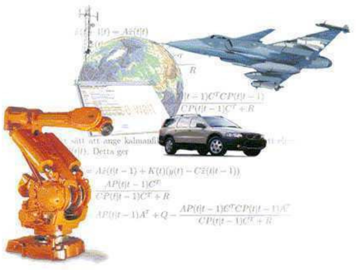
Method to Estimate the Position and Orientation of a Triaxial Accelerometer Mounted to an Industrial Manipulator



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Outline

1. Introduction
2. Calculation of the orientation
3. Calculation of the position
4. Results

- Less rigid manipulators $\Rightarrow$ New control strategies necessary.
- One possible solution: Include an accelerometer in the estimation of the system states*.

$$
\begin{aligned}
\dot{x} &= f(x,u) \\
y &= g(x)
\end{aligned}
$$

- Requires good knowledge of the orientation and position of the accelerometer.

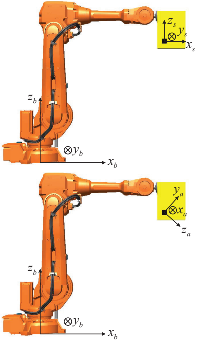
* Patrik Axelsson, **Evaluation of Six Different Sensor Fusion Methods for an Industrial Robot using Experimental Data**. In proceedings of the 10th International IFAC Symposium on Robot Control, 2012.

The acceleration of the end effector (including gravity),

$$\rho_s = h(q, \dot{q}, \ddot{q}, \theta_s)$$

Accelerometer measurement

$$\rho_a = g(\rho_s, \theta_a)$$



The acceleration of the end effector (including gravity),

$$\rho_s = h(q, 0, 0, \cdot)$$

Accelerometer measurement

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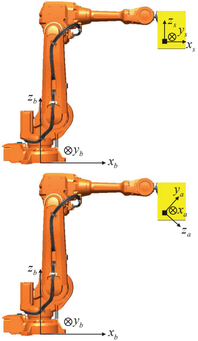
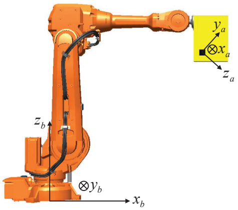
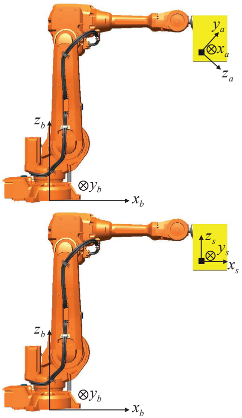


Calculation of the orientation - Step 1



- Looking for a transformation

$$\rho_s = \kappa \mathcal{R}_{a/s} \rho_a + \rho_0.$$



- Looking for a transformation

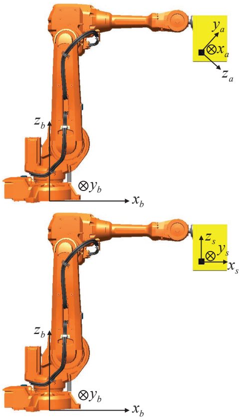
$$\rho_s = \kappa \mathcal{R}_{a/s} \rho_a + \rho_0.$$

- Define the residuals

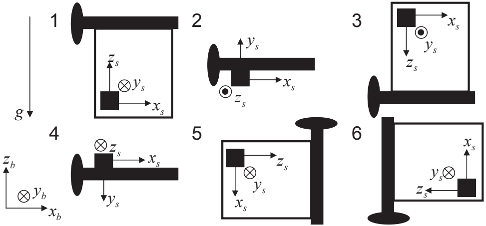
$$e_k = \rho_{s,k} - \kappa \mathcal{R}_{a/s} \rho_{a,k} - \rho_0.$$

- Optimisation problem:

$$\begin{aligned} \text{minimise} \quad & \sum_{k=1}^{N} ||e_k||^2 \\ \text{subject to} \quad & \det(\mathcal{R}_{a/s}) = 1 \\ & \mathcal{R}_{a/s}^{\mathsf{T}} = \mathcal{R}_{a/s}^{-1} \end{aligned}$$



- Measure the gravitation in different orientations.



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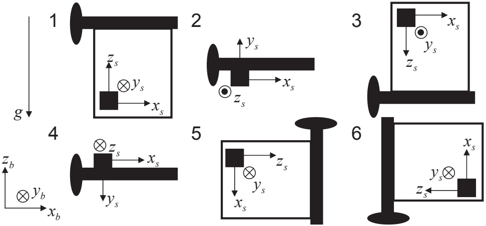


- Knows that ρ_s should resemble:

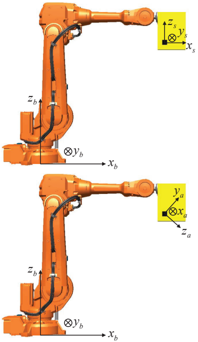
$$\rho_s^1 = \begin{pmatrix} 0 & 0 & g \end{pmatrix}^\mathsf{T}, \rho_s^2 = \begin{pmatrix} 0 & g & 0 \end{pmatrix}^\mathsf{T}, \rho_s^3 = \begin{pmatrix} 0 & 0 & -g \end{pmatrix}^\mathsf{T},$$
$$\rho_s^4 = \begin{pmatrix} 0 & -g & 0 \end{pmatrix}^\mathsf{T}, \rho_s^5 = \begin{pmatrix} -g & 0 & 0 \end{pmatrix}^\mathsf{T}, \rho_s^6 = \begin{pmatrix} g & 0 & 0 \end{pmatrix}^\mathsf{T}.$$

The acceleration of the end effector (including gravity),

$$\rho_s = h(q, \omega, 0, \theta_s)$$

Accelerometer measurement

$$\rho_a = g(\rho_s, \theta_a)$$



- Rotate joint 1 with constant velocity.
- Gives an acceleration pointing in to the center of the rotation in a plan perpendicular to the gravity.

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- The analytical expressions depends on the unknown position parameters.

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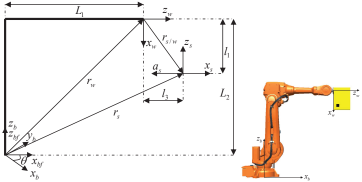
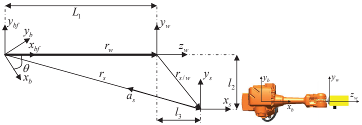
- Gives an acceleration pointing in to the center of the rotation in a plan perpendicular to the gravity.
- Calculate an analytical expression for the acceleration and compare with the measurements.
- The analytical expressions depends on the unknown position parameters.
- The orientation of the accelerometer is assumed to be known, i.e., the transformation from $Ox_ay_az_a$ to $Ox_sy_sz_s$ is known.
- Orientate the accelerometer such that the gravity is aligned with one of the axis in $Ox_sy_sz_s$.

Calculation of the position





- The position of the accelerometer is given by r_s

$$[r_s]_b = \left[Q_{bf/b}\right]_b \left([r_w]_{bf} + [r_{s/w}]_{bf}\right).$$

- Differentiation twice w.r.t. time gives the acceleration

$$[a_s]_b = S(\omega)S(\omega) \left[Q_{bf/b}\right]_b \left([r_w]_{bf} + [r_{s/w}]_{bf}\right).$$

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Calculation of the position

- Comparison with the measurements give

$$\begin{pmatrix} 0 & -\dot{\theta}^2 \\ \dot{\theta}^2 & 0 \end{pmatrix} \begin{pmatrix} l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} a_{s,x}^M + \dot{\theta}^2 L_1 \\ a_{s,y}^M \end{pmatrix}.$$

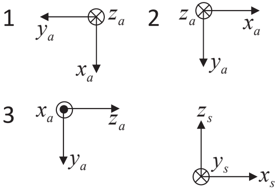
- Comparison with the measurements give

$$\begin{pmatrix} 0 & -\dot{\theta}^2 \\ \dot{\theta}^2 & 0 \end{pmatrix} \begin{pmatrix} l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} a^M_{s,x} + \dot{\theta}^2 L_1 \\ a^M_{s,y} \end{pmatrix}.$$

- Two more robot configurations give

$$\underbrace{\begin{pmatrix} 0 & 0 & -\dot{\theta}^2_{c1} \\ 0 & \dot{\theta}^2_{c1} & 0 \\ \dot{\theta}^2_{c2} & 0 & 0 \\ 0 & \dot{\theta}^2_{c2} & 0 \\ 0 & 0 & -\dot{\theta}^2_{c3} \\ \dot{\theta}^2_{c3} & 0 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}}_{l} = \underbrace{\begin{pmatrix} a^M_{s,x,c1} + \dot{\theta}^2_{c1} L_1 \\ a^M_{s,y,c1} \\ a^M_{s,z,c2} + \dot{\theta}^2_{c2} L_3 \\ a^M_{s,y,c2} \\ a^M_{s,x,c3} + \dot{\theta}^2_{c3} L_1 \\ a^M_{s,z,c3} \end{pmatrix}}_{b}.$$

- The accelerometer is mounted in 3 positions and orientations.



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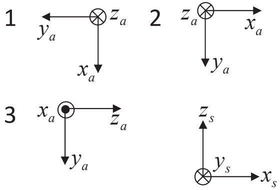

- The rotation matrix should resemble

$$\mathcal{R}_{a/s}{}^{1} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, \mathcal{R}_{a/s}{}^{2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$\mathcal{R}_{a/s}{}^{3} = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}.$$

Results

- The rotation difference between the “true” and estimated rotation matrix can be calculated using quaternions.

Test	1	2	3
ϑ	1.4°	1.8°	2.4°

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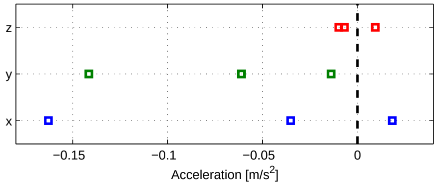
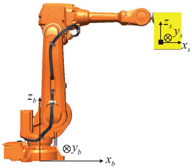
- Transform the measurements and compare with the expected result.





Estimated position

Test	Est. pos. ($\hat{l}$) [cm]	$\Delta = \hat{l} - l^M$ [cm]	Std. for $\hat{l}$ [cm]
1	$\begin{pmatrix}35.2 & 6.3 & 15.5\end{pmatrix}^T$	$\begin{pmatrix}0.2 & 2.3 & -1.0\end{pmatrix}^T$	$\begin{pmatrix}0.4 & 0.5 & 0.5\end{pmatrix}^T$
2	$\begin{pmatrix}14.2 & 5.8 & 16.9\end{pmatrix}^T$	$\begin{pmatrix}-0.3 & -1.2 & 1.8\end{pmatrix}^T$	$\begin{pmatrix}0.3 & 0.3 & 0.3\end{pmatrix}^T$
3	$\begin{pmatrix}29.2 & 1.6 & 5.9\end{pmatrix}^T$	$\begin{pmatrix}2.2 & 1.6 & 0.4\end{pmatrix}^T$	$\begin{pmatrix}0.4 & 0.4 & 0.4\end{pmatrix}^T$

Conclusions

- Method to find position and orientation of an accelerometer mounted to an industrial manipulator.
  - Step 1: Calculating transformation from the actual frame (unknown orientation) to a desired frame (known orientation)
    - Static measurements of the gravity component.
  - Step 2: Calculating the position of the accelerometer in a robot fixed frame.
    - Measurements when joint 1 moves with constant velocity.
- Evaluated on experimental data.
- Orientation error: 1 to 2 degrees.
- Position error: 0.5 to 1 cm.
- Sufficient for tool position estimation using Bayesian techniques.

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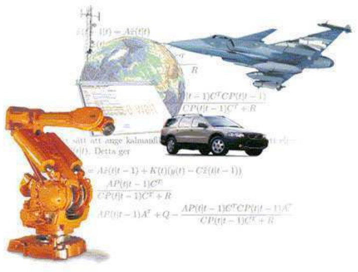

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