Learning Target Dynamics While Tracking Using Gaussian Processes

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Introduction

- General-purpose motion models in tracking

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- What if other behaviours or influences are visible?

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  - Wind affecting flying vehicles

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    - Wind affecting flying vehicles
    - Currents in the water affecting animals and ships

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  - Currents in the water affecting animals and ships
- Approximate initial model

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- Joint state estimation and learning of influences

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- Shared influences between multiple targets
- Time-varying influences

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Dynamic System with an Unknown Influence

$$\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0,\, \mathbf{P}_0),$$

Dynamic System with an Unknown Influence

$$
\begin{aligned}
&\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0), \\
&\mathbf{y}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{e}_k, && k = 1, \ldots, K
\end{aligned}
$$

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&\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{f}_{k-1} + \mathbf{v}_k, && k = 1, \dots, K
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\mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{f}_{k-1} + \mathbf{v}_k, && k = 1, \dots, K \\
\mathbf{f}_k &= \big(f^1(\mathbf{z}_k^f), \dots, f^J(\mathbf{z}_k^f)\big)^T
\end{aligned}
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\mathbf{f}_k &= \left(f^1(\mathbf{z}_k^f), \dots, f^J(\mathbf{z}_k^f)\right)^T \\
f^j(\mathbf{z}) &\sim \mathcal{GP}(0,\, K(\mathbf{z}, \mathbf{z}')), && j = 1, \dots, J
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f^j(\mathbf{z}) &\sim \mathcal{GP}(0,\, K(\mathbf{z}, \mathbf{z}')), && j = 1, \ldots, J
\end{aligned}
$$

The noise is given by

$$
\mathbf{v}_k \sim \mathcal{N}(\mathbf{0},\, \mathbf{Q}_k) \qquad \text{and} \qquad \mathbf{e}_k \sim \mathcal{N}(\mathbf{0},\, \mathbf{R}_k).
$$

Dynamic System with an Unknown Influence

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\mathbf{x}_0 &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\, \mathbf{P}_0), \\
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\mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{f}_{k-1} + \mathbf{v}_k, & k &= 1, \ldots, K \\
\mathbf{f}_k &= \left(f^1(\mathbf{z}_k^f), \ldots, f^J(\mathbf{z}_k^f)\right)^T \\
\mathcal{F} &\sim \mathcal{N}(\mathbf{0},\, \tilde{\mathbf{K}}_{ff}), & \mathcal{F} &= (\mathbf{f}_1^T, \ldots, \mathbf{f}_K^T)^T
\end{aligned}
$$

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$$
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$$

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Fully Independent Conditional

Inducing points $\mathbf{z}_l^u$, for $l = 1, \ldots, L$ with values $\mathbf{u}_l = (f^1(\mathbf{z}_l^u), \ldots, f^J(\mathbf{z}_l^u))^T$ represented by $\mathcal{U} = (\mathbf{u}_1^T, \ldots, \mathbf{u}_L^T)^T$.

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Original GP is

$$\begin{pmatrix} \mathcal{F} \\ \mathcal{U} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fu} \\ \mathbf{K}_{uf} & \mathbf{K}_{uu} \end{pmatrix} \otimes \mathbf{I}_J \right),$$

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Fully independent conditional (FIC) approximation is

$$\begin{pmatrix} \mathcal{F} \\ \mathcal{U} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \mathbf{Q}_{ff} + \mathbf{\Lambda} & \mathbf{K}_{fu} \\ \mathbf{K}_{uf} & \mathbf{K}_{uu} \end{pmatrix} \otimes \mathbf{I}_J \right),$$

where $\mathbf{Q}_{ab} = \mathbf{K}_{au}\mathbf{K}_{uu}^{-1}\mathbf{K}_{ub}$ and $\mathbf{\Lambda} = \operatorname{diag}(\mathbf{K}_{ff} - \mathbf{Q}_{ff})$.

Fully Independent Conditional

Inducing points $\mathbf{z}_l^u$, for $l = 1, \dots, L$ with values $\mathbf{u}_l = (f^1(\mathbf{z}_l^u), \dots, f^J(\mathbf{z}_l^u))^T$ represented by $\mathcal{U} = (\mathbf{u}_1^T, \dots, \mathbf{u}_L^T)^T$.

Fully independent conditional (FIC) approximation is

$$\begin{pmatrix} \mathcal{F} \\ \mathcal{W} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \mathbf{Q}_{ff} + \mathbf{\Lambda} & \mathbf{K}_{fu}\mathbf{K}_{uu}^{-1} \\ \mathbf{K}_{uu}^{-1}\mathbf{K}_{uf} & \mathbf{K}_{uu}^{-1} \end{pmatrix} \otimes \mathbf{I}_J \right),$$

where $\mathbf{Q}_{ab} = \mathbf{K}_{au}\mathbf{K}_{uu}^{-1}\mathbf{K}_{ub}$ and $\mathbf{\Lambda} = \operatorname{diag}(\mathbf{K}_{ff} - \mathbf{Q}_{ff})$.

Using $\mathcal{W} = (\mathbf{w}_1^T, \dots, \mathbf{w}_L^T)^T = \tilde{\mathbf{K}}_{uu}^{-1}\mathcal{U}$.

Approximation of the Influence Function

Original GP model is

$$
\begin{aligned}
\mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{f}_{k-1} + \mathbf{v}_k, \\
\mathcal{F} &\sim \mathcal{N}(\mathbf{0},\, \tilde{\mathbf{K}}_{\mathit{ff}}),
\end{aligned}
$$

Approximation of the Influence Function

Sparse GP approximation gives

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{f}_{k-1} + \mathbf{v}_k,$$

$$\begin{pmatrix} \mathcal{F} \\ \mathcal{W} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \mathbf{Q}_{ff} + \mathbf{\Lambda} & \mathbf{K}_{fu}\mathbf{K}_{uu}^{-1} \\ \mathbf{K}_{uu}^{-1}\mathbf{K}_{uf} & \mathbf{K}_{uu}^{-1} \end{pmatrix} \otimes \mathbf{I}_J \right)$$

Approximation of the Influence Function

Marginalizing out $\mathcal{F}$ results in

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k (\tilde{\mathbf{K}}_{fu}^{k-1} \mathcal{W} + \mathbf{v}_k^f) + \mathbf{v}_k,$$
$$\mathcal{W} \sim \mathcal{N}(\mathbf{0}, \tilde{\mathbf{K}}_{uu}^{-1}),$$

where $\mathbf{v}_k^f \sim \mathcal{N}(\mathbf{0}, \tilde{\Lambda}_k)$ and $\Lambda_k = [\mathbf{\Lambda}]_{kk}$.

Approximation of the Influence Function

Adding a prior to the GP gives

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k(\tilde{\mathbf{K}}_{fu}^{k-1}\mathcal{W} + \mathbf{v}_k^f) + \mathbf{v}_k,$$
$$\mathcal{W} \sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),$$

where $\mathbf{v}_k^f \sim \mathcal{N}(\mathbf{0}, \tilde{\Lambda}_k)$ and $\Lambda_k = [\mathbf{\Lambda}]_{kk}$.

Using $\bar{\mathcal{W}}_0 = \tilde{\mathbf{K}}_{uu}^{-1}\bar{\mathcal{U}}_0$ as the prior.

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Time-Varying Influence

Original GP model is

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k(\tilde{\mathbf{K}}_{fu}^{k-1}\mathcal{W} + \mathbf{v}_k^f) + \mathbf{v}_k,$$
$$\mathcal{W} \sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),$$

Time-Varying Influence

Time-varying influence is given by adding dynamics to the function

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k(\tilde{\mathbf{K}}_{fu}^{k-1} \mathcal{W}_{k-1} + \mathbf{v}_k^f) + \mathbf{v}_k,$$

$$\mathcal{W}_0 \sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),$$

$$\mathcal{W}_k = \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w$$

Time-Varying Influence

$$
\begin{aligned}
\mathbf{x}_k &= \mathbf{A}_k\mathbf{x}_{k-1} + \mathbf{B}_k(\tilde{\mathbf{K}}_{fu}^{k-1}\mathcal{W}_{k-1} + \mathbf{v}_k^f) + \mathbf{v}_k, \\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}), \\
\mathcal{W}_k &= \mathbf{G}_k\mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

where $\mathbf{v}_k^w \sim \mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$

Time-Varying Influence

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\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}), \\
\mathcal{W}_k &= \mathbf{G}_k\mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

where $\mathbf{v}_k^w \sim \mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$ and

$$
\begin{aligned}
\mathbf{G} &= e^{-\alpha T}\mathbf{I}, \\
\boldsymbol{\mu}_k &= (1 - e^{-\alpha T})\bar{\mathcal{W}}_0, \\
\boldsymbol{\Sigma}_k &= (1 - e^{-2\alpha T})\tilde{\mathbf{K}}_{uu}^{-1}.
\end{aligned}
$$

Multiple Systems with Shared Influence

Original GP model is

$$
\begin{aligned}
\mathbf{x}_0 &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\, \mathbf{P}_0),\\
\mathbf{y}_k &= \mathbf{C}_k \mathbf{x}_k + \mathbf{e}_k,\\
\mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k(\tilde{\mathbf{K}}_{fu}^{k-1} \mathcal{W}_{k-1} + \mathbf{v}_k^f) + \mathbf{v}_k,\\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),\\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

Multiple Systems with Shared Influence

Multiple systems are modelled by the extension

$$
\begin{aligned}
\mathbf{x}_0^i &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\ \mathbf{P}_0),\\
\mathbf{y}_k^i &= \mathbf{C}_k \mathbf{x}_k^i + \mathbf{e}_k^i,\\
\mathbf{x}_k^i &= \mathbf{A}_k \mathbf{x}_{k-1}^i + \mathbf{B}_k(\tilde{\mathbf{K}}_{iu}^{k-1} \mathcal{W}_{k-1} + \mathbf{v}_k^{if}) + \mathbf{v}_k^i,\\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),\\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
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\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),\\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

where $\mathbf{z}_k^i$ is the input for process $i = 1, \ldots, I$.

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State-Dependent Influence

Original GP model is

$$\begin{aligned}
\mathbf{x}_0^i &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\ \mathbf{P}_0), \\
\mathbf{y}_k^i &= \mathbf{C}_k \mathbf{x}_k^i + \mathbf{e}_k^i, \\
\mathbf{x}_k^i &= \mathbf{A}_k \mathbf{x}_{k-1}^i + \mathbf{B}_k \Big(\tilde{\mathbf{K}}_{iu}^{k-1} \mathcal{W}_{k-1} + \\
&\qquad \mathbf{v}_k^{if} \Big) + \mathbf{v}_k^i, \\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}), \\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}$$

State-Dependent Influence

State dependence is modelled by the extension

$$
\begin{aligned}
\mathbf{x}_0^i &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\, \mathbf{P}_0),\\
\mathbf{y}_k^i &= \mathbf{C}_k \mathbf{x}_k^i + \mathbf{e}_k^i,\\
\mathbf{x}_k^i &= \mathbf{A}_k \mathbf{x}_{k-1}^i + \mathbf{B}_k \Big(\tilde{\mathbf{K}}_{\cdot u}(\mathbf{D}_k \mathbf{x}_{k-1}^i) \mathcal{W}_{k-1} + \\
&\qquad \mathbf{v}_k^{if}(\mathbf{D}_k \mathbf{x}_{k-1}^i) \Big) + \mathbf{v}_k^i,\\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),\\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

State-Dependent Influence

Resulting in the Gaussian process motion model(GPMM)

$$
\begin{aligned}
\mathbf{x}_0^i &\sim \mathcal{N}(\bar{\mathbf{x}}_0,\ \mathbf{P}_0),\\
\mathbf{y}_k^i &= \mathbf{C}_k \mathbf{x}_k^i + \mathbf{e}_k^i,\\
\mathbf{x}_k^i &= \mathbf{A}_k \mathbf{x}_{k-1}^i + \mathbf{B}_k \Big(\tilde{\mathbf{K}}_{\cdot u}(\mathbf{D}_k \mathbf{x}_{k-1}^i) \mathcal{W}_{k-1} + \\
&\qquad \mathbf{v}_k^{if}(\mathbf{D}_k \mathbf{x}_{k-1}^i) \Big) + \mathbf{v}_k^i,\\
\mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}),\\
\mathcal{W}_k &= \mathbf{G}_k \mathcal{W}_{k-1} + \mathbf{v}_k^w
\end{aligned}
$$

where $\mathbf{z}_k^i = \mathbf{D}_k \mathbf{x}_{k-1}^i$.

Extended Kalman Filter

- Non-linear dependence on state in kernel

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- Almost linear-Gaussian

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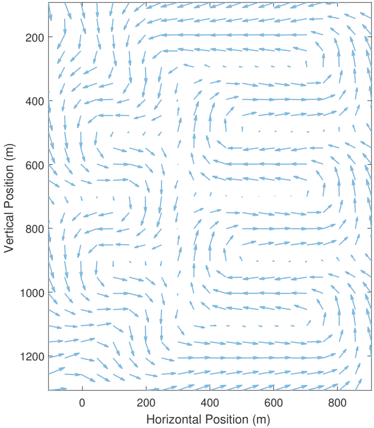
Extended Kalman Filter

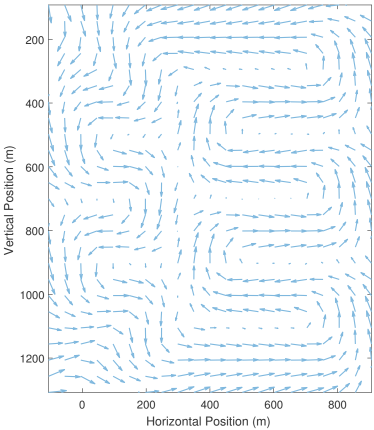
- Non-linear dependence on state in kernel
- Almost linear-Gaussian
- Extended Kalman filter(EKF)
- Noisy input compensated for implicitly by filter
- Enormous state space so approximations are needed

Velocity Field Simulation



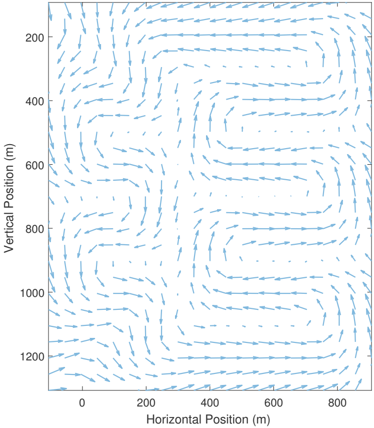
- 200 targets

Velocity Field Simulation



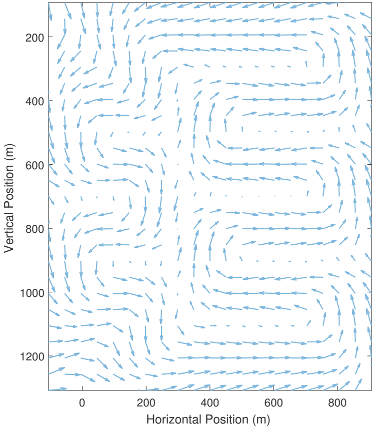
- 200 targets
- 150 time steps

Velocity Field Simulation



- 200 targets
- 150 time steps
- Velocity given by function

Velocity Field Simulation



- 200 targets
- 150 time steps
- Velocity given by function
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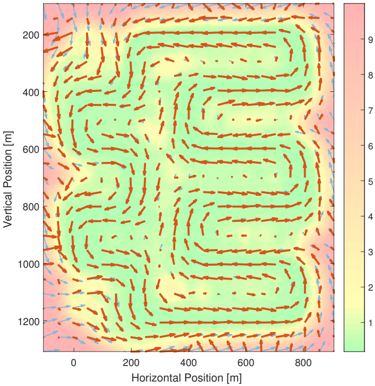
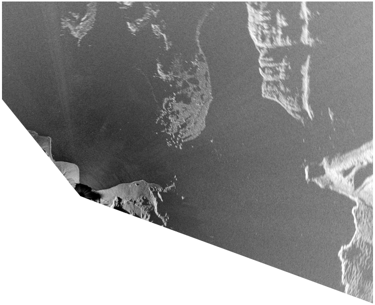
Velocity Field Simulation



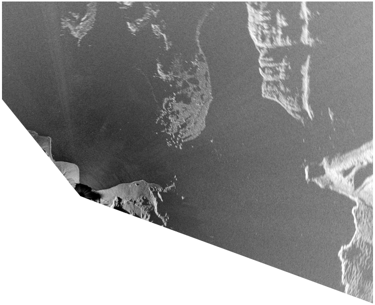
- 200 targets
- 150 time steps
- Velocity given by function
- Targets are identified
- Velocity field estimated well

Sea Ice Tracking



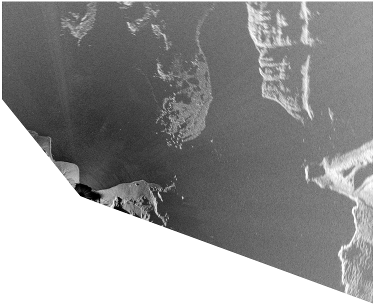
- Radar station detecting ice

Sea Ice Tracking



- Radar station detecting ice
- Measurements for long tracks are extracted from simple tracker

Sea Ice Tracking



- Radar station detecting ice
- Measurements for long tracks are extracted from simple tracker
- Acceleration caused by currents are modelled by GP

Sea Ice Tracking

- Radar station detecting ice
- Measurements for long tracks are extracted from simple tracker
- Acceleration caused by currents are modelled by GP
- Prediction errors are reduced

Conclusions

Theory is presented on

- a Gaussian process motion model

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Theory is presented on

- a Gaussian process motion model
- a number of variations of the model

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Theory is presented on

- a Gaussian process motion model
- a number of variations of the model
- tractable estimation for the model

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Theory is presented on

- a Gaussian process motion model
- a number of variations of the model
- tractable estimation for the model

Demonstration through two applications

Future Work

Possible future work is

- splitting up the input space

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- analysis of the impact of approximations

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- splitting up the input space
- analysis of the impact of approximations
- marginalized particle filter implementation

Thank you for listening!

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