Influences of the load centre of gravity on heavy vehicle acceleration

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Abstract: The aim of this paper is to analyse the influence of the load centre of gravity on heavy vehicle acceleration. This analysis is done through a method in which a vehicle centre of gravity map is used. A model for the driving force is presented for bus, truck and tractor-semitrailer combinations. The proposed model takes into consideration the resistance forces (drag, rolling resistance, translation and rotation acceleration, climbing resistance) and the 4 × 2 traction system. The positions of the vehicle centre of gravity as a function of the position of the load centre of gravity are determined. The vehicle acceleration is calculated based on the position of the load centre of gravity. This study analyses the acceleration of one of the Mercedes-Benz do Brasil tractor-semitrailer vehicle. A comparison of the acceleration for different maximum adhesion coefficients and ramps are presented, showing new results. An example showing the variations of the load centre of gravity position with the acceleration time and distance is provided. The load centre of gravity position is important for vehicle safety and the efficiency and economy in the transportation of the load.

Keywords: centre of gravity, heavy vehicle modelling, heavy vehicle simulation, vehicle acceleration.


Nomenclature

1. index for the tractor
2. index for the semitrailer
a. acceleration of the vehicle
A. matrix in the equation system \( A \times \mathbf{x} = \mathbf{b} \)
A. frontal area of the vehicle
\( A_{NS} \). reaction of the normal force on the fifth-wheel

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reaction of the longitudinal force on the fifth-wheel

vector in the equation system $A \cdot X = B$

centre of gravity of the tractor with load $q$, $c_{g1} = (c_{g1x}, c_{g1y}, c_{g1z})$

centre of gravity of the semitrailer with load $q$, $c_{g2} = (c_{g2x}, c_{g2y}, c_{g2z})$

centre of gravity of the fully loaded tractor, $c_{g1}^* = (c_{g1x}^*, c_{g1y}^*, c_{g1z}^*)$

centre of gravity of the fully loaded semitrailer, $c_{g2}^* = (c_{g2x}^*, c_{g2y}^*, c_{g2z}^*)$

centre of gravity of the empty tractor, $c_{g1e} = (c_{g1xe}, c_{g1ye}, c_{g1ze})$

centre of gravity of the empty semitrailer, $c_{g2e} = (c_{g2xe}, c_{g2ye}, c_{g2ze})$

centre of gravity of the tractor with load $q$ in directions $x$, $y$ and $z$, respectively

centre of gravity of the semitrailer with load $q$ in directions $x$, $y$ and $z$, respectively

coefficient of air resistance

coefficient of rolling resistance

adherence force

maximum adherence force

total traction force available from engine

total traction force acting in the ground plane in the tyre contact path

total resistance force

tractor front axle tractive force acting in the ground plane in the tyre contact path

tractor rear axle tractive force acting in the ground plane in the tyre contact path

acceleration of gravity

grade of the ramp

height of the aerodynamic drag force

equivalent mass of non-driving tractor front wheels

equivalent mass of non-driving semitrailer rear wheels

moment of inertia of the final drive

equivalent moment of inertia of the engine
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\[ I_1 \]
moment of inertia of the transmission

\[ I_w \]
moment of inertia of each wheel

\[ I_{w1} \]
moment of inertia of the tractor front wheels

\[ I_{w2} \]
moment of inertia of the tractor rear wheels

\[ I_{w22} \]
moment of inertia of the semitrailer rear wheels

\[ l_1 \]
wheelbase of the tractor

\[ l_2 \]
distance between the fifth-wheel and the central rear axle of the semitrailer

\[ m \]
vehicle total mass

\[ n \]
engine speed

\[ n_{tr2} \]
number of rear axles of the semitrailer

\[ N_5 \]
normal force on the fifth-wheel

\[ N_f \]
numerical ratio of the final drive

\[ N_{t1} \]
normal force on the tractor front axle

\[ N_{t2} \]
normal force on the tractor rear axle

\[ N_{tr2} \]
normal force on the semitrailer rear axle

\[ N_t \]
numerical ratio of the transmission

\[ N_{tf} \]
numerical ratio of the combined transmission and the final drive

\[ N_{tot} \]
total normal force

\[ r \]
effective rolling radius of the tyre (dynamic tyre radius)

\[ R_5 \]
longitudinal force on the fifth-wheel

\[ R_a \]
aerodynamic drag force

\[ R_a \]
total climbing resistance force

\[ R_{g1} \]
tractor climbing resistance force

\[ R_{g2} \]
semitrailer climbing resistance force

\[ R_{t1} \]
tractor inertia force

\[ R_{t2} \]
semitrailer inertia force

\[ R_{tr1} \]
rotating inertia force for tractor front wheels

\[ R_{tr2} \]
rotating inertia force for tractor rear wheels

\[ R_{tr2} \]
rotating inertia force for semitrailer rear wheels
20  

$R_{ad}$  

total rotating inertia force of the driving wheels

$R_{nd}$  

total rotating inertia force of the non-driving wheels

$R_r$  

total rolling resistance force

$R_{rf1}$  

rolling resistance force on the tractor front axle

$R_{rr1}$  

rolling resistance force on the tractor rear axle

$R_{rn2}$  

rolling resistance force on the semitrailer rear axle

$T_{f1}$  

tractor front axle tractive force available from the engine

$T_{r1}$  

tractor rear axle tractive force available from the engine

$T_e$  

drive engine torque

$v$  

instantaneous velocity of the vehicle

$W$  

vehicle total weight

$W_1$  

tractor weight

$W_2$  

semitrailer weight

$W_{2e}$  

fully loaded semitrailer weight

$W_{1e}$  

empty tractor weight

$W_{2e}$  

empty semitrailer weight

$W_{5e}$  

maximum weight on the fifth-wheel

$W_{res}$  

static weight on the empty tractor front axle (tractor without semitrailer)

$W_{res2}$  

static weight on the empty tractor front axle (tractor and semitrailer together)

$W_{res2}^*$  

static weight on the fully loaded tractor front axle (tractor and semitrailer together)

$W_{res}$  

static weight on the empty tractor rear axle (tractor without semitrailer)

$W_{res2}$  

static weight on the empty tractor rear axle (tractor and semitrailer together)

$W_{res2}^*$  

static weight on the fully loaded tractor rear axle (tractor and semitrailer together)

$W_{res3}$  

static weight on the empty semitrailer rear axle (tractor and semitrailer together)
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\( W_{r2s1} \)  
static weight on the fully loaded semitrailer rear axle (tractor and semitrailer together)

\( x \)  
longitudinal direction

\( x_s \)  
distance of the fifth-wheel to the back axle of the tractor (fifth-wheel offset)

\( x_a \)  
position initial for the centre of gravity of the load in the direction \( x \)

\( x_b \)  
position final for the centre of gravity of the load in the direction \( x \)

\( X \)  
unknown vector in the equation system \( A \cdot X = B \)

\( y \)  
lateral direction

\( z \)  
vertical direction

\( z_s \)  
height of the fifth-wheel in relation to the pavement

\( z_a \)  
position initial for the centre of gravity of the load in the direction \( z \)

\( z_b \)  
position final for the centre of gravity of the load in the direction \( z \)

\( \gamma \)  
factor of the inertia of the rotating parts of vehicle

\( \eta_i \)  
efficiency of the final drive

\( \eta_t \)  
efficiency of the transmission

\( \eta_{sf} \)  
combined efficiency of the transmission and final drive

\( \mu \)  
used adhesion coefficient

\( \mu_0 \)  
maximum adhesion coefficient

\( \mu_{rl} \)  
tractor rear axle adhesion coefficient

\( \theta \)  
road grade

\( \rho \)  
air density

1 Introduction

Vehicle acceleration performance is influenced by the location of the load. Several authors have investigated this kind of influence on vehicle dynamics. Bohn et al. (1981) developed a simulation for the effects of liquid cargo steady-state shifting on braking performance and cornering manoeuvres. Ranganathan and Yang (1996) also investigated the braking characteristics of a tractor-tank-semitrailer vehicle incorporating the influence of liquid load shift occurring within the partially filled tank. Ervin et al. (1985) gave a literature review on the sloshing of fluids in cargo containers with regard to safety. The latter authors also showed the results of an inquiry into world-wide regulatory restraints on cargo tank truck design and operation as well as the risk of rollover occurring during operation of slosh-loaded cargo tank trucks. Canale and Ruffino (1992)
studied the influence of the centre of gravity (CG) position on braking performance of passenger cars. Fernandes (1994) investigated the positions of the centre of gravity for the tractor-semitrailers and the influence of load position on braking performance. Schindler (1991) showed some measurement results obtained by application of the test procedures for the evaluation handling characteristics of heavy commercial vehicles which point out the influence of different loading conditions on the driving behaviour of two trucks with a vehicle gross weight of about 14 tons. Winkler (1986) examined the influence of trailer loading on braking performance, rearward amplification, and yaw damping of typical twin 28-foot doubles combination vehicles.

A number of researchers have investigated the influence of hill climbing on acceleration performance. Abbas (1982) described the acceleration capability of five-axle trucks along grades of a gradient from $-7$ per cent to $+7$ per cent on straight sections of roads under free traffic conditions. Gillespie (1986) presented a study about the acceleration performance of heavy trucks starting on grades. Khan et al. (1990) developed a model that supports a detailed simulation of vehicle performance on grade. In this model, forces resisting vehicle motion and the tractive effort of the vehicle are incorporated. Navarro et al. (1997) investigated the time, the distance travelled, the acceleration, and the velocity changes while overtaking on a grade.

An analysis of the dynamic performance of heavy vehicles is presented in Ervin et al. (1984) and Fancher and Mathew (1987). Ervin et al. (1984) assessed, by means of computerised analysis and review of existing literature, aspects of the dynamic performance of long truck combinations such as backing up, braking performance, issues related to brake system air delivery, low-speed offtracking, high-speed offtracking, stability issues related to rapid steering manoeuvres, roll stability and yaw stability of the vehicle. These authors compare the performance characteristics of the vehicle configurations in areas thought to have implications for operating efficiency and traffic safety. Fancher and Mathew (1987) provided a compilation of the effects of the mechanical properties of vehicle components and configurations on the braking and steering of heavy trucks. They describe the braking and steering performance of straight trucks and tractor-semitrailers and present performance measures for driving situations involving constant deceleration braking, low- and high-speed offtracking, steady turning, initiation of curved paths, obstacle evasion (quick lane changes), and braking while turning.

Navarro (1997) presented a study about acceleration performance and also an analysis of commercial vehicles' fuel consumption. The resistance forces involved in the vehicle movement, the ideal forces, in which all the motor axles of the vehicle use the same adhesion coefficient, the real forces with transmission system, the parameters to the development of acceleration performance and the vehicle fuel consumption prevision are determined in that work.


In Section 2 of this paper there is a description of a model for single and combination commercial vehicles with $4 \times 2$ traction systems, i.e. traction only in the rear axle. The proposed model considers forces, such as drag resistance, rolling resistance, translation and rotation acceleration and climbing resistance. Section 3 presents a vehicle CG map, i.e. the positions of the vehicle’s centre of gravity as a function of the position of the load’s centre of gravity.
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A method for computing the heavy vehicle acceleration using a vehicle CG map is presented. New results are obtained through its use. This method consists of calculating the heavy vehicle acceleration by using the procedure described in Section 4, for several engine speeds and engaged gears, in association with the use of the vehicle CG map.

In Section 5, the acceleration of one of the Mercedes-Benz do Brasil tractor-semi-trailer vehicle is determined for several loads, such as, empty, a partially loaded and a fully loaded vehicle. This acceleration is computed for different maximum adhesion coefficients (dry road $\mu_0 = 0.7$ and wet water (slippery) road $\mu_0 = 0.3$) and grade of ramps ($G = 0\%$ and $G = 3\%$). An example showing the variations of the load centre of gravity position with the acceleration time and distance is provided.

2 Model for single vehicles with two-axles and combination vehicles

The ideal condition is a situation in which the available driving force is maximum, that is, when all the driving axles utilise the same coefficient of road adhesion. In this condition, the maximum coefficient of road adhesion will be reached by all the axles simultaneously. The term real condition is adopted in this work to describe a situation in which the forces acting on the vehicle are determined by a specific installed engine and transmission system. It is assumed that the real forces are the same as the ideal ones, so that it is possible to obtain the maximum performance of the vehicle characteristics. In real conditions, for vehicles with traction in more than one axle, it is necessary to know the distribution of the traction force according to the transmission system installed in the vehicle.

![Figure 1: Force diagram of a combination vehicle with two axles in the tractor.](image)

The mathematical model developed in this study is for both single commercial vehicles (buses, trucks) and combination commercial vehicles (tractor-semi-trailers). Figure 1 illustrates a combination vehicle whose tractor has two axles. The model analysed is suitable for the vehicles which Karamihes and Gillespie (1993) named 3-axle tractor-semi-trailer, or 4-axle tractor-semi-trailer or 5-axle tractor-semi-trailer. According to Gillespie (1992), for acceleration analysis, the vehicles can be represented as one lumped mass located at its centre of gravity with appropriate mass and inertia.
properties. The axle loadings on a vehicle are expressed by a system of the form \( A \cdot X = B \) determined with application of Newton’s second law of motion. The vehicle weight acts at its centre of gravity, so when the vehicle is on a grade its weight has two components: one perpendicular to the road surface and another parallel to it. The effect of the vehicle acceleration is represented by an equivalent inertial force (d’Alembert force), acting at the centre of gravity, opposite to the direction of the acceleration. The normal forces on the tyres represent the dynamic loads carried on the wheels. Tractive forces and rolling resistance forces act on the tyre during ground contact. Aerodynamic force acts on the body of the vehicle at a point above ground. Forces connecting tractor with the semitrailer act on the fifth-wheel. The additional forces necessary to accelerate the rotating masses can be determined using the rotating mass factor. The vehicle axis directions are in accordance with ISO8855 (x-axis: forward, y-axis: vehicle-left and z-axis: up). For single vehicles, the forces \( N_{1z}, N_{2y}, R_{1}, W_{1}, R_{2z}, R_{1n2}, R_{2n2} \) and \( R_{1z} \) should be ignored in the system \( A \cdot X = B \) presented in this section. For vehicles with traction \( 4 \times 2 \), two forms are presented for the system \( A \cdot X = B \). In the first \( A \cdot X \) \((N_{1z}, N_{2}, R_{1}, N_{11}, N_{1}, F_{x1}) = B, \) the normal forces \((N_{1z}, N_{r1}, N_{r1})\), fifth-wheel forces \((N_{r}, R_{1})\) and traction force \((F_{x1})\) are unknown parameters determined from the acceleration. In the second \( A \cdot X \) \((N_{1z}, N_{r1}, N_{1}, N_{r1}, F_{x1}, a) = B, \) \( N_{1z}, N_{r1}, N_{r1}, N_{r1}, F_{x1}, a \) and acceleration \( a \) are unknown parameters determined once the adhesion coefficient is specified (see Equation (30), later). The solution \( X = A^{-1} \cdot B \) is numerically calculated. Limpert (1992) presented a similar analysis of braking forces acting on single and combination commercial vehicles, in which the resistance forces were not taken into consideration.

The tractor inertia, semitrailer inertia, air resistance, tractor climbing resistance and semitrailer climbing resistance are calculated according to the following equations, respectively:

\[
R_{1z} = \frac{W_{1} \cdot a}{8} \tag{1}
\]

\[
R_{2z} = \frac{W_{2} \cdot a}{8} \tag{2}
\]

\[
R_{a} = \frac{\rho \cdot v^{3} \cdot C_{d} \cdot A}{2} \tag{3}
\]

\[
R_{g1} = W_{g} \cdot \sin \theta \tag{4}
\]

\[
R_{g2} = W_{g} \cdot \sin \theta \tag{5}
\]

where \( A \) is the vehicle frontal area as defined in Hucho (1998) and \( \theta \) is the road grade given by:

\[
\theta = \arctan \left( \frac{G}{100} \right) \tag{6}
\]

where \( G \) is the grade of the ramp. Note that in the model, following Wong (1993), the force due to the road gradient, in the performance analysis of road vehicles, is taken to be solely a aerodynamic drag force \( R_{a} \), acting at a point above the ground indicated by the
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height $h_c$. The aerodynamic drag force might also be represented by a longitudinal force of the same magnitude in the ground plane with an associated moment (the aerodynamic pitching moment) equivalent to $R_a$ times $h_c$ (Gillespie, 1992). According to Taborek (1957b), the air resistance $R_a$ acts approximately in the vertical position of the centre of gravity of the vehicle. In the proposed model it is considered that the geometry of the load of the semitrailer does not affect the vehicle frontal area, i.e., the height of the load does exceed the height of the tractor. So, the vehicle frontal area is the same as the tractor frontal area and the line of action of the $R_a$ passes approximately through the vertical position of the centre of gravity of the tractor.

The rolling resistance forces on the tractor front axle, on the tractor rear axle and on the semitrailer rear axles are expressed by the following equations, respectively:

$$ R_{r1} = fN_{r1} $$  \hspace{1cm} (7)

$$ R_{r1} = fN_{r1} $$  \hspace{1cm} (8)

$$ R_{r2} = fN_{r2} $$  \hspace{1cm} (9)

where $f$ is the coefficient of rolling resistance as a function of the vehicle velocity and $N_{r1}$, $N_{r1}$ and $N_{r2}$ are normal forces. The force and moment equations for the semitrailer are expressed by Equations (10) to (12). The force and moment equations for the tractor are expressed by Equations (13) to (15). The vertical reaction force on the fifth-wheel ($A_{v5}$) is equal to the vertical force on the fifth-wheel ($N_5$) and the horizontal reaction force on the fifth-wheel ($A_{h5}$) is equal to the vertical force on the fifth-wheel ($R_5$).

Equation (10) expresses moments in the semitrailer about the point B (Figure 1), the point of connection of the fifth-wheel to the semitrailer. The vertical balance force in the semitrailer is represented by Equation (11). Moment balance in the semitrailer about the point A located where the semitrailer tyres contact the ground is computed by Equation (12). Equation (13) expresses moments in the tractor about the point C located where the tractor rear tyres contact the ground. Moment balance in the tractor about the point D located where the tractor front tyres contact the ground is computed by Equation (14). The force balance along the longitudinal axis of the tractor is represented by Equation (15).

$$ \Sigma M_B = 0 \Rightarrow $$

$$ N_{r2}(l_2 + z_2) - c_{g2}W_c\cos\theta - R_{v2}(c_{g2} - z_2) - R_{g2}(c_{g2} - z_2) + R_{tr2}z_2 \Sigma = 0 $$  \hspace{1cm} (10)
\[
\sum F_Z = 0 \Rightarrow \\
N_{\text{re}} - W \cos \theta + N_Z = 0 \tag{11}
\]
\[
\Sigma M_A = 0 \Rightarrow \\
R_{xZ} - N_{\text{re}} l_2 + W_2 (l_2 - c g_{2x}) \cos \theta - R_{gZ} g_{2x} - R_{gZ} g_{2x} = 0 \tag{12}
\]
\[
\Sigma M_c = 0 \Rightarrow \\
W_1 (l_1 - c g_{1x}) \cos \theta - R_{g1} g_{1z} - R_{g1} g_{1z} - R_{g1} g_{1z} - R_{g1} g_{1z} + N_Z x_2 - N_{\text{re}} l_1 = 0 \tag{13}
\]
\[
\Sigma M_D = 0 \Rightarrow \\
N_{\text{re}} l_1 - N_2 (l_1 - x_2) - R_{xZ} - R_{g1} g_{1z} - R_{g1} g_{1z} - W_1 c g_{1c} \cos \theta - R_{g1} g_{1z} = 0 \tag{14}
\]
\[
\Sigma F_X = 0 \Rightarrow \\
F_{xtr1} - J N_{\text{re}} - f N_{\text{re}} - R_{\text{re}} - R_{g1} - R_{g2} - R_x = 0 \tag{15}
\]

The axle traction forces are expressed according to the Equations (16) and (17), respectively. \(T_{\text{re}}\) and \(T_{\text{st}}\) are forces available from the engine, i.e. they are axle forces without loss in the inertia of rotating parts (engine, transmission and wheels). The forces \(T_{\text{re}}\) and \(T_{\text{st}}\) are reduced, respectively, of the rotating inertia forces \(R_{\text{re}}\) and \(R_{\text{st}}\) that depend on the case of traction (4 \times 2 - rear axle traction, 4 \times 4 - rear and front axle traction). The total traction force \(F_{\text{tr}}\) available from the engine (without inertial losses) is expressed by the Equation (18) and also by the Equation (46, later).

\[
F_{xtr1} = T_{\text{re}} - R_{\text{re1}} \tag{16}
\]

\[
F_{xtr1} = T_{\text{re}} - R_{\text{re1}} \tag{17}
\]

\[
F_{\text{tot}} = T_{\text{re}} + T_{\text{st}} \tag{18}
\]

Equations (1) to (18) are general, i.e. they are valid for traction 4 \times 2 and 4 \times 4. For the case 4 \times 2 (rear axle traction), the rotating inertia forces are \(R_{\text{re1}}\) (Equation (19)) and \(R_{\text{re2}}\) (Equation (20)) for non-driving wheels and \(R_{\text{re1}}\) (Equation (21)) for driving wheels. For this case (4 \times 2), \(R_{\text{re1}}\) represents the losses of the tractive force due to the inertia of the engine, the transmission system and the tractor rear wheels. The equivalent inertia of each component is amplified by the square of the numerical gear ratio between the component and the wheels (Gillespie, 1992). In these expressions, \(r\) is a constant effective rolling radius of the tyre as defined in Genta (1997). Equation (21) takes into account the efficiency of the transmission and the final drive due to losses in the gears (Green, 1962).

\[
R_{\text{re1}} = \frac{R_{\text{st1}}}{r^2 a} \tag{19}
\]

\[
R_{\text{re2}} = \frac{R_{\text{st2}}}{r^2 a} \tag{20}
\]
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\[ R_{\text{tr1}} = \frac{(I_c + I_f)N_{r2}^2 \eta_\text{f} + I_dN_{r1}^2 \eta_\text{t}}{r^2} a \]  

(21)

The total rotating inertia of the driving wheels is represented as:

\[ R_{\text{tot}} = R_{\text{tr1}} \]  

(22)

For non-driving wheels it is represented as:

\[ R_{\text{ind}} = R_{\text{tr1}} + R_{\text{ar2}} \]  

(23)

Equation (15) depends on the traction system and on the analyses condition (ideal or real). In the real case \( 4 \times 2 \) the vehicle has one traction axle, consequently the ideal condition is similar to the real one. In this case the front axle force \( (T_f) \) is equal to zero and Equations (16) and (17) become:

\[ F_{\text{sfl}} = -R_{\text{tr1}} \]  

(24)

\[ F_{\text{sr1}} = T_r - R_{\text{tr1}} = \mu_{\text{tr}} N_{r1} \]  

(25)

where \( \mu_{\text{tr}} \) is the adhesion coefficient on the tractor front axle. In the non-driving axles, such as the tractor front axle, the available force \( F_{\text{sfl}} \) does not work as a traction force \( (4 \times 2) \), but as an inertia force of the rotating parts. The available total dynamic driving force is represented by:

\[ F_{\text{sr1}} = F_{\text{sfl}} \]  

(26)

There are two cases of the system \( AX = B \) to be analysed according to the unknown parameters of the vector \( X \). The first case, expressed by Equation (27), is for \( 4 \times 2 \) traction system, where: vehicle instantaneous velocity \( (v) \), weight \( (W) \), centre of gravity, vehicle geometry, acceleration \( (a) \) – see Equation 52, later) are known parameters and normal forces \( (N_{r2},N_{r1},N_{r1}) \), fifth-wheel forces \( (N_r,R_s) \) as well as traction force \( (F_{\text{sfl}}) \) are unknown parameters:

\[ AX = (N_{r2},N_{r1},N_{r1},N_{r1},F_{\text{sfl}}) = B \]  

(27)

where

\[
X = \begin{pmatrix}
N_{r2} \\
N_{r1} \\
R_s \\
N_{f1} \\
F_{\text{sfl}}
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
l_2 + z_5f & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & -l_2 & z_5 & 0 & 0 \\
0 & x_5 & -z_5 & -l_1 & 0 \\
0 & -(l_1 - x_5) & -z_5 & 0 & l_1 \\
0 & 0 & -l & -f & -f \\
\end{pmatrix}
\]
\[ B = \begin{pmatrix}
W_2 c g_{2z} \cos \theta + R_{1z} \left( c g_{2z} - z_5 \right) + R_{2z} \left( c g_{2z} - z_5 \right) - R_{1v2} z_5 \\
W_2 \cos \theta \\
-W_2 (l_2 - c g_{2z}) \cos \theta + R_{12} c g_{2z} + R_{22} c g_{2z} \\
-W_1 (l_1 - c g_{1z}) \cos \theta + R_{11} c g_{1z} + R_{11} c g_{1z} + R_{22} c g_{1z} \\
R_{11} c g_{1z} + R_{22} c g_{1z} + W_1 c g_{1z} \cos \theta + R_{22} c g_{1z} \\
R_{1v1} + R_{1v1} + R_{1v1} + R_{u1}
\end{pmatrix} \]

With the vector \( X \), we compute the used adhesion coefficient on the tractor rear traction axle as:

\[ \mu_{r1} = \frac{F_{xv1}}{N_{r1}} \]

and the total normal force as:

\[ N_{tot} = N_{r1} + N_{r1} + N_{r2} \tag{29} \]

The second case, expressed by Equation (30), is for \( 4 \times 2 \) traction system, where vehicle instantaneous velocity (\( v \)), weight (\( W \)), centre of gravity, vehicle geometry and adhesion coefficient (\( \mu_{r1} \)) are known parameters and normal forces (\( N_{r1}, N_{r2}, N_{r3} \)), fifth-wheel forces (\( N_5, R_3 \)), traction force (\( F_{xv1} \)) as well as acceleration (\( a \)) are unknown parameters.

\[ A \cdot X(N_{r2}, N_5, R_3, N_{r1}, F_{xv1}, a) = B \tag{30} \]

where

\[ X = \begin{pmatrix}
N_{r2} \\
N_5 \\
R_3 \\
N_{r1} \\
N_{r1} \\
F_{xv1} \\
a
\end{pmatrix}, \quad
A = \begin{pmatrix}
(l_2 + z_5 f) & 0 & 0 & 0 & 0 & 0 - W_2 (c g_{2z} - z_5) + i_{wr2} z_5 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & -l_2 & z_5 & 0 & 0 & 0 - W_2 c g_{2z} \\
0 & x_5 & -z_5 & -l_1 & 0 & 0 - W_1 c g_{1z} \\
0 & -l_1 - x_5 & z_5 & 0 & l_1 & 0 - W_1 c g_{1z} \\
0 & 0 & -1 - f & -f & 1 & -i_{w} f - W_1 \\
0 & 0 & 0 & -\mu_{r1} & 1 & 0
\end{pmatrix} \]
and the equivalent mass of non-driving wheels are computed by the Expressions (31) and (32) derived from the Equations (19) and (20).

\[
i_{wt1} = \frac{l_{w1}}{r^2} \quad (31)
\]

\[
i_{wt2} = \frac{l_{w2}}{r^2} \quad (32)
\]

3 Positions of the centre of gravity for semitrailer

In tractor-semitrailer combination vehicles the load in the tractor, such as the driver's weight, is small and consequently is neglected here. Therefore, we consider only the influences of the semitrailer load on the vehicle acceleration performance. The tractor CG \( c_{g1} \) is taken to be fixed. The position of the semitrailer CG \( c_{g2} = (c_{g2x}, c_{g2y}, c_{g2z}) \) is specified by: the distance from the fifth-wheel \( (c_{g2z}) \), the distance from the centre of the semitrailer rear wheel tread \( (c_{g2y}) \) and the distance from the ground \( (c_{g2x}) \). For a homogeneous lateral load distribution, i.e. load CG along \( y \) located in the same position of the empty semitrailer CG in the direction \( y \), the \( c_{g2y} \) position does not influence the acceleration of vehicles in straight line movement.

There are three different conditions for computing the \( c_{g2} \) according to the load \( q \) \((0 \leq q \leq (W_{t1s} - W_{t2s}))\): empty vehicle \((q = 0)\), vehicle maximum load and load \( q \) varying from zero to maximum. These cases were investigated by Canale and Ruffino (1992) and Fernandes (1994) in braking studies. Ranganathan and Yang (1996) developed an approach to compute the impact of liquid load shift on the braking behaviour of liquid tank vehicles.

For an empty vehicle the CG position \( c_{g2e} = (c_{g2e}, c_{g2y}, c_{g2z}) \) is calculated as a function of the static weight on the axles and for a load \( q \) equal zero, i.e. \( c_{g2} (q = 0) = c_{g2e} \). For the tractor without a semitrailer, the weight \( (W_{t1e}) \) is computed according to Equation (33), using the static weight on the tractor front axle \( (W_{f1e}) \) and the static weight on the tractor rear axle \( (W_{r1e}) \).

\[
W_{t1e} = W_{f1e} + W_{r1e} \quad (33)
\]
For the semitrailer, the static weight is calculated with tractor and semitrailer together using Equation (34), where \( n_{t2} \) is the number of rear axles of the semitrailer, \( W_{f12}^* \) is the static weight on the tractor front axle, \( W_{r12}^* \) is the static weight on the tractor rear axle and \( W_{r21}^* \) is the static weight on the semitrailer rear axle.

\[
W_{2e} = W_{f12}^* + W_{r12}^* - W_{1e} + n_{t2} W_{r21}^*	ag{34}
\]

For the fully loaded vehicle, the maximum static weights on the axes of the vehicle (\( W_{f12}^* \), \( W_{r12}^* \), \( W_{r21}^* \)) are calculated with tractor and semitrailer together and are limited by the vehicle, road and federal laws. The semitrailer CG position \( c_{g2}^* \) (\( c_{g2x}^* \), \( c_{g2y}^* \), \( c_{g2z}^* \)) is calculated as a function of the maximum static weight on the axes and for a maximum load \( q \), i.e., \( c_{g2}(q = (W_{2e}^* - W_{1e})) \equiv c_{g2}^* \). The maximum weight of the semitrailer (\( W_{2e}^* \)) is computed using Equation (35).

\[
W_{2e}^* = W_{f12}^* + W_{r12}^* - W_{1e} + n_{t2} W_{r21}^*	ag{35}
\]

In the case of a varying load, the semitrailer CG \( c_{g2} \) is computed as a function of the position of the CG of a given load \( q \), i.e., \( c_{g2}(0 \leq q \leq (W_{2e}^* - W_{1e})) \equiv c_{g2} \). Figure 1 illustrates the position of the centre of gravity for a load \( q \) in the directions \( x \) and \( z \) (\( x_q \) and \( z_q \)). The CG of the load \( q \) in direction \( x \) (\( x_q \)) varies from \( x_{q1} \) to \( x_{q2} \) (\( x_{q1} \leq x_q \leq x_{q2} \)). The CG of the load \( q \) in direction \( z \) (\( z_q \)) varies from \( z_{q1} \) to \( z_{q2} \) (\( z_{q1} \leq z_q \leq z_{q2} \)). In the direction \( x \), there are two cases for the calculation of \( c_{g2x} \): load CG in the position \( x_q \) and load CG in the position \( x_b \) (Equation (36)). The values \( x_q \) and \( x_b \) are limited by the geometry of the semitrailer.

\[
c_{g2x} = \frac{c_{g2x} W_{2e}^* + x_q q}{W_{2e}^* + q}	ag{36}
\]

In the direction \( z \), there are also two cases for the calculation of \( c_{g2z} \): load CG in the position \( z_q \) and load CG in the position \( z_b \) (Equation (37)). The values \( z_q \) and \( z_b \) are limited by the height of the bridges as well as by the semitrailer geometry.

\[
c_{g2z} = \frac{c_{g2z} W_{2e}^* + z_q q}{W_{2e}^* + q}	ag{37}
\]

In the direction \( x \), there are two more possibilities for the calculation of \( c_{g2x} \) for a given load \( q \): maximum weight on the fifth-wheel (\( W_{5x}^* \) - expressed by the Equation (38)) and maximum weight on the rear axles of the semitrailer (\( W_{r21}^* \)). These possibilities are computed according to the Equations (39) and (40), respectively.

\[
W_{5x}^* = W_{f12}^* + W_{r12}^* - W_{1e}	ag{38}
\]

\[
c_{g2x} = l_2 \frac{W_{2e}^* + q - W_{5x}^*}{W_{2e}^* + q}	ag{39}
\]

\[
c_{g2x} = l_2 \frac{n_{t2} W_{r21}^*}{W_{2e}^* + q}	ag{40}
\]

A specific load \( q \) can be calculated for two conditions: (a) the semitrailer CG position \( c_{g2x} \), for the load CG position \( x_q \), is equal to semitrailer CG position \( c_{g2x} \), for
Influences of the load centre of gravity on heavy vehicle acceleration

maximum weight on the fifth-wheels ($W_{5w}$) or (b) the semitrailer CG position ($c_{g2}$), for the load CG position $x_1$, is equal to semitrailer CG position ($c_{g2}$), for maximum weight on the rear axle semitrailer ($W_{23a}$). In the first condition the load $q$ is computed based on the Equation (41), whereas for the second one the load $q$ is computed based on the Equation (42).

\[ q = \frac{l_2W_{5w} - W_{2a}(l_2 - c_{g2})}{l_2} \]  
\[ q = \frac{l_2n_2W_{23a} - W_{2a}c_{g2}}{x_0} \]

(41)  
(42)

4 Dynamic forces and acceleration of the vehicle

This section presents a calculation procedure for the dynamic forces in the vehicle acceleration as a function of the adherence force and of the transmission system. The engine force is limited by the maximum adherence force and is determined by the conditions of the tyre adherence of the traction axles. For the case $4 \times 2$, the adherence force $F_{ad}$ is calculated by the product of the dynamic load by the adherence coefficient as:

\[ F_{ad} = \mu_t N_{rt} \]  

(43)

A total slip can happen in case the used adherence coefficient ($\mu_t$) of the rear traction axle becomes larger than the maximum adherence coefficient ($\mu_0$). In the traction limit, i.e., when the vehicle uses the maximum adherence coefficient ($\mu_t = \mu_0$) without total slipping, the maximum adherence force $F_{ad_{\text{max}}}$ is calculated using Equation (43) with the substitution of the coefficient $\mu_t$ by the coefficient $\mu_0$. In the acceleration calculations, the smallest force is used between the total traction force, which is available on the pavement ($F_{ad}$, Equation (26)), and the maximum adherence force ($F_{ad_{\text{max}}}$).

The calculation of the force $F_{ad_{\text{max}}}$ depends on the dynamic load on the traction axles. The calculation steps, for vehicle with $4 \times 2$ traction, manual transmission and the same type of tyres on the axles, are described below. This calculation is accomplished for a specific acceleration and it should be repeated for each engaged gear in the whole range of the engine speed.

(a) total ratio ($N_d$) for a specific engaged gear is represented by:

\[ N_d = N_f N_r \]  

(44)

(b) total efficiency ($\eta_d$) for a specific engaged gear is represented by:

\[ \eta_d = \eta_f \eta_r \]  

(45)

(c) total traction force ($F_{tot}$) is calculated as:

\[ F_{tot} = \frac{T_n N_d \eta_d}{r} \]  

(46)
where $T_e$ is the engine torque and $r$ is the effective rolling radius of the tyre.

(d) vehicle instantaneous velocity ($v$) for a specific engaged gear is calculated according to Genta (1997) by the Equation (47).

$$v = \frac{n_2 \pi r}{N_{df}}$$  \hspace{1cm} (47)

where $n$ is the engine speed.

(e) factor of the inertia of the rotating parts of vehicle ($\gamma$) for a engaged gear is calculated using the Equations (19), (20) and (21), and is represented by:

$$\gamma = 1 + \frac{I_{w1} + I_{wr1} + I_{wr2} + (I_e + I_s)N_f^2 \frac{\eta_{rl}}{\eta_{le}} + I_s N_f^2 \frac{\eta_{rl}}{\eta_{le}}}{mr^2}$$  \hspace{1cm} (48)

(f) vehicle total resistance force ($F_{rot}$) is calculated as a function of the vehicle velocity, road grade $\theta$ and weight $W$. This force is composed of air resistance (Equation (3)), total rolling resistance (Equation (50)) and total climbing resistance (Equation (51)) and is represented by:

$$F_{rot} = R_a + R_t + R_g$$  \hspace{1cm} (49)

where

$$R_a = R_{ef1} + R_{ef2} + R_{w1} + f(W_t + W_s)\cos \theta = fW\cos \theta$$  \hspace{1cm} (50)

$$R_g = W\sin \theta$$  \hspace{1cm} (51)

(g) acceleration ($a$) of the vehicle as a function of the installed systems (engine, gearshift, differential, tyres) is calculated according to the expression:

$$a = \frac{F_{tot} - F_{rot}}{\gamma m}$$  \hspace{1cm} (52)

(h) inertia forces $R_{i1}, R_{i2}, R_{ef1}, R_{ef2}, R_{ir1}, R_{ir2}$ are calculated according to the Equations (1), (2), (19), (20), and (21) respectively.

(i) dynamic normal forces of the vehicle are calculated with the acceleration of the item (g) and according to the Equation (27).

(j) adhesion coefficient used on the rear axle is calculated according to the Equation (28).

(k) it is verified if the used coefficient $\mu_2$ is larger than the maximum adhesion coefficient $\mu_2$; in the case it is not, the traction force $F_{tra}$ is the same as the total force $F_{rot}$ and the calculation procedure is over. In the case $\mu_2$ is smaller than the used adhesion coefficient, the rear traction axle will have a tendency of slipping and the calculation procedure follows in the next step.

(l) new acceleration of the vehicle, new dynamic normal forces and new traction forces are computed in the axles according to the Equation (30) in the traction limit. This new acceleration is limited by the position of the vehicle centre of gravity.
(m) new inertia forces \( R_{fl1}, R_{fl2}, R_{rl1}, R_{rl2} \) are calculated with the acceleration of the item (l) according to the Equations (1), (2), (19), (20) and (21), respectively.

(n) maximum adherence force \( F_{\text{admax}} \) is calculated according to the Equation (53), in which \( \mu_i \) assumes the value of \( \mu_0 \) for the rear traction axle in the traction limit.

\[
F_{\text{admax}} = \mu_0 N_{rl}
\]  

(53)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>4.6 m</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.66 m</td>
</tr>
<tr>
<td>( z_3 )</td>
<td>1.21 m</td>
</tr>
<tr>
<td>( n_{fl2} )</td>
<td>3 with 4 wheels each</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>7 m</td>
</tr>
<tr>
<td>( r )</td>
<td>0.55 m</td>
</tr>
<tr>
<td>( C_D )</td>
<td>0.8</td>
</tr>
<tr>
<td>( \rho )</td>
<td>1.225 kg/m³</td>
</tr>
<tr>
<td>( f )</td>
<td>0.008</td>
</tr>
<tr>
<td>( A )</td>
<td>6.5 m²</td>
</tr>
<tr>
<td>( I_e )</td>
<td>2.34 kg.m²</td>
</tr>
<tr>
<td>( I_w )</td>
<td>7 kg.m²</td>
</tr>
</tbody>
</table>

Table 1 Data for vehicle analysis.

5 Results

5.1 Characteristics of the vehicle

The analysed vehicle is a tractor-semitrailer whose main characteristics are presented in Table 1, according to information given by the tractor and semitrailer manufacturers. The model of the tractor is LS1935 (Mercedes-Benz do Brasil) with traction system \( 4 \times 2 \). The tractor has one front axle with two wheels and one rear axle with four wheels. The semitrailer has three rear axles with four wheels each. Hucho (1998) showed that the coefficient of air resistance \( C_D \) varies according to kind of commercial vehicle. For the tractor-semitrailer vehicles, the range of values is approximately from 0.65 to 0.9. According to Barnard (1996), for vehicle velocities below 27.78 m.s⁻¹ the rolling resistance coefficient can be treated as a constant. The values for the coefficients \( C_D \) and \( f \) were computed experimentally thought the method described in White and Korst (1972). The frontal area of the vehicle \( A \) was estimated based on the dimensions of the vehicle design. The sum \( I_{e+I_w} \) in the Equation (21) represents the equivalent moment of the inertia for components rotating at engine speed and \( I_e \) takes into account the flywheel
and the clutch. Following Taborek (1957a), the moment of inertia of the transmission parts, the gears and the shafts were neglected. The air density is assumed to be at the sea level.

Table 2 Engine torque.

<table>
<thead>
<tr>
<th>Engine speed (n [\text{rad.s}^{-1}])</th>
<th>Torque (T_b [\text{Nm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.6</td>
<td>811.6</td>
</tr>
<tr>
<td>89.5</td>
<td>966.9</td>
</tr>
<tr>
<td>104.7</td>
<td>1,781.1</td>
</tr>
<tr>
<td>115.2</td>
<td>1,767.2</td>
</tr>
<tr>
<td>119.3</td>
<td>1,777.6</td>
</tr>
<tr>
<td>139.2</td>
<td>1,708.9</td>
</tr>
<tr>
<td>159.1</td>
<td>1,638.8</td>
</tr>
<tr>
<td>178.9</td>
<td>1,504.1</td>
</tr>
<tr>
<td>198.8</td>
<td>1,369.2</td>
</tr>
</tbody>
</table>

Table 2 shows the engine maximum torque curve with the values reduced for the experimental environmental conditions (sea level). This table contains some engine speed points in the range from 69.6 rad.s\(^{-1}\) to 198.8 rad.s\(^{-1}\) with maximum torque value equal to 1,781.1 Nm at 104.7 rad.s\(^{-1}\). Table 3 illustrates the ratio of the transmission \(N_t\) according to engaged gear, where \(R\) is a reduction ratio and \(L\) is a smaller reduction ratio. The transmission efficiency \((\eta_t)\) vary from 0.96 to 1 according to the numerical ratio \(N_t\). The final drive ratio \((N_f)\) is equal to 3.77 and its efficiency \((\eta_f)\) is equal to 0.92.

Table 3 Ratio of the transmission.

<table>
<thead>
<tr>
<th>gear</th>
<th>(N_t)</th>
<th>gear</th>
<th>(N_t)</th>
<th>gear</th>
<th>(N_t)</th>
<th>gear</th>
<th>(N_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 or 1R</td>
<td>13.676</td>
<td>5 or 3R</td>
<td>6.727</td>
<td>9 or 5R</td>
<td>3.357</td>
<td>13 or 7R</td>
<td>1.651</td>
</tr>
<tr>
<td>2 or 1L</td>
<td>11.635</td>
<td>6 or 3L</td>
<td>5.723</td>
<td>10 or 5L</td>
<td>2.856</td>
<td>14 or 7L</td>
<td>1.405</td>
</tr>
<tr>
<td>3 or 2R</td>
<td>9.397</td>
<td>7 or 4R</td>
<td>4.788</td>
<td>11 or 6R</td>
<td>2.307</td>
<td>15 or 8R</td>
<td>1.175</td>
</tr>
<tr>
<td>4 or 2L</td>
<td>7.995</td>
<td>8 or 4L</td>
<td>4.074</td>
<td>12 or 6L</td>
<td>1.963</td>
<td>16 or 8L</td>
<td>1.0</td>
</tr>
</tbody>
</table>

5.2 Centre of gravity

The centre of gravity position of the vehicle is determined for the empty, fully loaded and partially loaded vehicle. The CG is calculated for the tractor and semitrailer separately. The weight for the empty tractor (without the semitrailer) is \(W_{le} = 73,771\) N. The static weight on the tractor front axle is \(w_{fle} = 44,341\) N and the static weight on the tractor rear axle is \(w_{rle} = 29,430\) N. The CG position of the empty tractor \((c_{gle})\) is:

\(\diamond\) distance from front axle \((x)\): \(c_{gle} = 1.93\) m;
Influences of the load centre of gravity on heavy vehicle acceleration

- distance from centre of front wheel tread of the tractor (y): \( c_{g_{tiey}} = 0 \text{ m} \);
- height from road surface (z): \( c_{g_{tiez}} = 0.95 \text{ m} \).

The weight for the empty semitrailer (without the tractor) is \( W_{2e} = 71,368 \text{ N} \). The static weight on the tractor front axle (with the semitrailer) is \( W_{tiez2} = 47,039 \text{ N} \), the static weight on the tractor rear axle (with the semitrailer) is \( W_{tiez1} = 39,240 \text{ N} \) and the static weight on the rear axles of the semitrailer is \( W_{2est} = 19,620 \text{ N} \). The CG position of the empty semitrailer \( (c_{g_{te}}) \) is:
- distance from fifth-wheel (x): \( c_{g_{te}} = 6.133 \text{ m} \);
- distance from centre of tread of the semitrailer rear axle: (y): \( c_{g_{tey}} = 0 \text{ m} \);
- height from road surface (z): \( c_{g_{tez}} = 1.4 \text{ m} \).

The weight for the fully loaded vehicle is \( W = 407,115 \text{ N (41.5 ton)} \). The load in the tractor, such as the driver weight and any others will be neglected. In this case, tractor CG position is equal to empty tractor CG position. The fully loaded semitrailer CG position is determined by the maximum weight on the axles vehicle and by the load CG position.

The maximum static weight of the tractor-semitrailer combination is distributed on the front axle of the tractor \( W_{tiez1}^{*} = 58,860 \text{ N} \), on the rear axle of the tractor \( W_{tiez2}^{*} = 98,100 \text{ N} \) and on the axles of the semitrailer \( W_{2est}^{*} = 83,385 \text{ N} \). So, the weight for the fully loaded semitrailer is \( W_{2e}^{*} = 333,344 \text{ N} \) (Equation (35)) and \( W_{2s}^{*} = 83,189 \text{ N} \) (Equation (38)).

A region for the load CG in the semitrailer in the directions x and z is limited, respectively, by \( x_{p}, x_{b} \) and \( z_{p}, z_{b} \). In direction y, for a homogeneous lateral load distribution, the semitrailer CG \( c_{g_{2y}} \) is equal zero. In the direction x, a physical load distribution will be considered so that the load CG will be between \( x_{p} = 2 \text{ m} \) and \( x_{b} = 7 \text{ m} \). In the direction z, for the lower edge of the load located in the surface of the semitrailer \( (z = 1.5 \text{ m from the pavement}) \), two conditions for load distribution will be considered. In the first one the load has a height of the 0.5 m beyond of the semitrailer surface. In this case, the load CG is located at 0.25 m (homogeneous load) from the semitrailer surface and \( z_{c} = 1.75 \text{ m} \). In the second one, a height limit of the loading equal to 4 m (average height of bridges) is considered. In this case, the load CG is located at (4-1.5)/2 = 1.25 m (homogeneous load) from the semitrailer surface and \( z_{c} = 2.75 \text{ m} \).

The centre of gravity position for the fully loaded semitrailer alone \( (c_{g_{2e}}) \) is:
- distance from fifth-wheel (maximum weight on the axles) (x): \( c_{g_{2x}}^{*} = 5.253 \text{ m} \);
- distance from centre of tread of the semitrailer rear axle: (y): \( c_{g_{2y}}^{*} = 0 \text{ m} \), in the case of a homogeneous lateral load distribution;
- height from road surface (z) - (load CG position \( z_{c} = 1.75 \text{ m} \) from the road surface): \( c_{g_{2e}}^{*} = 1.675 \text{ m} \).
height from road surface \((z)\) - (load CG position \(z_b = 2.75\) m from the road surface):
\[ c_{g_{22}} = 2.461 \text{ m}. \]

For the partially loaded vehicle, the CG position is computed as a function of the load \(q\). The load CG position in the direction \(x\) is limited from \(x_a = 2\) m to \(x_b = 7\) m and the load CG position in the direction \(z\) is limited from \(z_a = 1.75\) m to \(z_b = 2.75\) m.

Figure 2 shows the position of the centre of gravity of the vehicle in the direction \(x\) as a function of the vehicle loading. For each load \(q\) there are four possibilities for the centre of gravity. These possibilities are limited by the semitrailer geometry \((x_a, x_b)\) and by the maximum axle weights \((W_{5x}^*\) and \(n_{2x}W_{22x}^*\)). The region valid in Figure 2 is the interior one among the curves \(x_p, x_p, W_{5x}^*\), and \(n_{2x}W_{22x}^*\). For the vehicle empty \((W_{2e} = 71,368\) N), the four possibilities for \(c_{g_{2x}}\) are 6.133 m \((x_a = 2\) m\), 6.133 m \((x_b = 7\) m\), -1.159 m \((W_{5x}^* = 83,189\) N\), 24.536 m \((n_{2x}W_{22x}^* = 250,155\) N\), but only the position \(c_{g_{2x}} = 6.133\) m is valid. The negative value \((c_{g_{2x}} = -1.159\) m\) and a value superior to the length of semitrailer \((c_{g_{2x}} = 24.536\) m\) show that an empty vehicle will never be limited by the maximum weights in the axles. For the fully loaded vehicle \((W_{2}^* = 333,344\) N\), the four possibilities for \(c_{g_{2x}}\) are 2.885 m \((x_a = 2\) m\), 6.806 m \((x_b = 7\) m\), 5.253 m \((W_{5x}^* = 83,189\) N\), 5.253 m \((n_{2x}W_{22x}^* = 250,155\) N\), but only the position \(c_{g_{2x}} = 5.253\) m is valid.

Figure 2 also shows values for a specific load under two conditions. In the first, the semitrailer CG position \((c_{g_{2x}})\) for the load CG position \(x_a\) is equal to semitrailer CG position \((c_{g_{2x}})\), for maximum weight on the fifth-wheels \((W_{5x}^*)\). For this case, the load \(q\) is equal to 104,094 N and the vehicle weight is \(W_2 = 175,462\) N. The four possibilities for \(c_{g_{2x}}\) with the load \(q\) are 3.681 m \((x_a = 2\) m\), 6.647 m \((x_b = 7\) m\), 3.681 m \((W_{5x}^* = 83,189\) N\), 9.980 m \((n_{2x}W_{22x}^* = 250,155\) N\), but only the positions CG \(c_{g_{2x}} = 3.681\) m \((x_a = 2\) m\), \(W_{5x}^* = 83,189\) N\) and \(c_{g_{2x}} = 6.547\) m \((x_b = 7\) m\) are valid. In the second condition, the semitrailer CG position \((c_{g_{2x}})\), for the load CG position \(x_a\) is equal to semitrailer CG position \((c_{g_{2x}})\), for maximum weight on the rear axle semitrailer \((W_{22x}^*)\).

Then, the load \(q\) is equal to 187,626 N and the vehicle weight is \(W_2 = 259,082\) N. For this load, the four possibilities for \(c_{g_{2x}}\) are 3.139 m \((x_a = 2\) m\), 6.761 m \((x_b = 7\) m\), 4.752 m \((W_{5x}^* = 83,189\) N\), 6.761 m \((n_{2x}W_{22x}^* = 250,155\) N\), but only the positions CG \(c_{g_{2x}} = 6.761\) m \((x_a = 7\) m\), \(n_{2x}W_{22x}^* = 250,155\) N\) and \(c_{g_{2x}} = 4.752\) m \((W_{5x}^* = 83,189\) N\) are valid.

Figure 3 shows the position of the centre of gravity of the vehicle in the direction \(z\) as a function of the vehicle loading. For each load \(q\) there are two possibilities for the centre of gravity. These possibilities are limited by the height of the bridges as well as by the semitrailer geometry \((z_a\) and \(z_b\)). The valid region in Figure 3 is the interior one between the curves for \(z_a\) and \(z_b\). For the vehicle empty \((W_{2e} = 71,368\) N\), the two possibilities for \(c_{g_{2z}}\) are 1.4 m \((z_a = 1.75\) m\) and 1.4 m \((z_b = 2.75\) m\), so the valid position is \(c_{g_{2z}} = 1.4\) m.

For fully loaded vehicle \((W_2^* = 333,344\) N\), the two possibilities for \(c_{g_{2z}}\) are 1.675 m \((z_a = 1.75\) m\) and 2.461 m \((z_b = 2.75\) m\). Figure 3 also illustrates the \(c_{g_{2z}}\) for other loads \(q\), such as, load \(q = 104,094\) N, \(c_{g_{2z}} = 1.608\) m \((z_a = 1.75\) m\), \(c_{g_{2z}} = 2.201\) m \((z_b = 2.75\) m\) and load \(q = 187,626\) N, \(c_{g_{2z}} = 1.654\) m \((z_a = 1.75\) m\), \(c_{g_{2z}} = 2.378\) m \((z_b = 2.75\) m\).
5.3 Dynamics forces in the vehicle acceleration

Graphs of acceleration versus velocity of the vehicle are used in order to analyse the influence of the load CG position on the vehicle acceleration. There are four groups (G1, G2, G3, G4) for the position of the CG according to the loading of the vehicle (empty, fully loaded, partially loaded). Two conditions of load $q$ will be analysed for partially loaded vehicle, i.e. $q = 104,094$ N (Equation (41)) and $q = 187,626$ N (Equation (42)).
The load for the tractor will be neglected in all the four groups, i.e., the weight will be constant and equal to $W_1 = 73,771$ N with tractor CG $c_{g_{1x}} = 1.93$ m, $c_{g_{1y}} = 0$, $c_{g_{1z}} = 0.95$ m. The semitrailer CG position in the direction $y$ will also be constant $c_{g_{2y}} = 0$ m, because this position does not influence straight movement. The four groups are:

(a) empty vehicle, $q = 0$, $W_2 = W_{2e} = 71,368$ N

G1: $c_{g_{2x}} = c_{g_{2e}} = (c_{g_{2ex}} = 6.133$ m; $c_{g_{2ey}} = 1.4$ m)

(b) fully loaded vehicle, $q = (W_2^* - W_{2e}) = 261,976$ N, $W_2^* = 333,344$ N

G2a: $c_{g_{2x}} = c_{g_{2x}}^* = (c_{g_{2ex}}^* = 5.253$ m ($W_{s_1}^* = 83,189$ N, $n_{r_1}W_{r_2}^* = 250,155$ N); $c_{g_{2e}}^* = 1.675$ m, ($z_a = 1.75$ m))

G2b: $c_{g_{2x}} = c_{g_{2x}}^* = (c_{g_{2ex}}^* = 5.253$ m ($W_{s_1}^* = 83,189$ N, $n_{r_2}W_{r_2}^* = 250,155$ N); $c_{g_{2e}}^* = 2.461$ m, ($z_b = 2.75$ m))

(c) partially loaded vehicle, $q = (W_2 - W_{2e}) = 104,094$ N, $W_2 = 175,462$ N

G3a: $c_{g_{2x}} = (c_{g_{3x}} = 3.681$ m, ($x_a = 2$ m, $W_{s_1}^* = 83,189$ N); $c_{g_{2e}} = 1.608$ m, ($z_a = 1.75$ m))

G3b: $c_{g_{2x}} = (c_{g_{3x}} = 3.681$ m, ($x_a = 2$ m, $W_{s_1}^* = 83,189$ N); $c_{g_{2e}} = 2.201$ m, ($z_b = 2.75$ m))

G3c: $c_{g_{2x}} = (c_{g_{3x}} = 6.547$ m, ($x_a = 7$ m); $c_{g_{2e}} = 1.608$ m, ($z_a = 1.75$ m))

G3d: $c_{g_{2x}} = (c_{g_{3x}} = 6.547$ m, ($x_a = 7$ m); $c_{g_{2e}} = 2.201$ m, ($z_b = 2.75$ m))

(d) partially loaded vehicle, $q = (W_2 - W_{2e}) = 187,626$ N, $W_2 = 259,082$ N

G4a: $c_{g_{2x}} = (c_{g_{3x}} = 6.761$ m, ($x_a = 7$ m, $n_{r_1}W_{r_2}^* = 250,155$ N); $c_{g_{2e}} = 1.654$ m, ($z_a = 1.75$ m))

G4b: $c_{g_{2x}} = (c_{g_{3x}} = 6.761$ m, ($x_a = 7$ m, $n_{r_2}W_{r_2}^* = 250,155$ N); $c_{g_{2e}} = 2.378$ m, ($z_b = 2.75$ m))

G4c: $c_{g_{2x}} = (c_{g_{3x}} = 4.752$ m, ($W_{s_1}^* = 83,189$ N); $c_{g_{2e}} = 1.654$ m, ($z_a = 1.75$ m))

G4d: $c_{g_{2x}} = (c_{g_{3x}} = 4.752$ m, ($W_{s_1}^* = 83,189$ N); $c_{g_{2e}} = 2.378$ m, ($z_b = 2.75$ m))

There are two subgroups (SG1, SG2) for each group, according to the ramp grades and the conditions of the tyre and pavement:

(a) grade of ramp $G = 0$

SG1a: the vehicle can use its total traction force available from the engine without reaching the traction limit, theoretically for a $\mu_0 = \infty$

SG1b: maximum adhesion coefficient between tyres and pavement $\mu_0 = 0.7$

SG1c: maximum adhesion coefficient between tyres and pavement $\mu_0 = 0.3$

(b) grade of ramp $G = 3$

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The context suggests a theoretical or academic discussion on the load distribution and vehicle performance under different loading conditions.
SG2a: the vehicle can use its total traction force available from the engine without reaching the traction limit, theoretically for $\mu_0 = \infty$.

SG2b: maximum adhesion coefficient between tyres and pavement $\mu_0 = 0.7$

SG2c: maximum adhesion coefficient between tyres and pavement $\mu_0 = 0.3$

<table>
<thead>
<tr>
<th>$a$ [m.s$^{-2}$]</th>
<th>$SG1a$</th>
<th>$SG1b$</th>
<th>$SG1c$</th>
<th>$SG2a$</th>
<th>$SG2b$</th>
<th>$SG2c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>4.388</td>
<td>2.03</td>
<td>0.741</td>
<td>4.253</td>
<td>1.742</td>
<td>0.453</td>
</tr>
<tr>
<td>G2a</td>
<td>2.402</td>
<td>1.924</td>
<td>0.705</td>
<td>2.199</td>
<td>1.632</td>
<td>0.413</td>
</tr>
<tr>
<td>G2b</td>
<td>2.402</td>
<td>1.809</td>
<td>0.688</td>
<td>2.199</td>
<td>1.517</td>
<td>0.395</td>
</tr>
<tr>
<td>G3a</td>
<td>3.308</td>
<td>3.183</td>
<td>1.193</td>
<td>3.139</td>
<td>2.891</td>
<td>0.902</td>
</tr>
<tr>
<td>G3b</td>
<td>3.308</td>
<td>3.057</td>
<td>1.175</td>
<td>3.139</td>
<td>2.765</td>
<td>0.884</td>
</tr>
<tr>
<td>G3c</td>
<td>3.308</td>
<td>1.225</td>
<td>0.43</td>
<td>3.139</td>
<td>0.934</td>
<td>0.14</td>
</tr>
<tr>
<td>G3d</td>
<td>3.308</td>
<td>1.177</td>
<td>0.423</td>
<td>3.139</td>
<td>0.886</td>
<td>0.133</td>
</tr>
<tr>
<td>G4a</td>
<td>2.759</td>
<td>0.850</td>
<td>0.285</td>
<td>2.569</td>
<td>0.559</td>
<td>0</td>
</tr>
<tr>
<td>G4b</td>
<td>2.759</td>
<td>0.805</td>
<td>0.279</td>
<td>2.569</td>
<td>0.514</td>
<td>0</td>
</tr>
<tr>
<td>G4c</td>
<td>2.759</td>
<td>2.368</td>
<td>0.877</td>
<td>2.569</td>
<td>2.076</td>
<td>0.585</td>
</tr>
<tr>
<td>G4d</td>
<td>2.759</td>
<td>2.243</td>
<td>0.859</td>
<td>2.569</td>
<td>1.951</td>
<td>0.567</td>
</tr>
</tbody>
</table>

Table 4 shows the maximum acceleration in m.s$^{-2}$ of the vehicle in the first engaged gear when it presents an engine speed $n = 104.72$ rad.s$^{-1}$, a vehicle instantaneous velocity $v = 1.114$ m.s$^{-1}$ (Equation (47)) and a maximum engine torque $T_e = 1.780$ Nm. This table contains vehicle acceleration as a function of the vehicle weight (from empty to maximum load), the load CG position defined by the groups and the operational conditions defined by the subgroups. For typical values of $\mu_0$ (0.7 and 0.3) and G (0% and 3%), on the one hand, the groups G3a, G3b, G4c, G4d belong to the best boundary (bigger acceleration) for load CG position, and they are better than the empty vehicle. On the other hand, the groups G3c, G3d, G4a, G4b belong to the worst boundary (smaller acceleration) for load CG position, and they are worse than the fully loaded vehicle. There are the configurations G4a–SG2c and G4b–SG2c in which the vehicle cannot accelerate on the ramp of the grade G = 3% with maximum adhesion coefficient $\mu_0 = 0.3$. In the subgroups SG1a and SG2a ($\mu_0 = \infty$), the vehicle acceleration depends on the weight and does not depend on the CG position.
Figure 4 Acceleration graph for the group G2a-SG2a (full load, $\mu_0 = \infty$, $G = 3\%$).

Figure 5 Acceleration graph for the group G2a-SG2b (full load, $\mu_0 = 0.7$, $G = 3\%$).
Figure 6 Acceleration graph for the group G2a-SG2c (full load, $\mu_0 = 0.3$, $G = 3\%$).

For each one of the sixty-six combinations it is possible to construct graphs for the vehicle acceleration. Figures 4, 5 and 6 show graphs of the vehicle acceleration for the sixteen engaged gears and groups-subgroups combinations G2a-SG2a, G2a-SG2b, G2a-SG2c, respectively. In these graphs, the vehicle total weight is $W = 407,115$ N, the semitrailer weight is $W_a = 333,344$ N, the load is $q = 261,976$ N, the semitrailer CG in the direction $x$ is $cg_{2a} = 5.253$ m, the semitrailer CG in the direction $z$ is $cg_{2a} = 1.675$ m and the grade of ramp is $G = 3\%$. In these figures the maximum vehicle velocity is reached in the thirteenth engaged gear at 14.8 m.s$^{-1}$. Figure 4 illustrates the vehicle acceleration for a theoretical adhesion coefficient $\mu_0 = \infty$. The value for the vehicle maximum acceleration is 2.199 m.s$^{-2}$. Figure 5 illustrates the vehicle acceleration limited by maximum adhesion coefficient between the tyres and the pavement $\mu_0 = 0.7$. In this figure, the vehicle acceleration at 1.11 m.s$^{-1}$ is 1.632 m.s$^{-2}$. Figure 6 illustrates the vehicle acceleration limited by maximum adhesion coefficient between the tyres and the pavement $\mu_0 = 0.3$. In this figure, the vehicle acceleration at 1.11 m.s$^{-1}$ is 0.413 m.s$^{-2}$.

Tables 5 and 6 illustrate the forces acting on the vehicle and the adhesion coefficient $\mu_1$ as a function of vehicle accelerations. These tables show the acceleration range for the vehicle for the following conditions: fully loaded, riding on a road with a grade of ramp $G = 3\%$ and $\mu_0 = \infty$. Table 5 represents the vehicle riding in the first engaged gear and the maximum adhesion coefficient is approximately 0.86. Table 6 represents the vehicle riding in the thirteenth engaged gear and the range of acceleration is reduced. The total climbing resistance and the total rolling resistance are constant and equal to 12,208 N and 3,255 N respectively. The total normal force is 406,932 N. The factor of the inertia of the rotating parts of the vehicle $\gamma$ for first, thirteenth and sixteenth engaged gears are equal to 1.449, 1.017 and 1.013 respectively.
Table 5 Forces acting on the fully loaded vehicle using first engaged gear.

<table>
<thead>
<tr>
<th>$A$ [m.s$^{-2}$]</th>
<th>$V$ [m.s$^{-1}$]</th>
<th>$R_{i1}$ [N]</th>
<th>$R_{i2}$ [N]</th>
<th>$R_{in1d}$ [N]</th>
<th>$R_{in2d}$ [N]</th>
<th>$R_{in}$ [N]</th>
<th>$F_{x1}$ [N]</th>
<th>$M_{e1}$ [N]</th>
<th>$N_{f1}$ [N]</th>
<th>$N_{r1}$ [N]</th>
<th>$N_{r2}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.86</td>
<td>0.75</td>
<td>6,482</td>
<td>29,292</td>
<td>280</td>
<td>15,788</td>
<td>2</td>
<td>51,520</td>
<td>0.456</td>
<td>41,688</td>
<td>112,986</td>
<td>252,258</td>
</tr>
<tr>
<td>1</td>
<td>0.89</td>
<td>7,527</td>
<td>34,014</td>
<td>325</td>
<td>18,333</td>
<td>2</td>
<td>57,333</td>
<td>0.502</td>
<td>40,175</td>
<td>114,193</td>
<td>252,564</td>
</tr>
<tr>
<td>1.51</td>
<td>1.03</td>
<td>11,388</td>
<td>51,460</td>
<td>492</td>
<td>27,737</td>
<td>3</td>
<td>78,808</td>
<td>0.664</td>
<td>34,586</td>
<td>118,649</td>
<td>253,697</td>
</tr>
<tr>
<td>2.19</td>
<td>1.17</td>
<td>16,486</td>
<td>74,492</td>
<td>713</td>
<td>40,151</td>
<td>4</td>
<td>107,157</td>
<td>0.860</td>
<td>27,208</td>
<td>124,532</td>
<td>255,192</td>
</tr>
<tr>
<td>2.19</td>
<td>1.28</td>
<td>16,427</td>
<td>74,229</td>
<td>710</td>
<td>40,009</td>
<td>5</td>
<td>106,836</td>
<td>0.858</td>
<td>27,292</td>
<td>124,465</td>
<td>255,175</td>
</tr>
<tr>
<td>2.12</td>
<td>1.42</td>
<td>15,965</td>
<td>72,139</td>
<td>690</td>
<td>38,883</td>
<td>7</td>
<td>104,264</td>
<td>0.841</td>
<td>27,961</td>
<td>123,932</td>
<td>255,039</td>
</tr>
<tr>
<td>2.06</td>
<td>1.56</td>
<td>15,497</td>
<td>70,023</td>
<td>670</td>
<td>37,743</td>
<td>8</td>
<td>101,666</td>
<td>0.824</td>
<td>28,638</td>
<td>123,392</td>
<td>254,902</td>
</tr>
<tr>
<td>1.99</td>
<td>1.69</td>
<td>14,992</td>
<td>67,744</td>
<td>648</td>
<td>36,514</td>
<td>9</td>
<td>98,857</td>
<td>0.805</td>
<td>29,368</td>
<td>122,810</td>
<td>254,754</td>
</tr>
<tr>
<td>1.87</td>
<td>1.83</td>
<td>14,081</td>
<td>63,626</td>
<td>609</td>
<td>34,294</td>
<td>11</td>
<td>93,790</td>
<td>0.770</td>
<td>30,687</td>
<td>121,758</td>
<td>254,487</td>
</tr>
<tr>
<td>1.75</td>
<td>1.97</td>
<td>13,138</td>
<td>59,638</td>
<td>568</td>
<td>31,999</td>
<td>12</td>
<td>88,550</td>
<td>0.734</td>
<td>32,051</td>
<td>120,671</td>
<td>254,210</td>
</tr>
<tr>
<td>1.62</td>
<td>2.11</td>
<td>12,170</td>
<td>54,991</td>
<td>526</td>
<td>29,640</td>
<td>14</td>
<td>83,164</td>
<td>0.696</td>
<td>33,453</td>
<td>119,553</td>
<td>253,926</td>
</tr>
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</table>

Table 6 Forces acting on the fully loaded vehicle using thirteenth engaged gear.

<table>
<thead>
<tr>
<th>$a$ [m.s$^{-2}$]</th>
<th>$v$ [m.s$^{-1}$]</th>
<th>$R_{i1}$ [N]</th>
<th>$R_{i2}$ [N]</th>
<th>$R_{in1d}$ [N]</th>
<th>$R_{in2d}$ [N]</th>
<th>$R_{in}$ [N]</th>
<th>$F_{x1}$ [N]</th>
<th>$H_{r1}$ [N]</th>
<th>$N_{f1}$ [N]</th>
<th>$N_{r1}$ [N]</th>
<th>$N_{r2}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.059</td>
<td>9.56</td>
<td>428</td>
<td>1,933</td>
<td>18</td>
<td>21</td>
<td>291</td>
<td>18,133</td>
<td>0.171</td>
<td>50,393</td>
<td>106,057</td>
<td>250,482</td>
</tr>
<tr>
<td>0.049</td>
<td>10.69</td>
<td>404</td>
<td>1,827</td>
<td>17</td>
<td>20</td>
<td>365</td>
<td>18,077</td>
<td>0.170</td>
<td>50,412</td>
<td>106,045</td>
<td>250,475</td>
</tr>
<tr>
<td>0.039</td>
<td>11.83</td>
<td>309</td>
<td>1,394</td>
<td>13</td>
<td>15</td>
<td>446</td>
<td>17,626</td>
<td>0.166</td>
<td>50,534</td>
<td>105,951</td>
<td>250,447</td>
</tr>
<tr>
<td>0.029</td>
<td>12.97</td>
<td>210</td>
<td>950</td>
<td>9</td>
<td>10</td>
<td>536</td>
<td>17,170</td>
<td>0.162</td>
<td>50,657</td>
<td>105,857</td>
<td>250,418</td>
</tr>
<tr>
<td>0.01</td>
<td>14.11</td>
<td>104</td>
<td>471</td>
<td>5</td>
<td>5</td>
<td>635</td>
<td>16,678</td>
<td>0.158</td>
<td>50,791</td>
<td>105,754</td>
<td>250,387</td>
</tr>
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</table>

Tables 7, 8 and 9 present time (t) and distance (d) in the vehicle acceleration using engine maximum torque curve, illustrated in Table 2. The time of shift gears is 0.7 s. The shift gears always occur in the maximum engine speed ($n = 198.8$ rad.s$^{-1}$).
Table 7 Time and distance for the vehicle accelerate from 0 to 13.89 m.s\(^{-1}\) as functions of the groups and subgroups.

<table>
<thead>
<tr>
<th>t(s)/d[m]</th>
<th>SG1a</th>
<th>SG1b</th>
<th>SG1c</th>
<th>SG2a</th>
<th>SG2b</th>
<th>SG2c</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>15.1/109.8</td>
<td>16.2/112.6</td>
<td>26.8/179.4</td>
<td>16.7/126.2</td>
<td>18.2/130.1</td>
<td>39.3/268.6</td>
</tr>
<tr>
<td>G2a</td>
<td>28.3/324.2</td>
<td>28.5/234.3</td>
<td>32.8/245.5</td>
<td>71.3/742.6</td>
<td>71.6/742.9</td>
<td>81.0/769.0</td>
</tr>
<tr>
<td>G2b</td>
<td>28.3/324.2</td>
<td>28.6/234.4</td>
<td>33.1/246.4</td>
<td>71.3/742.6</td>
<td>71.7/743.0</td>
<td>81.8/771.8</td>
</tr>
<tr>
<td>G3a</td>
<td>20.1/156.1</td>
<td>20.1/156.1</td>
<td>22.4/261.9</td>
<td>26.2/223.1</td>
<td>26.3/223.1</td>
<td>29.9/232.5</td>
</tr>
<tr>
<td>G3b</td>
<td>20.1/156.1</td>
<td>20.1/156.1</td>
<td>22.5/262.3</td>
<td>26.2/223.1</td>
<td>26.3/223.1</td>
<td>30.0/233.1</td>
</tr>
<tr>
<td>G3c</td>
<td>20.1/156.1</td>
<td>22.2/161.4</td>
<td>40.6/276.0</td>
<td>26.2/223.1</td>
<td>29.6/231.5</td>
<td>113.3/801.9</td>
</tr>
<tr>
<td>G3d</td>
<td>20.1/156.1</td>
<td>22.5/162.2</td>
<td>41.1/279.7</td>
<td>26.2/223.1</td>
<td>30.0/233.0</td>
<td>119.1/843.8</td>
</tr>
<tr>
<td>G4a</td>
<td>24.3/196.2</td>
<td>28.0/205.8</td>
<td>57.5/395.4</td>
<td>40.2/374.2</td>
<td>47.1/393.1</td>
<td>-</td>
</tr>
<tr>
<td>G4b</td>
<td>24.3/196.2</td>
<td>28.5/207.6</td>
<td>58.6/403.0</td>
<td>40.2/374.2</td>
<td>48.3/397.7</td>
<td>-</td>
</tr>
<tr>
<td>G4c</td>
<td>24.3/196.2</td>
<td>24.5/196.3</td>
<td>27.8/204.8</td>
<td>40.2/374.2</td>
<td>40.3/374.3</td>
<td>46.5/390.9</td>
</tr>
<tr>
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<td>24.5/196.3</td>
<td>27.9/205.5</td>
<td>40.2/374.2</td>
<td>40.4/374.4</td>
<td>46.9/392.4</td>
</tr>
</tbody>
</table>

Table 7 shows the time (t) and the distance (d) for the vehicle accelerates from 0 to 13.89 m.s\(^{-1}\) as functions of the groups and subgroups. The sequences of gears used range from 1 to 12. In the road with \(\mu_0 = 0.3\), there are semitrailer CG positions in which the performance of the empty vehicle and fully loaded vehicle are respectively worse and better than when it has a partial load. Table 8 shows that this fact happens even on dry roads (\(\mu_0 = 0.7\)) when the vehicle accelerates in low velocities. This table presents the time (t) and the distance (d) for the vehicle accelerates from 0 to 5.56 m.s\(^{-1}\) as functions of the groups and subgroups. The sequences of gears used range from 1 to 7.

In practice, the driver needs to number of the sixteen gears. Figure 7 illustrates an acceleration graphic for the configuration G2b-SG1b. In this configuration, the vehicle total weight is \(W = 407.115\) N, the semitrailer CG in the direction x is \(c_{g_x} = 5.253\) m, the semitrailer CG in the direction z is \(c_{g_z} = 2.461\) m, the maximum adhesion coefficient between tyres and pavement is \(\mu_0 = 0.7\) and the grade of ramp is \(G = 0%\). Figure 7 shows that initial gear should be the fourth one, and it also illustrates the best sequence of gears in the vehicle acceleration. Table 9, which is based on Figure 7, presents the time and the distance for the vehicle accelerates using the best sequence of gears in several ranges of velocities (from 0 to 5.56 m.s\(^{-1}\), 0-11.11 m.s\(^{-1}\), 0-16.67 m.s\(^{-1}\), 0-22.22 m.s\(^{-1}\), 0-27.78 m.s\(^{-1}\), 5.56-11.11 m.s\(^{-1}\), 5.56-16.67 m.s\(^{-1}\), 5.56-22.22 m.s\(^{-1}\), 11.11-16.67 m.s\(^{-1}\), 11.11-22.22 m.s\(^{-1}\), 16.67-22.22 m.s\(^{-1}\)).
Table 8 | Time and distance for the vehicle accelerate from 0 to 5.56 m.s\(^{-1}\) as functions of the groups and subgroups.

<table>
<thead>
<tr>
<th>t[s]/d[m]</th>
<th>SG1a</th>
<th>SG1b</th>
<th>SG1c</th>
<th>SG2a</th>
<th>SG2b</th>
<th>SG2c</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>5.9/19.6</td>
<td>6.9/22.1</td>
<td>11.7/35.4</td>
<td>6.0/20.0</td>
<td>7.4/23.3</td>
<td>16.5/48.8</td>
</tr>
<tr>
<td>G2a</td>
<td>7.9/26.7</td>
<td>8.1/26.9</td>
<td>12.1/36.4</td>
<td>8.8/30.3</td>
<td>9.1/30.6</td>
<td>17.7/52.0</td>
</tr>
<tr>
<td>G2b</td>
<td>7.9/26.7</td>
<td>8.1/26.9</td>
<td>12.3/36.9</td>
<td>8.8/30.3</td>
<td>9.2/30.7</td>
<td>18.3/53.6</td>
</tr>
<tr>
<td>G3a</td>
<td>6.6/22.3</td>
<td>6.7/22.3</td>
<td>8.9/27.5</td>
<td>7.0/23.5</td>
<td>7.0/23.6</td>
<td>10.4/31.6</td>
</tr>
<tr>
<td>G3b</td>
<td>6.6/22.3</td>
<td>6.7/22.3</td>
<td>8.9/27.5</td>
<td>7.0/23.5</td>
<td>7.0/23.6</td>
<td>10.5/32.0</td>
</tr>
<tr>
<td>G3c</td>
<td>6.6/22.3</td>
<td>8.7/27.1</td>
<td>17.1/50.5</td>
<td>7.0/23.5</td>
<td>10.2/31.0</td>
<td>44.3/126.4</td>
</tr>
<tr>
<td>G3d</td>
<td>6.6/22.3</td>
<td>8.9/27.6</td>
<td>17.4/51.1</td>
<td>7.0/23.5</td>
<td>10.5/31.9</td>
<td>46.4/132.2</td>
</tr>
<tr>
<td>G4a</td>
<td>7.3/24.6</td>
<td>10.7/32.6</td>
<td>23.8/68.9</td>
<td>7.9/26.9</td>
<td>14.2/42.2</td>
<td>-</td>
</tr>
<tr>
<td>G4b</td>
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<td>11.1/33.7</td>
<td>24.2/70.1</td>
<td>7.9/26.9</td>
<td>15.0/44.6</td>
<td>-</td>
</tr>
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<td>G4c</td>
<td>7.3/24.6</td>
<td>7.4/24.7</td>
<td>10.5/32.1</td>
<td>7.9/26.9</td>
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<td>7.9/26.9</td>
<td>8.1/27.1</td>
<td>14.0/41.8</td>
</tr>
</tbody>
</table>

Table 9 | Time and distance in the vehicle acceleration (G2b-SG1b, full load, \(\mu_0 = 0.7\), G = 0%).

<table>
<thead>
<tr>
<th>velocity[m.s(^{-1})]</th>
<th>t[s]</th>
<th>d[m]</th>
<th>gears</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5.56</td>
<td>4.8</td>
<td>15.5</td>
<td>4-7</td>
</tr>
<tr>
<td>0-11.11</td>
<td>15.9</td>
<td>109.0</td>
<td>5-9-11</td>
</tr>
<tr>
<td>0-16.67</td>
<td>34.5</td>
<td>370.4</td>
<td>5-9-11-13</td>
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<tr>
<td>0-22.22</td>
<td>65.6</td>
<td>981.6</td>
<td>5-9-11-13-14-15</td>
</tr>
<tr>
<td>0-27.78</td>
<td>118.6</td>
<td>2321.4</td>
<td>5-9-11-13-14-15-16</td>
</tr>
<tr>
<td>5.56-11.11</td>
<td>10.5</td>
<td>91.2</td>
<td>9-12</td>
</tr>
<tr>
<td>5.56-16.67</td>
<td>29.1</td>
<td>352.5</td>
<td>9-11-13</td>
</tr>
<tr>
<td>5.56-22.22</td>
<td>60.7</td>
<td>967.3</td>
<td>9-12-14-15</td>
</tr>
<tr>
<td>11.11-16.67</td>
<td>18.6</td>
<td>261.2</td>
<td>13</td>
</tr>
<tr>
<td>11.11-22.22</td>
<td>49.7</td>
<td>872.4</td>
<td>13-14-15</td>
</tr>
<tr>
<td>16.67-22.22</td>
<td>30.5</td>
<td>601.1</td>
<td>14-15</td>
</tr>
</tbody>
</table>
6 Conclusions

The influence of the load CG position on heavy vehicle acceleration, specifically tractor-semi-trailer vehicle with 4 × 2 traction system, was presented in this paper. A steady state model mathematical, a vehicle CG position and a computational procedure are utilised for investigating the vehicle acceleration. Computer simulations are developed for different CG positions of the load. For a specific loading vehicle, the load CG position influences the performance of the vehicle, mainly in the velocity changes while overtaking on a hill. Depending on the load CG position the following situations may occur: the empty vehicle presenting worse performance than when it has got some load or the fully loaded vehicle presenting better performance than when it has got a smaller load. Such influences were shown in the calculation of the following quantities: vehicle maximum acceleration, and time and distance during vehicle acceleration. The study presented in this paper permits one to foresee limitations the driver is likely to face while transporting a certain load on a predetermined road. The load CG position influences the vehicular security and the efficiency and economy in the transportation of the load. The acceleration performance study can be used to determine the best position for load transportation. It is recommended that a partial load be transported as close to the fifth-wheel as possible. This load should also be distributed in a way to make it as low as possible.

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References


Influences of the load centre of gravity on heavy vehicle acceleration


