$, by a finite number of particles. However, asymptotically the approximated pdf converges to the true one, [15].

IV. Application

A. Tracking Model

In the dynamic model that is used here both the target's center dynamics as well as its extent are modeled within the dynamical model. In the application under consideration the target is assumed only to have a significant extent in one dimension. The state vector is defined as:

$$x_t = \begin{pmatrix} \mathsf{x}_t & \mathsf{y}_t & \dot{\mathsf{x}}_t & \dot{\mathsf{y}}_t & L_t \end{pmatrix}^T, \quad (19)$$

where $(\mathsf{x}_t, \mathsf{y}_t)$ is the position of the center of the target and $(\dot{\mathsf{x}}_t, \dot{\mathsf{y}}_t)$ is the velocity of the center of the target, and L_t is the extension or length of the target.

Note that although in this application the shape of target is known to the filter, the actual size is not and is to be inferred from the data. Furthermore, it is assumed that the orientation is along the direction of the velocity vector, i.e., the heading angle. If this is not the case, the state can be extended with another state, in order to estimate the orientation.

The systems dynamics are:

$$x_{t+1} = f(x_t, m_t) + g(x_t, m_t, w_t), \quad (20)$$

with

$$f(x_t, m_t) = \begin{pmatrix} 1 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} x_t, \quad (21)$$

where T is the update time. The process noise input model is given by

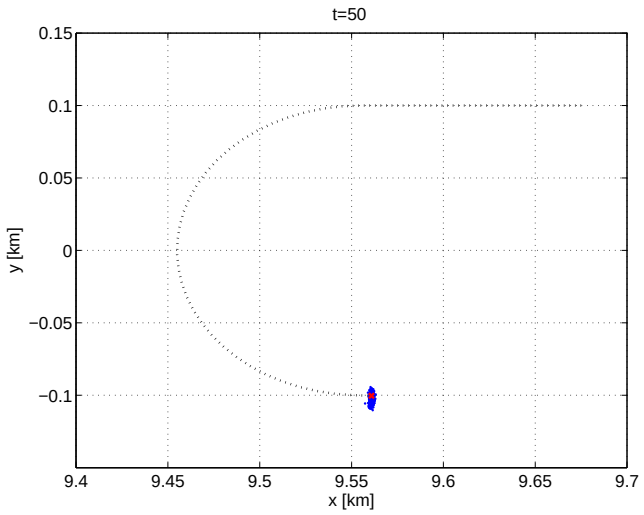
$$g(x_t, m_t, w_t) = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ \sigma_{\dot{x}} & 0 & 0 \\ 0 & \sigma_{\dot{y}} & 0 \\ 0 & 0 & \sigma_l \end{pmatrix} w_t, \quad (22)$$

where $w_t = \begin{pmatrix} w_{k_1} & w_{k_2} & w_{k_3} \end{pmatrix}^T$, and $\{w_{k_n}\}_{n=1}^3 \sim \mathcal{N}(0,1)$.

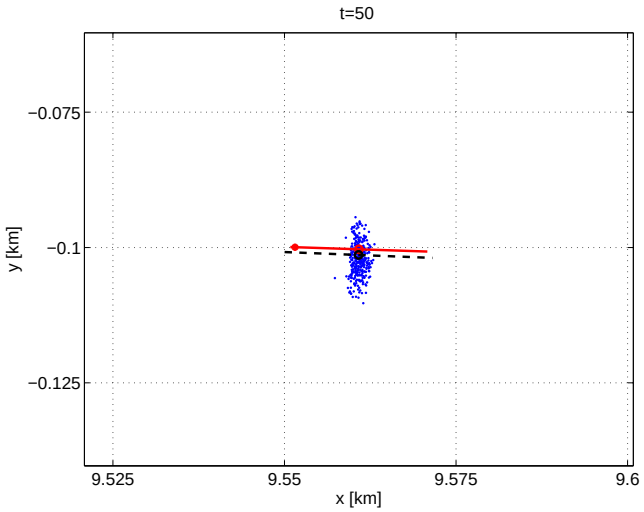
The modal state of the system evolves according to (15c), which can be described by a transitional probability matrix, with

$$\Pi_{ij} = \begin{pmatrix} 1-P_b & P_b \\ P_d & 1-P_d \end{pmatrix} = \begin{pmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{pmatrix}. \quad (23)$$

This transition model represents transitions from target absence to presence and vice versa.



(a) Entire trajectory and the PF cloud at time $t = 50$.



(b) Detail - PF cloud with true target extent (solid) and estimated target extent (dashed).

Fig. 3. The scenario for the extended target, consisting of a straight moving part and a turn with constant angular velocity.

The relation between the Cartesian coordinates in which the system state is given and the radar coordinates is given through the nonlinear transformation,

$$r_t = \sqrt{\mathsf{x}_t^2 + \mathsf{y}_t^2}, \quad (24a)$$

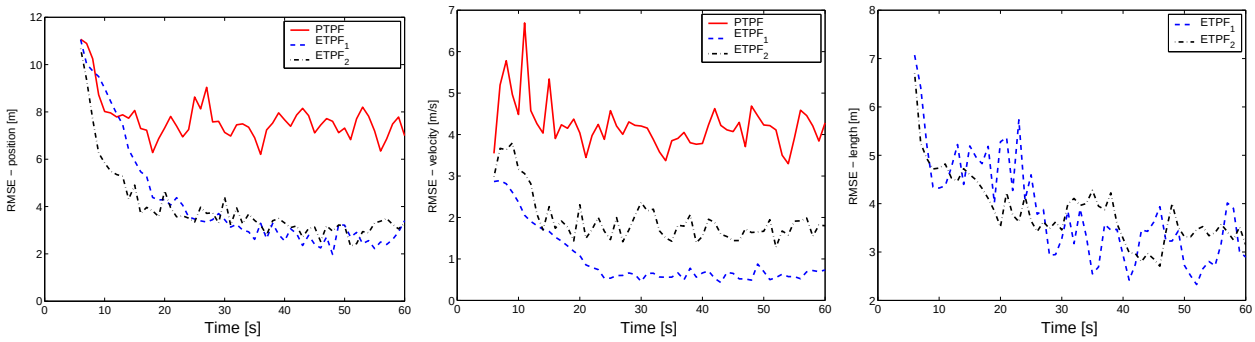
$$d_t = \dot{r}_t = \frac{\mathsf{x}_t\dot{\mathsf{x}}_t + \mathsf{y}_t\dot{\mathsf{y}}_t}{\sqrt{\mathsf{x}_t^2 + \mathsf{y}_t^2}}, \quad (24b)$$

$$b_t = \arctan\left(\frac{\mathsf{y}_k}{\mathsf{x}_k}\right), \quad (24c)$$

for range, Doppler, and bearing, respectively.

B. Simulations

To illustrate the extended target TBD application a PF is employed on simulated measurements from a target with a physical spatial extent, such that the target occupies multiple resolution cells. A *point target particle filter* (PTPF) will be compared to a PF with an extended target model, in this



(a) RMSE position - straight. (b) RMSE velocity - straight. (c) RMSE length - straight.

Fig. 4. Tracking performance based on 100 Monte Carlo simulations for a straight moving target. RMSE for position, velocity, and target length are depicted.

TABLE I

RADAR SENSOR PARAMETERS AND UPDATE RATE.

Number of cells			**Resolution**		
N_r	N_d	N_b	Range [m]	Doppler [m/s]	Bearing [mrad]
500	16	11	2	4.25	0.5

TABLE II

PROCESSING PARAMETERS FOR THE POINT TARGET PF (PTPF) AND THE EXTENDED TARGET PF (ETPF).

	N	$\sigma_{\mathbf{x}}$[m]	$\sigma_{\mathbf{y}}$[m]	$\sigma_{\dot{\mathbf{x}}}$[m/s]	$\sigma_{\dot{\mathbf{y}}}$[m/s]	σ_l[m]
PTPF	2000	4	4	8	8	-
ETPF_1	2000	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	2
ETPF_2	2000	4	4	8	8	2

example referred to as *extended target particle filter* (ETPF). In the proceeding three types of filters are compared. These are the PTPF and two versions of the ETPF. The two versions of the ETPF differ in the amount of process noise that is used in the models.

Setup: In the simulations the target is assumed to be extended in one dimension, with an extent of 20 m. Again, it is emphasized, that this not known to the filter. As can be seen from Table I, the target may depending on its orientation, occupy as much as 10 range cells. Additionally, no prior knowledge on spatial distribution is assumed. Therefore, the density $p(\tilde{x}|x)$ is assumed to be a uniform over the target extent. Furthermore, the target appears at time $t = 6$ at $[9.65,\ 0]$ km, and is initially moving at a constant velocity of $[-10,\ 0]$ m/s towards the sensor. The dynamics of the target is captured by a constant velocity model.

The standard deviations for the process noise inputs are listed in Table II. The maximum target accelerations are assumed to be $a_{\text{x,max}} = 4\,\text{m/s}^2$ and $a_{\text{y,max}} = 4\,\text{m/s}^2$. The update time for the radar is assumed to be $T = 1$ s and the average target SNR has been set to 10 dB and known to the filter. It is emphasized that this assumption can be relaxed on, straightforwardly by including the average SNR or equivalently target *radar cross section* (RCS) into the filter, see e.g. [1].

All filters have been implemented through a more or less standard particle filter and $N = 2000$ particles have been used in each of the filters. This number has been obtained experimentally. It has been observed that increasing the number of particles to values greater than $N = 2000$ did not significantly improve the results, thus 2000 particles are sufficient for the application at hand.

Measurement data for both a straight moving object and a turning object have been generated. For the turning object the lateral acceleration was 1 m/s^2.

Results: In Fig 3 (a) an illustration of the scenario for a maneuvering target is given, where the trajectory is shown and the PF cloud at time $t = 50$ is indicated. In Fig 3 (b) a more detailed representation of the true target extent together with its estimate is shown, as well as the particle cloud.

It has been observed that if the "plain" point target model is used on extended target data the filter often diverges. The reason for this is that the target extent induces virtual accelerations. These virtual accelerations, a_v, could in "worst case" amount to

$$a_v = \frac{2L}{T^2}. \tag{25}$$

Thus, if the level of process noise for the point target model is not adjusted, taking into account the above virtual accelerations, divergence occurs frequently. ¿From the target extent and (25), it follows that the maximal virtual acceleration is $a_v = 40\,\text{m/s}^2$. Note that the true maximum target accelerations (without the virtual acceleration term), $a_{\text{x,max}}$ and $a_{\text{y,max}}$, are only a tenth of the maximum virtual acceleration, a_v, for this setup.

The ETPF is tested with a level of process noise, accounting only for true target accelerations. This filter is referred to as ETPF_1. Also a version of the ETPF is applied, for which the process noise has been matched to the same level as the one used in the point target filter. This filter is referred to as ETPF_2. This model allows us to isolate the performance gain due only to the adaption of the measurement model. Simulation results in terms of *root mean square error* (RMSE) performance of the different filters over 100 Monte Carlo runs both for a straight moving target as well as for the maneuvering target are shown in Fig 4 and Fig 5. It is evident from these performance figures that the extended target model algorithm results in a significantly improved performance in terms of

accuracy, compared to the point target model algorithm. This holds even for the case where the process noise level of the extended target filter ($ETPF_2$) has been increased to match the level of the process noise in the point target filter. Furthermore, it can also be seen that the target extent is estimated relatively well.

V. Conclusions

A track before detect algorithm for extended targets has been proposed. The algorithm has been implemented through a particle filter. The performance of the extended target algorithm has shown to be superior to the algorithm based on a point target model assumption. Furthermore it has been shown that the algorithm is capable of estimating the target extent well enough to have a good overall performance, i.e., a performance that is significantly better than the performance under the point target model assumption.

References

[1] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*. Artech House, 2004.

[2] D. J. Salmond and H. Birch, "A particle filter for track-before-detect," in *Proceedings of American Control Conference*, vol. 5, 2001, pp. 3755–3760.

[3] Y. Boers and H. Driessen, "Particle filter based detection for tracking," in *Proceedings of American Control Conference*, 2001.

[4] J. Torstensson and M. Trieb, "Particle filtering for track before detect applications," Master's Thesis LiTH-ISY-EX-3683, Department of Electrical Engineering. Linköping University, Linköping, Sweden, 2005.

[5] M. Skolnik, *Introduction to Radar Systems*, 2nd ed. Mc Graw-Hill Book Company, 1980.

[6] S. S. Blackman, *Multiple-Target Tracking with Radar Applications*. Norwood, MA: Artech House, 1986.

[7] S. S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Artech House, 1999.

[8] Y. Boers and J. N. Driessen, "A particle filter multi target track before detect application." *IEE Proceedings of Radar, Sonar and Navigation*, vol. 151, no. 6, 2004.

[9] K. Gilholm and D. Salmond, "A spatial distribution model for tracking extended objects." *IEE Proceedings of Radar, Sonar and Navigation*, 2005, To appear.

[10] Y. Bar-Shalom and X. R. Li, *Estimation and Tracking: Principles, Techniques, and Software*. Artech House, 1993.

[11] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*, ser. Mathematics in Science and Engineering. Academic Press, 1970, vol. 64.

[12] B. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ: Prentice Hall, 1979.

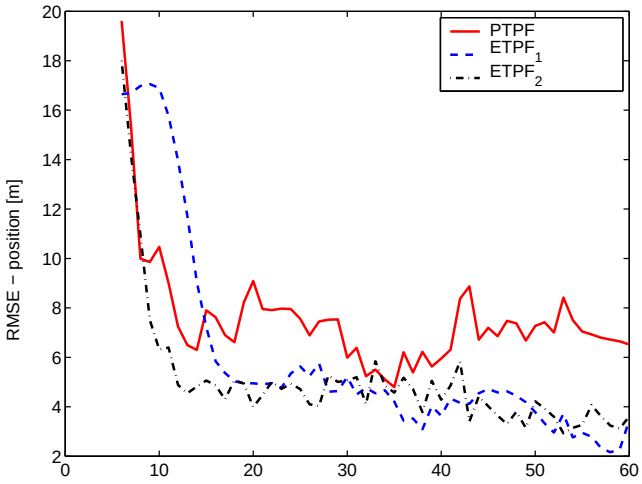
[13] T. Kailath, A. Sayed, and B. Hassibi, *Linear Estimation*, ser. Information and System Sciences. Upper Saddle River, New Jersey: Prentice Hall, 2000.

[14] N. J. Gordon, D. J. Salmond, and A. Smith, "A novel approach to nonlinear/non-Gaussian Bayesian state estimation," in *IEE Proceedings on Radar and Signal Processing*, vol. 140, 1993, pp. 107–113.

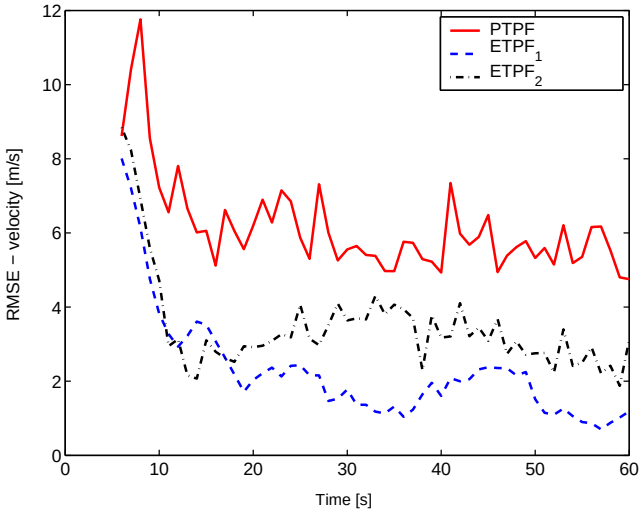
[15] A. Doucet, N. de Freitas, and N. Gordon, Eds., *Sequential Monte Carlo Methods in Practice*. Springer Verlag, 2001.

[16] N. Bergman, "Recursive Bayesian estimation: Navigation and tracking applications," Linköping Studies in Science and Technology. Dissertations No. 579, Linköping University, Linköping, Sweden, 1999.

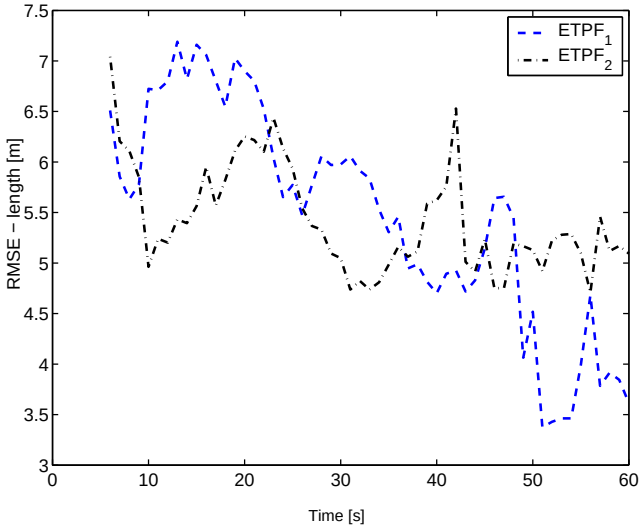
[17] R. Karlsson, "Particle filtering for positioning and tracking applications," Ph.D. dissertation, Linköping University, Linköping, Sweden, 2005, Linköping Studies in Science and Technology. Dissertations No. 924.



(a) RMSE position - turning.



(b) RMSE velocity - turning.



(c) RMSE length - turning.

Fig. 5. Tracking performance based on 100 Monte Carlo simulations for a maneuvering target. RMSE for position, velocity, and target length are depicted.