Learning Target Dynamics While Tracking Using Gaussian Processes

Clas Veibäck, Jonatan Olofsson, Tom Rune Lauknes and Gustaf Hendeby

Problem Description

Common general-purpose motion models do not take into consideration behaviours, such as

- preferred paths in open terrain or indoors;
- velocity or acceleration profiles in racing events; and
- water currents affecting animals and ships.

The method described here learns these influences online to improve tracking performance.

Joint inference of the states and learning of the unknown influence $\mathbf{f}(\mathbf{z})$ are considered for the model

$$\begin{aligned} &\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0,\ \mathbf{P}_0), \\ &\mathbf{x}_k = \mathbf{A}_k\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{f}(\mathbf{D}_k\mathbf{x}_{k-1}) + \mathbf{v}_k, \qquad k = 1, \dots, K \\ &\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{e}_k. \end{aligned}$$

Influence Model

The influence is modelled as a Gaussian process,

$$\begin{aligned} &\mathbf{f}_k(\mathbf{z}) = \big(f^1(\mathbf{z}_k^f), \dots, f^J(\mathbf{z}_k^f)\big)^T, \\ &f^j(\mathbf{z}) \sim \mathcal{GP}(0,\ K(\mathbf{z}, \mathbf{z}')), \qquad j = 1, \dots, J, \end{aligned}$$

with covariance function $K(\mathbf{z}, \mathbf{z}')$. Using the *fully independent conditional* approximation with the inducing points $\mathbf{z}_l^u$, $l = 1, \dots, L$, this reduces to

$$\begin{aligned} &\mathbf{f}_k(\mathbf{z}) = \tilde{\mathbf{K}}_{\cdot u}(\mathbf{z})\mathcal{W} + \mathbf{v}_k^f(\mathbf{z}), \qquad [\mathbf{K}_{\cdot u}(\mathbf{z})]_{1l} = K(\mathbf{z}, \mathbf{z}_l^u), \\ &\mathcal{W} \sim \mathcal{N}(\mathbf{0}, \tilde{\mathbf{K}}_{uu}^{-1}), \qquad [\mathbf{K}_{uu}]_{lm} = K(\mathbf{z}_l^u, \mathbf{z}_m^u), \end{aligned}$$

where $\mathbf{v}_k^f(\mathbf{z})$ is Gaussian noise and $\tilde{\mathbf{A}} = \mathbf{A} \otimes \mathbf{I}_J$. Including multiple targets and allowing the function to vary over time gives the *Gaussian process motion model* (GPMM).

Inference & Learning

- Estimation using an *extended Kalman filter* (EKF).
- Gridded inducing points.
- Approximations to improve speed.

Gaussian Process Motion Model

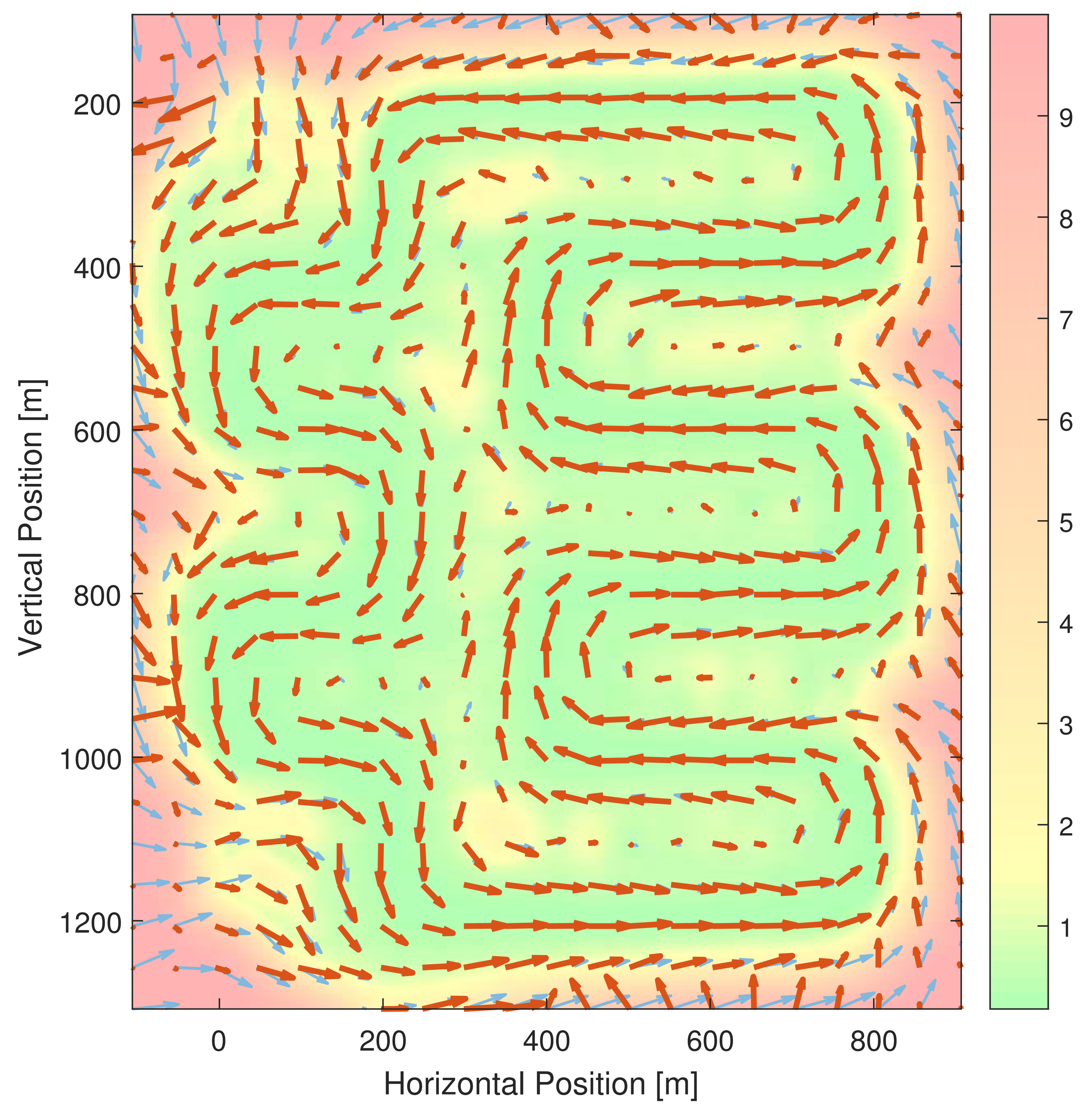
A model is proposed on the form

$$\begin{aligned} \mathbf{x}_0^i &\sim \mathcal{N}(\bar{\mathbf{x}}_0^i,\ \mathbf{P}_0^i), \\ \mathbf{x}_k^i &= \mathbf{A}_k\mathbf{x}_{k-1}^i + \mathbf{B}_k\Big(\tilde{\mathbf{K}}_{\cdot u}(\mathbf{D}_k\mathbf{x}_{k-1}^i)\mathcal{W}_{k-1} + \mathbf{v}_k^{if}(\mathbf{D}_k\mathbf{x}_{k-1}^i)\Big) + \mathbf{v}_k^i, \\ \mathbf{y}_k^i &= \mathbf{C}_k\mathbf{x}_k^i + \mathbf{e}_k^i, \\ \mathcal{W}_0 &\sim \mathcal{N}(\bar{\mathcal{W}}_0, \tilde{\mathbf{K}}_{uu}^{-1}), \\ \mathcal{W}_k &= \mathbf{G}_k\mathcal{W}_{k-1} + \mathbf{v}_k^w, \end{aligned}$$

where $\mathbf{G} = e^{-\alpha T}\mathbf{I}$ and $\mathbf{v}_k^w \sim \mathcal{N}((1-e^{-\alpha T})\bar{\mathcal{W}}_0, (1-e^{-2\alpha T})\tilde{\mathbf{K}}_{uu}^{-1})$.

Velocity Field Simulation

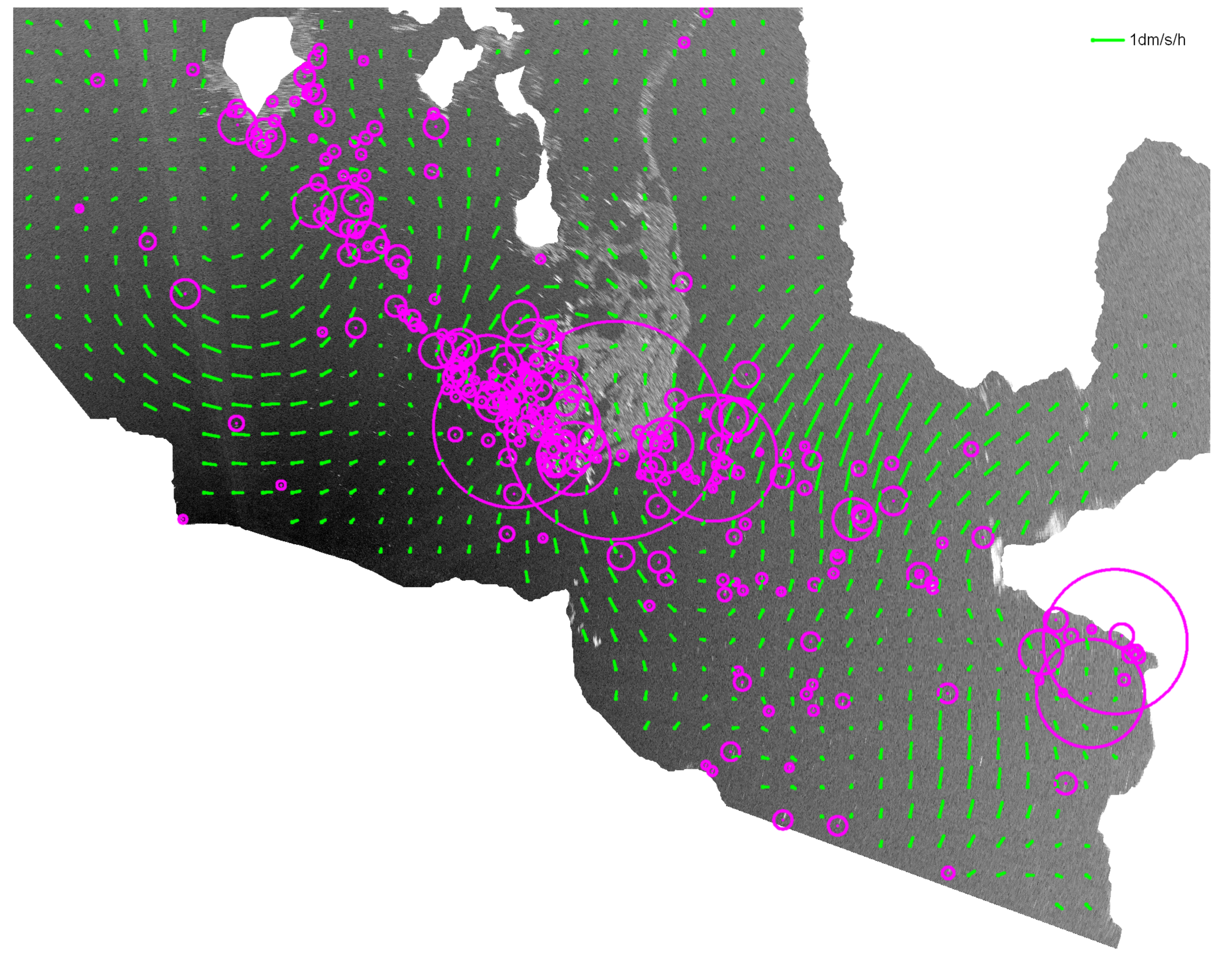
Simulation of 200 targets in a static velocity field. Blue arrows show the true function and red arrows show the estimated function. The background colour indicates uncertainty. The function is estimated well where data is available.



Estimated velocity field for 200 targets.

Sea Ice Tracking Application

Sea ice radar data with targets modelled as constant velocity in a time-varying acceleration field. The green arrows show the estimated acceleration.



Estimated velocity field and tracks.

Summary

- Inference and learning for dynamical systems in one framework.
- Learning of particular influence or behaviour.
- Time-varying functions.
- Fixed computation time.

Future Work

- Improve scalability.
- Determine hyperparameters.
- Analysis of approximations.
- Data association.

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