On Extended Target Tracking Using PHD Filters

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Multiple target tracking

- Find the location of multiple targets
  - Unknown number.
  - The targets are not always detected.
  - Noisy measurements.
  - Clutter.
  - Difficult data association.

- Early RADAR-airplane-tracking assumed that targets produce at most one measurement.
Multiple target tracking

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- Early RADAR-airplane-tracking assumed that targets produce at most one measurement.

- Modern sensors have higher resolution, multiple measurements per target.

Need framework that handles multiple measurements per target.
One measurement per target is often not valid, e.g.,

- laser sensors, camera images, or automotive radar.

**Definition:**

Extended targets are targets that potentially give rise to more than one measurement per time step.
• Random finite set (RFS) of targets $X_k = \left\{ \xi_k^{(i)} \right\}_{i=1}^{N_{x,k}}$

\[ \xi_{k+1} = f_k \left( \xi_k^{(i)}, w_k^{(i)} \right) \quad w_k^{(i)} \text{ – process noise.} \]

• RFS of measurements $Z_k = \left\{ z_k^{(j)} \right\}_{j=1}^{N_{z,k}}$

\[ z_k^{(j)} = h_k \left( \xi_k^{(i)}, e_k^{(j)} \right) \quad e_k^{(j)} \text{ – measurement noise.} \]

Aim:

Compute target set estimate $\hat{X}_{k|k}$ using measurement sets $Z^k$. 
The number of target measurements must be modelled. Gilholm et al. (2005) suggests using a Poisson model. For the $i$:th target at time step $k$,

$$N_{z,k}^{(i)} \in \text{POIS} \left( \gamma_k^{(i)} \right).$$

Effective probability of target detection

$$p_{D,\text{eff}} = \left(1 - e^{-\gamma_k^{(i)}}\right) p_D.$$

ET-PHD filter under Gilholm’s model given by Mahler in 2009.
Prediction of PHD-intensity performed identically to standard PHD-filter

\( D_{k|k-1}(\xi|Z) \) is predicted PHD-intensity. Corrected PHD-intensity

\[
D_{k|k}(\xi|Z) = L_{Z_k}(\xi) D_{k|k-1}(\xi|Z),
\]

where measurement pseudo-likelihood is given by

\[
L_{Z_k}(\xi) = 1 - \left( 1 - e^{-\gamma(\xi)} \right) p_D(\xi) + e^{-\gamma(\xi)} p_D(\xi) \sum_{p \not\in Z_k} \omega_p \sum_{W \in p} \frac{\gamma(\xi)^{|W|}}{d_W} \cdot \prod_{z \in W} \frac{\phi_z(\xi)}{\lambda_k c_k(z)}.
\]

[Mahler, 2009]
We will focus on the following aspects of ET-PHD:

- Measurement set partitioning.
- Target measurement rate.
- Probability of detection.
- Two PHD intensity approximations:
  1. Gaussian mixture.
  2. Gaussian inverse Wishart mixture.
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Partitioning the measurements

- In each time step $Z_k$ must be partitioned.
- A partition $p$ is a division of the set $Z_k$ into non-empty subsets, called cells $W$.
- Important because more than one measurement can stem from the same target.
Partitioning the measurements — example

Partition the measurement set \( Z_k = \{ z_k^{(1)}, z_k^{(2)}, z_k^{(3)} \} \)
Partitioning method

- Number of possible partitions for $n$ measurements given by $n$:th Bell number $B_n$.

- The Bell numbers $B_n$ increase very fast when $n$ increases, e.g. $B_3 = 5$, $B_5 = 52$ and $B_{10} = 115975$. 

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Partitioning method

- Number of possible partitions for \( n \) measurements given by \( n \):th Bell number \( B_n \).

- The Bell numbers \( B_n \) increase very fast when \( n \) increases, e.g. \( B_3 = 5 \), \( B_5 = 52 \) and \( B_{10} = 115975 \).

- Necessary to approximate the full set of partitions with a subset of partitions.

- Intuition: Measurements are from same source if they are close, with respect to some measure or distance.
Partitioning method

- Method: Measurements are in same cell \( W \) if distance is “small”.
- Partitions \( p_i \) where cells contain measurements where distance to nearest measurement is \(< d_i \).
- Limit to partitions for thresholds \( d_i \) that satisfy

\[
d_{\text{min}} \leq d_i < d_{\text{max}}
\]
Partitioning method

- Method: Measurements are in same cell $W$ if distance is “small”.
- Partions $p_i$ where cells contain measurements where distance to nearest measurement is $< d_i$.
- Limit to partitions for thresholds $d_i$ that satisfy

$$d_{\text{min}} \leq d_i < d_{\text{max}}$$

- If possible, use scenario knowledge to choose distance measure and to determine bounds.
- Important to choose $d_{\text{min}}$ and $d_{\text{max}}$ conservatively.
Partitioning example

\[ p_1 = \{ W_1^1, W_2^1, W_3^1 \} \]

\[ p_2 = \{ W_1^2, W_2^2, W_3^2, W_4^2 \} \]

- Reasonable to discard most partitions as highly unlikely.
- Additional methods given in paper.
We will focus on the following aspects of ET-PHD:

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- Target measurement rate.
- Probability of detection.
- Two PHD intensity approximations:
  1. Gaussian mixture.
  2. Gaussian inverse Wishart mixture.
• The measurement rate is a function of the target state, $\gamma (\xi)$.
• Approximated as function of the target estimate

$$\gamma (\xi_k) \approx \gamma (\hat{\xi}_k|k)$$

• Important to have reasonable estimates of the true rates.
The measurement rate is a function of the target state, $\gamma(\xi)$.

Approximated as function of the target estimate

$$\gamma(\xi_k) \approx \gamma(\hat{\xi}_k|k)$$

Important to have reasonable estimates of the true rates.

Sometimes possible to design a model for $\gamma(\hat{\xi}_k|k)$

Can be modelled as Gamma distributed and estimated online

$$p\left(\gamma_k \mid Z^k\right) = \text{GAM}\left(\gamma_k \mid \alpha_k|k, \beta_k|k\right)$$
• Gamma distribution is conjugate prior for Poisson measurements.
• Simple exponential forgetting with effective window length $w_e$ is used for prediction.
• Possible to estimate multiple rates simultaneously
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Probability of detection

- Probability of detection is function of target state, $p_D(\xi)$.
- Approximated as function of the target estimate
  $$p_D(\xi_k) \approx p_D(\hat{\xi}_{k|k})$$
- This allows for a non-homogeneous probability of detection
  \[ \Rightarrow \] possible to handle target occlusion.
  \[ \Rightarrow \] also brings problems: bias in the number of the targets

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PHD intensity approximations

- At time step $k$ the PHD-intensity is approximated as a weighted mixture of distributions,

$$D_{k|k}(\xi|Z) = \sum_{i=1}^{J_{k|k}} w^{(i)}_{k|k} p\left(\xi_k; \zeta^{(i)}_{k|k}\right).$$

- $\xi_k$ is the extended target state. Kinematical states, states that govern shape and size, etc.
- $\zeta^{(i)}_{k|k}$ is the distribution parameter for the $i$:th mixture component.
\[ D_{k|k} (x|Z) = \sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)} N \left( x_k ; m_{k|k}^{(i)}, P_{k|k}^{(i)} \right) \]

- \( \xi_k = x_k \) and \( \zeta_{k|k}^{(i)} = \left( m_{k|k}^{(i)}, P_{k|k}^{(i)} \right) \).
- The extended target state is a vector \( x \) that contains all states, e.g. kinematical states and parameters for shape and size.
- Measurement model

\[ p (z_k|x_k) = \mathcal{N} (z_k ; h_k (x_k), R_k) \]
Gaussian mixture – a simple case

- Example state vector

\[ \mathbf{x} = [x \ y \ v_x \ v_y]^T \]

- Measurement model

\[ p(\mathbf{z}_k | \mathbf{x}_k) = \mathcal{N}(\mathbf{z}_k ; H_k \mathbf{x}_k, R_k) \]

\[ H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

- Extension implicitly assumed circular with constant radius.
Experiment data

- SICK laser range sensor used to collect data.
- Multiple human targets, at most 3 at the same time.
- Measurements of stationary objects removed beforehand.
- Difficult to handle occlusion when targets are close.
- “Multiplicatively biased” cardinality estimate on the edge of low probability of detection areas.
Difficult to handle occlusion when targets are close.

“Multiplicatively biased” cardinality estimate on the edge of low probability of detection areas.

⇒ Estimate the shape and size for each target.
Gaussian inverse Wishart mixture

\[ D_{k|k} (\xi | Z) = \sum_{i=1}^{J_{k|k}} w_{k|k}^{(i)} \mathcal{N} \left( x_k; m_{k|k}^{(i)}, P_{k|k}^{(i)} \otimes X_k \right) \mathcal{IW} \left( X_k; v_{k|k}^{(i)}, V_{k|k}^{(i)} \right) \]

- \( \xi_k = (x_k, X_k) \) and \( \zeta_{k|k}^{(i)} = (m_{k|k}^{(i)}, P_{k|k}^{(i)}, v_{k|k}^{(i)}, V_{k|k}^{(i)}) \).
- The extended target state decomposes to the kinematical state vector \( x \) and the extension state matrix \( X \).
- Extension shape is modelled as elliptic.
- Measurement model

\[ p (z_k | \xi_k) = \mathcal{N} (z_k; h_k (x_k), X_k) \]
• Example state vector

\[ x = [x \ y \ v_x \ v_y]^T \]

• Measurement model

\[ p(z_k | x_k) = \mathcal{N}(z_k ; H_k x_k, X_k) \]

\[ H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]
- Occlusion when targets are close is no longer difficult.
- Cardinality estimate on the edge of low probability of detection areas still “biased”, but less so.
- Overall the performance is improved.
Estimating the extension

- The GIW model applied to human and bike in laser data.
  Reasonable approximation of shape.

- Less suitable shape model for cars measured with laser sensor.
  Need for models that do not assume a certain shape.

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• Consider a number of measurements
• Target can be estimated as an extended target, without considering shape.
Estimating the Shape

- Consider a number of measurements
- Target can be estimated as an extended target, without considering shape.
- Considering shape – How to describe relation between point measurements and shape?
• Consider a number of measurements
• Target can be estimated as an extended target, without considering shape.
• Considering shape – How to describe relation between point measurements and shape?
• Reflection point – Data association problem
Measurement Generating Points

Measurements

\[ Z_k = \{ z_k^{(1)}, \ldots, z_k^{(3k)} \} \]

- Point observations \( z_k^{(m)} \)
- number of measurements \( 3k \) not fixed, due to:
  - detection uncertainty
  - spurious measurements
  - unknown number of reflection points

Measurement generating points

\[ S_k = \{ s_k^{(1)}, \ldots, s_k^{(s_k)} \} . \]

- A MGP \( s \in \mathbb{R} \) on the boundary
- number of MGPs \( s_k \) is not fixed
- A MGP is defined on a one dimensional coordinate axis
- restricted to MGP-space \([s_{\min}, s_{\max}]\)
Modeling MGP

- $S - \text{RFS of all MGP on boundary}$
- $S_k \subset S$ detectable MGP
- as target moves new MGP appear $B_k(x_k) \subset S$

$$S_k = \left\{ \mathbf{F}_k(S_{k-1}) \right\} \cup B_k(x_k)$$

 surviving MGP

- RFS of measurements $Z_k$

$$Z_k = \bigcup_{s \in S_k} \left\{ \mathbf{H}_k(s, x_k) \right\} \cup C_k$$

target generated
• The target state is $x$ that contains e.g., kinematical states and parameters for shape and size.
• spline representation of target shape
• First order moment of RFS $S_k$ approximated by a PHD $D$ as a means to estimate $x_k$
• Rao-Blackwellized particle filter implementation for state $x_k$
  • particles for nonlinear states
  • KF for linear states
- The PHD filter is sensitive to low $p_D(\cdot)$.

- The Cardinalized PHD (CPHD) filter solves this by propagating the full probability mass function of the target cardinality.
### ET-PHD vs. ET-CPHD

**PHD**

$Z^T_k, Z^{FA}_k$ have non-homog. Poisson pdfs

\[
f(Z^T_k | x) = e^{-N^Z_k} \prod_{z_k \in Z^T_k} \gamma(\xi)p_z(z_k | \xi)
\]

\[
f(Z_{FA}) = e^{-N_{FA}^Z} \prod_{z_k \in Z_{FA}^Z} \lambda p_{FA}(z_k)
\]

Prior $f(X_k | Z_{0:k-1})$ is assumed Poisson

\[
e^{-N_{k|k-1}} \prod_{\xi_k \in X_k} N_{k|k-1} p_{k+1|k}(\xi_k)
\]

**Aim:**

Obtain the updated PHD $D_{k|k}(\xi_k)$ using the measurement set $Z_k$.

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**CPHD**

$Z^T_k, Z^{FA}_k$ have pdfs (i.i.d. cluster)

\[
f(Z^T_k | x) = N_k^Z! P_z(N^Z_k | \xi) \prod_{z_k \in Z^T_k} p_z(z_k | \xi)
\]

\[
f(Z_{FA}) = N_{FA}^Z! P_{FA}(N_{FA}^Z) \prod_{z_k \in Z_{FA}^Z} p_{FA}(z_k)
\]

Prior $f(Z_k | Z_{0:k-1})$ is assumed cluster process

\[
N_{k|k-1}! P_{k+1|k}(N_{k+1|k}) \prod_{\xi_k \in X_k} p_{k+1|k}(\xi_k)
\]

**Aim:**

Obtain the updated PHD $D_{k|k}(\xi_k)$ and updated posterior cardinality pmf $P_{k|k}(N_k)$ using the measurement set $Z_k$. 
ET-CPHD Formulas

- **PHD update**

\[
D_{k|k}(x) = \left( \kappa (1 - P_D(x) + P_D(x) G_Z(0)) + P_D(x) \frac{\sum_{\mathcal{P} \subset \mathcal{Z}} \sum_{\mathcal{W} \in \mathcal{P}} \sigma_{\mathcal{P},\mathcal{W}} \prod_{z' \in \mathcal{W}} \frac{p_{z'}(z'|x)}{p_{FA}(z')}}{\sum_{\mathcal{P} \subset \mathcal{Z}} \sum_{\mathcal{W} \in \mathcal{P}} \alpha_{\mathcal{P},\mathcal{W}} \beta_{\mathcal{P},\mathcal{W}}} \right) D_{k|k-1}(x)
\]

- **Cardinality update**

\[
P_{k|k}(n) = \frac{\sum_{\mathcal{P} \subset \mathcal{Z}} \sum_{\mathcal{W} \in \mathcal{P}} \alpha_{\mathcal{P},\mathcal{W}} G_{k|k-1}^{(n)} (0) \left( G_{FA}(0) \frac{\eta_{W}[0,1]}{|\mathcal{P}|} \frac{\rho[1]^{n-|\mathcal{P}|}}{(n-|\mathcal{P}|)!} \delta_{n} \geq |\mathcal{P}| + G_{FA}^{(|\mathcal{W}|)}(0) \frac{\rho[1]^{n-|\mathcal{P}|+1}}{(n-|\mathcal{P}|+1)!} \delta_{n} \geq |\mathcal{P}|-1 \right)}{\sum_{\mathcal{P} \subset \mathcal{Z}} \sum_{\mathcal{W} \in \mathcal{P}} \alpha_{\mathcal{P},\mathcal{W}} \beta_{\mathcal{P},\mathcal{W}}}
\]

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**ET-CPHD Formulas**

- **PHD update**

\[ D_{k|k}(x) = \left( \kappa (1 - P_D(x) + P_D(x) G_z(0)) + P_D(x) \frac{\sum_{P \not\subset Z} \sum_{W \in P} \sigma_{P,W} \prod_{z' \in W} p_{z'}(z'|x)}{\sum_{P \not\subset Z} \sum_{W \in P} \alpha_{P,W} p_{\beta_{P,W}}} \right) D_{k|k-1}(x) \]

- **Cardinality update**

\[ P_{k|k}(n) = \frac{\sum_{P \not\subset Z} \sum_{W \in P} \alpha_{P,W} G_{k|k-1}^{(n)} (0) \left( G_{FA}(0) \frac{\eta_{[0,1]}(0)}{|P|} \frac{\rho[1]^{n-|P|}}{(n-|P|)!} \delta_{n \geq |P|} + G_{FA}(0) \frac{\rho[1]^{n-|P|+1}}{(n-|P|+1)!} \delta_{n \geq |P|-1} \right)}{\sum_{P \not\subset Z} \sum_{W \in P} \alpha_{P,W} \beta_{P,W}} \]

- **Complexity is same order as ETT-PHD (single level of partitioning)**
  - Same partitioning method used.
  - Gaussian mixture implementation assuming that
    - The prior PHD is a Gaussian mixture
    - The measurement model is linear and Gaussian.
ET-PHD vs. ET-CPHD: Results

Comparison of Gaussian mixture implementations.

\[ P^0_D = 0.99 \]
ET-PHD vs. ET-CPHD: Results

Comparison of Gaussian mixture implementations.

\[ P_D^0 = 0.70 \]
ET-PHD vs. ET-CPHD: Summary

- The optimal CPHD filter is more or less of the same infeasible complexity as the PHD filter.

- Early simulation results show that the more robust characteristics of CPHD type algorithms apply also in the extended target case.

- More experiments are necessary for testing the advantages and the applicability of these algorithms.
• The sensor moves, the landmarks are stationary.
• Target tracking is similar to SLAM.
  • In case each landmark gives more than one measurement, the landmarks could be defined as extended objects, or
  • MGP could be interpreted as landmarks
• The extended target PHD and CPHD filters could be generalized to the SLAM problem.
We have presented

- Implementational aspects of the ET-PHD filter,
  - Measurement set partitioning.
  - Target measurement rate.
  - Probability of detection.
  - Two PHD intensity approximations.
- A model for estimating the shape and size of extended targets.
- A CPHD filter for extended targets.
Thank you for listening!

Any questions?