1 Purpose and aims

Vector-fields, such as the earth's magnetic-field and gravity-field, are highly informative sources for localization. These exemplified vector-fields are omnipresent, stable over-time, and variations in the fields, if measured accurately, provides a fingerprint highly correlated to the measurement location. Hence, they constitute a viable and robust source for localization in GPS denied environments.

Today, thanks to last decade's tremendous sensor technology development, high-performing and affordable accelerometer and magnetometer vector-sensor arrays can be constructed and integrated into portable devices and platforms. Similarly as a camera can take an image of the surrounding environment, these sensor arrays can take an image-like measurement of a vector-field — a type of measurement that, in the literature, is referred to as *tensor-field observation* as it constitutes an assembly of spatially separated vector-field observations. Just as in computer-vision based localization systems, these tensor-field observation can be used for odometry via feature tracking, absolute localization via feature matching, and simultaneous localization and mapping (SLAM) via on-the-fly constructions of feature maps. However, unlike visual imaging, magnetic-field and gravity-field sensing is not impaired by smoke, dust, fog, etc. Further, in contrast to the hard to model structure of a visual images, the imaged vector-field must comply with, easy to model, physical laws.

Based on these attractive properties, the purpose of the project is to explore how tensor-field observations, combined with inertial navigation[1], can enable accurate, reliable and cost-efficient "anywhere and anytime" self-localization. More precisely, the project aim to research and develop:

- Sensor-fusion and model-learning methods for tensor-field based odometry aided inertial navigation; see Fig. 1 and Fig. 2 for an illustration of the concept. This will reduce the inertial navigation error growth rate from cubic to linear in time, which in turn will enable significant longer stand-alone operation times before the location error gets too large.
- Sensor-fusion and map-representation methods tailored for joint tensor-field based odometry and SLAM. Compared to existing vector-field SLAM solutions, this will drastically extend the length of the allowable exploration phases where new areas are mapped, as well as increase the accuracy and reliability through more informative field measurement.

Possible use cases of the developed self-localization solutions include pedestrian indoor navigation, underwater navigation, autonomous vehicle navigation in mines, aerospace navigation, etc.

The project investigator (PI) I. Skog has an extensive track record of theoretical and applied research in the area of signal processing for localization, with several highly cited (+100 citations) papers within the field [1, 2, 3, 4, IS46] and a wide range of national and international collaborations; see Table 1. His unique ability of combining theoretical and applied research, together with his in-depth knowledge about magnetic and inertial sensors, guarantees results of high academic and industrial impact, and makes him well suited to lead the proposed project.

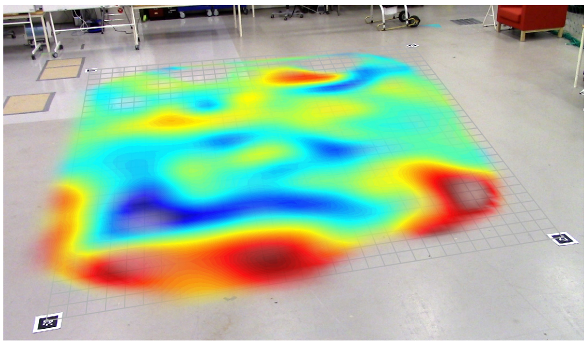

Fig. 1. Illustration of the magnetic-field potential variations inside a building (Courtesy: A. Solin, et. al. [5]).

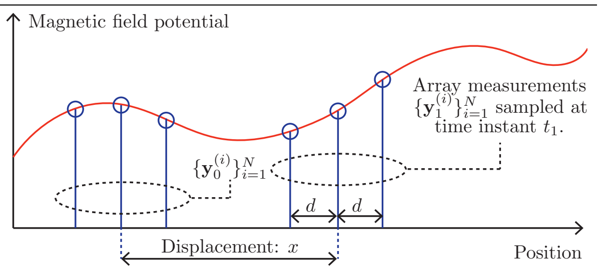

Fig. 2. Illustration of the tensor-field odometry method. Given the tensor-field observations $\{\mathbf{y}_0^{(i)}\}_{i=1}^N$ and $\{\mathbf{y}_1^{(i)}\}_{i=1}^N$, a model for the field is constructed and the displacement $\mathbf{x}$ that best fits this model is estimated and then used to calibrate the inertial navigation process.

[1] Today, inertial navigation is the backbone of most of self-localization systems, but to provide long stand-alone operation time with good accuracy high-cost sensors must be used.

2 State-of-the-art

Tensor-field odometry

State-of-the-art techniques for reducing the inertial navigation error drift rate is based upon sensor calibration [IS17, 6], imposing motion constraints [IS18, GH7], and fusing multiple inertial sensors [IS12, IS40, IS41]. In the context of visual aided inertial navigation, the inertial navigation error drift rate is commonly reduced using optical-flow and feature-point based odometry techniques [GH9, 7]. Recently, as an offspring from these methods, the idea of using tensor-field observations for odometry was proposed [8]. The idea is based upon the differential equation

$$\frac{d\mathbf{f}(\mathbf{r})}{dt} = \mathbf{f}(\mathbf{r}) \times \boldsymbol{\omega} + \frac{d\mathbf{f}(\mathbf{r})}{d\mathbf{r}}\,\mathbf{v}, \tag{1}$$

which relates the rate of change of the field $\mathbf{f}(\mathbf{r})$ to the rotation rate $\boldsymbol{\omega}$ of the sensor array, the Jacobian $d\mathbf{f}(\mathbf{r})/d\mathbf{r}$ of the field w.r.t. the position $\mathbf{r}$, and the velocity $\mathbf{v}$. Given the tensor-field observations, obtained from an array of spatially separated vector-field sensors, the Jacobian $d\mathbf{f}(\mathbf{r})/d\mathbf{r}$ can be estimated and the differential equation solved. That is, the velocity $\mathbf{v}$ can be estimated and used to aid the inertial navigation system (INS). This will reduce the inertial navigation error drift rate from cubic to linear in time. Thus it is a prospective game changer for inertial navigation based self-localization, as much longer stand-alone operation can be allowed before the position error gets too large and extern location information is needed to calibrate the system. To give an example, in a system using commercial-grade inertial sensors the stand-alone operation phase can then be extended from about a minute to tens of minutes. Noteworthy is that (1) holds for all vector fields in $\mathbb{R}^3$ and no map of the field is required beforehand to solve the equation. Hence, equation (1) cannot only be used for odometry using the magnetic or gravity field, but also with e.g., acoustic fields [GH10].

The sensor configuration and geometric requirements on the array that must be fulfilled to estimate the Jacobian depends the physical properties of the vector-field. For example, the free-space magnetic-field is both curl and divergence free, that is

$$\text{Curl-free field:} \quad \nabla_{\mathbf{r}} \times \mathbf{f}(\mathbf{r}) = 0 \tag{2a}$$

$$\text{Div-free field:} \quad \nabla_{\mathbf{r}} \cdot \mathbf{f}(\mathbf{r}) = 0 \tag{2b}$$

and the Jacobian $d\mathbf{f}(\mathbf{r})/d\mathbf{r}$ can be estimated with a planar array with only three vector-sensors [IS26]; noteworthy, the gravity-field is also curl-free.

Based upon (1) various nonlinear observers and extended Kalman filters for magnetic tensor-field odometry aided INS have been proposed [9, 10, 11]. The results presented in [9, 10, 11] show that, in situations with a high field-variations-to-noise ratio, such a tensor-field odometry aided inertial navigation system can indeed achieve a linear in time position error growth rate. However, in areas with a low field-variations-to-noise ratio, the proposed methods perform poorly. This is, inter alia, an effect of the Jacobian of the field being computed by direct differentiation of the sensor measurements, which amplifies the measurement noise. Hence, to utilize the full potential of the tensor-field odometry concept, new and refined sensor fusion methods that takes into account the physical laws of the field, are needed.

In the recent paper [IS26] by the PI and the co-PI, the tensor-field odometry problem was studied from a model estimation perspective; see Fig. 2 for a conceptional illustration. This enables the application of estimation theory to analyze the properties of the odometry problem and physical knowledge about the field to be incorporated in the estimation process, e.g., the constraints (2a) and (2b). To that end, a curl and divergence free polynomial model for the local magnetic-field was derived and used to create a maximum-likelihood estimator for the displacement (velocity) of the array. The experimental results presented in [IS26] indicate that the model-based approach has significantly higher accuracy and is more robust than the methods used in [8, 9, 10]. However, currently no sensor-fusion methods for model-based tensor-field odometry aided inertial navigation exists, nor does the problem fit into any of the standard sensor-fusion methods for inertial navigation.

Tensor-field SLAM

There exists a multitude of techniques for self-localization using inertial navigation in combination with feature-matching (fingerprinting) of observations from spatially varying fields, such as radio signal strength [GH8], magnetic-field [12, 13], gravity-field [14], etc. However, their scalability and usability is limited by the need of pre-existing feature maps, and the achievable accuracy is dependent on the resolution of the existing maps [15]. To avoid the need of pre-existing maps, and enable on-the-fly creation of high-resolution maps, a range of techniques for simultaneous localization and mapping (SLAM) have been developed [16]. That is, a function (map) that describes the vector-field $\mathbf{f}(\mathbf{r})$ as a function of the location $\mathbf{r}$, measured by the inertial navigation system, is incrementally built as new field measurements are collected. Once a previously map location is revisited, i.e., a loop-closure is done, the errors in the inertial navigation process is calibrated and the map refined.

Fundamental to successful operation of all SLAM systems is the existence of informative features and frequent loop-closures. The requirement to frequently revisit mapped areas is often in conflict with the operational goal of the localization system user. The allowable length of the exploratory phases, i.e., the time between loop-closures, depends on the uniqueness of the features and the error drift rate of the inertial navigation process. To increase the scalability and usability of current vector-field based SLAM solutions, there is a need for techniques that reduce the inertial navigation error drift rate and enable faster and more unique map-feature learning.

Today, the state-of-the-art methods for vector-field based SLAM are based upon modeling the underlying field $\mathbf{f}(\mathbf{r})$ using polynomial models [17, 18] or Gaussian processes [19, 20]. The Gaussian process based SLAM methods have shown impressive accuracy when used for indoor self-localization using variations in the local magnetic-field [20], but suffers from the earlier highlighted problems due to the inertial navigation error grow rate and the limited uniqueness of the map-features obtained with a single vector-field senors; a single vector-field sensor can be viewed as a camera with only one pixel and thus provide limited information about the vector-field and the potential location of the system.

A brute force approach to extend existing vector-field based SLAM methods to handle tensor-field observations is to simply include the observations from each vector-sensor in the array as an additional and spatially slightly offset measurements of the field. This would enable faster map learning, as field measurements from a larger area is obtained at every time instant; equivalent to having a multi-pixel camera taking an image of the surrounding. However, to get the full benefit of the tensor-field observations, i.e., use all the information in the image of the field, map representations methods with multi-scale resolution are needed — micro-scale resolution to support more unique features matching and odometry, and macro-scale resolution for coverage and extrapolation.

A challenge when representing the underlying field using a Gaussian process is that standard Gaussian process regression suffers from the curse of dimensionality, with a computational complexity that grows cubically in the number of data points. Therefore, various sparse Gaussian process approximations, such as inducing point representations [GH1], base function expansions [21], reduced-rank approximations of the kernel [5], etc., have been developed. To improve the accuracy of the learned maps and enable better extrapolation to unmapped areas, methods for including physical properties, such as curl and divergence freeness, of the underlying field into the Gaussian process, have been developed [5, 22]. Unfortunately, current sparse Gaussian process map-representation methods: (i) only incorporate parts of the physical properties of the field into the model [5]; (ii) create mismatches in the boundary between map segments [20]; and (iii) has limited short scale resolution to support tensor-field odometry. Hence, to fully utilize the benefits of tensor-field observations in the SLAM process, new and improved map-representation methods, along with associated inference strategies for sensor-fusion and model-learning that support both odometry and SLAM, are needed.

3 Significance and scientific novelty

Advancement of the research front & relation to previous research within the area

In its first phase, the project will develop sensor-fusion and learning techniques for model-based tensor-field odometry aided inertial navigation. This will advance the state-of-the-art techniques for tensor-field odometry systems by enabling them to operate at lower field-variation-to-noise ratios and increase their accuracy [IS26]. This, in turn, will enable the technology to be used in a more diverse set of environments and significantly reduce the error drift rate of the inertial navigation process.

In a model-based approach to tensor-field based odometry aided inertial navigation the observation function, which relates the tensor-field observations to the system dynamics, constitutes an unknown model of the local vector-field. Therefore, focus will be on developing techniques for real-time joint state-inference and fast learning of local and short lived observation models in dynamic systems. As a starting point for the development of these techniques, recent methods for joint state-inference and system identification in dynamic systems [GH1, GH2, 23], encoding of physical properties into regression models [5, IS26, 24], and multipole modelling of vector-fields [25, 26], will be used.

In the second phase, currently vector-field (map) modeling and regression techniques, see e.g., [20], will be revisited and extended to handle both micro- and macro-scale field variations. This will allow them to be used for both tensor-field odometry and SLAM within the same framework. Two approaches to develop models and regression methods with the desired properties will be explored. The first approach is to merge recent developed multi-resolution Gaussian process models [27, 28] with methods for encoding of physical properties into Gaussian process [5]. The second approach is to approximate the observed vector-field using a series of, by physical laws, motivated basis functions [25], and thereby obtain map representation that fulfills all the physical properties of the underlying field and are separable into micro-and macro-scale field variations.

In the final phase, based upon the in previous phases developed sensor-fusion, model-learning, and vector-field modeling techniques, methods for joint tensor-field based odometry and SLAM will be developed. This will significantly boost the accuracy, reliability, and scalability of the state-of-the-art techniques for vector-field SLAM by providing significantly longer exploration phases, and faster and more unique map-feature learning through more informative measurements. Targeted is hierarchial sensor-fusion and learning methods that first learn the micro-scale variations to support the odometry, and then adaptively decompose and sparsify (resample) these to a macro-scale resolution suitable for large-scale SLAM. As starting point for this development recent methods for joint state-inference and system identification in dynamic systems [GH1, GH2, 23] will be combined with recent developed multi-resolution Gaussian process and models [27, 28].

Scientific impact

Though framed within the context of localization and navigation, the tensor-field observations based sensor-fusion and model-learning methods developed, are all generic. Hence, beyond advancing the scientific field of localization and navigation, the research provides generic tools for processing of data from a type of sensing devices, i.e., vector sensor arrays, which is expected to grow in both number and availability. And thus, the developed methods and tools will contribute to the general advancement of the signal processing, automatic control, and machine learning research fields.

Industrial and societal impact

Accurate and scalable indoor self-localization is a long sought after capability, which once realized, will have far reaching economical and social impact. Magnetic-field based SLAM is considered one of the most promising technologies for realizing this capability in the near future [20]. The proposed research targets the key challenges — the frequently required loop-closures and the limited uniqueness of the observed map-features — associated with current magnetic-field SLAM technology, and will enable the technology to take a large leap in terms of accuracy and scalability. Further, as high-sensitive and affordable superconducting quantum interference magnetometer arrays become available

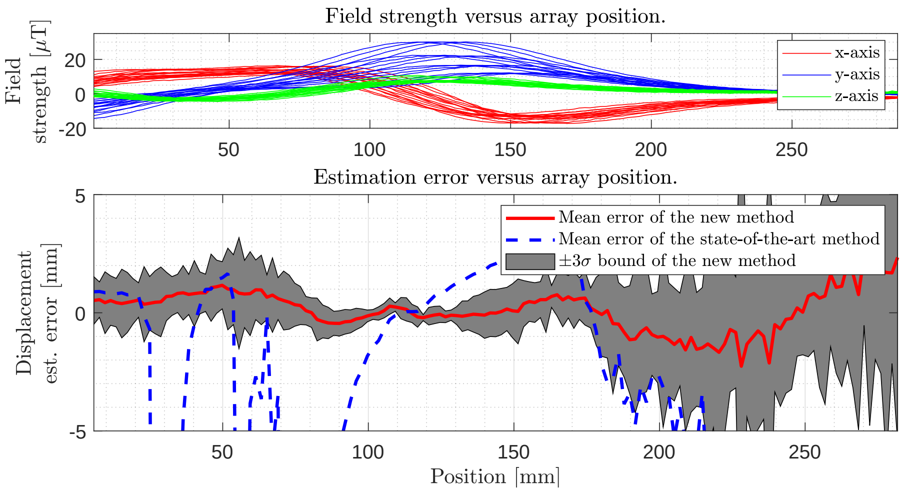

Fig. 3. Results from moving a miniaturized sensor array through an artificially created magnetic-field. Shown is the mean and standard deviation (in terms of the 3σ bound) of the displacement estimation error for the new model-based odometry method. Also shown is the mean estimation error of the state-of-the-art odometry method, which is based upon directly solving differential equation (1). The true displacement of the array was 10 $[mm]$. As seen the model-based approach outperforms the state-of-the-art method, which only works when there are large field variations.

the developed methods will also enable accurate and scalable self-localization in environments, such as the oceans, subjected to only subtle magnetic-field variations.

In a longer perspective the developed methods could, with the emergence of cold-atom accelerometer that can measure small gravity variations [29], enable gravity-field based odometry and SLAM solutions. This could be a game changer for, among other things, aerospace and marine vessels, as it would enable the development of fully self-contained and completely spoofing (jamming) resistant self-localization solutions.

4 Preliminary and previous results

A proof-of-concept study of the tensor-field based odometry concept have been conducted and shown encouraging results [IS26]. In the study, a miniaturized magnetic-field vector sensor array was moved through an artificially created magnetic-field, and a model-based sensor fusion strategy was used to estimate the displacement of the array. The results were an order of magnitude better than current state-of-the-art methods [8, 9, 10], which a requires high field-variations-to-noise ratio to work reliably. See Fig. 3 for an illustration.

5 Project description

5.1 Theory and method

Theories and methods from statistical signal processing, automatic control, and system identification, along with basic physical models for electromagnetic fields, gravity-fields, etc., will be applied in the research and development of the targeted technologies for tensor-field self-localization. More specifically, recent theories and methods for joint state-inference and system identification in Gaussian process state-space models [21, 23, GH1], sparse and multi-resolution vector-field representations [30, 27, 28], and tensor-field odometry [9, IS26], will be applied.

5.2 Time plan and implementation

The project is planned to start in January 2021 and end in December 2024, and is implemented in terms of the three work packages (WP) listed below. An indicative time plan for the project is shown in Fig 4. Parallel to the formal WPs of the project, a fourth WP, funded by the PI's other grants, focusing on experimental validation and evaluation of the developed methods, will be implemented.

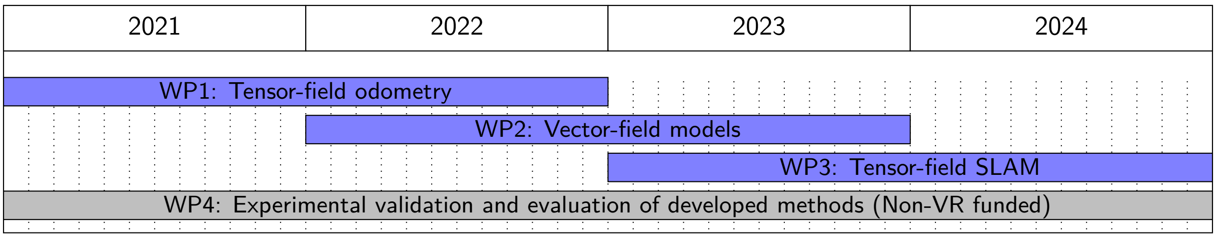

Fig. 4. Indicative time line for the project and the WPs. WP4 is funded via the PI's other grants.

WP1: Tensor-field based odometry

Goal: To develop methods for tensor-field based odometry aided inertial navigation.

Description: Tensor-field based odometry aided inertial navigation can reduce the inertial navigation error drift rate from cubic to linear in time. Current state-of-the-art techniques for tensor-field based odometry aided inertial navigation only work in regions with a high field-variations-to-noise ratio. Recently introduced model based approach to tensor-field based odometry has been shown also to works in regions with a low field-variations-to-noise ratio. However, as of today, no sensor-fusion methods for model-based tensor-field odometry aided inertial navigation exists.

Approach: In a model-based approach to tensor-field based odometry aided inertial navigation the observation function, which relates the tensor-field observations to the system dynamics, constitutes an unknown model of the local vector-field. Therefore, focus will be on developing techniques for real-time joint state-inference and fast learning of local and short lived observation models in dynamic systems, will be developed. As a starting point for the development of these techniques, recent methods for joint state-inference and system identification in dynamic systems [GH1, GH2, 23], encoding of physical properties into vector-field regression models [5, IS26, 24], and multipole modelling of vector-fields [25, 26], will be used.

Challenge: Compared to the typical systems identification setup, the model to be identified is local and only of interest over a short time window when solving the odometry problem, and no long term use of it exist. Thus, the developed state inference and learning methods must have high, close to momentary, adaptability, and model order selection and hyperparameter tuning must be done online.

WP2: Vector-field models

Goal: To develop multi-resolution vector-field modeling and regression techniques, where the models fulfills all the physical laws of the underlying field and that can be used for joint tensor-field based odometry and SLAM.

Description: Current sparse Gaussian process vector-field representation methods only incorporate parts of the physical properties of the field into the model [5]; creates mismatches in the boundary between model (map) segments [20]; and has limited micro-scale resolution needed to support tensor-field odometry.

Approach: Two approaches to develop models with the desired properties will be explored. The first approach is to merge recent developed multi-resolution Gaussian process models [27, 28] with methods for encoding of physical properties into Gaussian process [5]. The second approach is to approximating the observed vector-field using a series of, by physical laws, motivated basis functions, such as dipole models or gravity point masses models. This will enable the creation of physically interpretable vector-field models that fulfills all the physical laws of the underlying field and that can be naturally decomposed into microscopic and macroscopic scale field variations. To establish a deeper insight to the use of physical basis functions, the relationship between the basis function expansion and the choice of kernel function in a Gaussian process will be sought after [30]. Further, the recent presented results in [31], which shows that parts of the Jacobian of common vector-field are rotational invariant, will be explored to find alternative vector-field representations that are robust to uncertainties in attitude of the localization system.

Challenge: To create vector-field models that has sufficient resolution to support odometry and description of unique map-features, and at the same time are sparse enough to support large-scale mapping.

WP3: Tensor-field SLAM

Goal: To develop sensor-fusion and model-learning methods for joint tensor-field based odometry and SLAM.

Description: Current vector-field SLAM solutions can be directly extended to handle tensor observation. However, to fully exploit the capabilities enabled by joint tensor-field based odometry and SLAM, sensor-fusion strategies that allows joint state interference and learning of the field variations on both a microscopic and macroscopic scale, must be developed; microscopic scale learning for odometry and macroscopic scale learning for mapping.

Approach: The sensor-fusion techniques and vector-field models developed in WP1 and WP2, will be integrated into current vector-field SLAM frameworks [20]. To enable this, focus will be on modifying and extending recent methods for joint state-inference and system identification in dynamic systems [GH1, GH2, 23], to also work with multi-resolution Gaussian process models [27, 28].

Challenge: To create sensor fusion methods that first learn the micro scale variations to support tensor-field odometry, and then decompose and sparsify (resample) these to a macro-scale resolution optimized for large-scale SLAM and associated memory constraints.

5.3 Project organisation

The project will be lead by the PI and staffed by a PhD student, the PI, and the co-PI. The core part of the research will be conducted by the PhD student, which has been assigned a 100% activity level. This is motivate by that the nominal duration of the doctoral education being 4 years at a 100% activity level. The PI has been assigned a 10% activity level, to be used for research supervision and active participation in the research. The PI will also devote an additional 10% of his time to the experimental activities in WP4; activities funded by the PI's Security Link (Vinnova) grant. The total activity level of 20% is reasonable considering the PI's current teaching and research undertakings. Co-PI, G. Hendeby is assigned a 5% activity level, with the primary focus of being an expert advisor in question regarding sensor fusion strategies; he will also act as a co-supervisor for the PhD student.

6 Equipment

For validation and evaluation of the developed methods an in-house developed magnetic-field vector sensor array, adapted for indoor localizations, is available. As a gold-standard for reference data collection, a camera based tracking system from Qualisys is available. Further, via the PI's collaboration with the Swedish Defence Research Agency (FOI) opportunities exists for collecting experimental data from electromagnetic arrays onboard, or towed behind, marine vessels.

7 Need for research infrastructure

No need for additional research infrastructure exist.

8 International and national collaboration

The PI has a large national and international network and frequently collaborates with external researchers and industrial partners, see Table 1. For this project, the most significant collaboration partners are: Dr. Manon Kok (TU Delft) on Gaussian-process based magnetic-field SLAM; Dr. Johan Wahlström (Oxford University) on sensor-fusion for inertial sensor arrays [IS2]; and Prof. J. Jaldén (KTH) on the WASP project "Localization using large arrays of inertial sensors" [IS32].

9 Other applications or grants

The PI does not have, and has not applied for, any other research grants for the proposed research activities. However, the PI has received 1 MSEK in research funding for experimental activities and equipment within the area of magnetic-field positioning from the Vinnova funded Security-Link

Table 1: Summary of the PI's collaborations between 2012 and 2020 in terms of organizations and publications.

Organization	Joint Publications
Academia	
Oxford University, UK	[IS2, IS1]
Electrical Engineering Department, University of North Dakota, USA	[IS10]
MENRVA Research Group, Simon Fraser University, USA	[IS10]
Instituto de Telecomunicac, Universidade do Porto, Portugal	[IS9]
Indian Institute of Science, India	[IS19, IS46]
Universit degli Studi di Perugia, Italy	[IS3]
Consejo Superior de Investigaciones Cientificas, Spain	[IS43]
Washington University in St. Louis, USA	[IS12, IS11, IS10, IS5]
Stockholm University, SU	[IS14, IS15, IS42]
Dept. Automatic Control, KTH	[IS34]
Industry	
Ericsson Research, Sweden	[IS24]
Cambridge Mobile Telematics, USA	[IS1]
Recco AB, Sweden	[IS22]
FOI Swedish Defence Research Agency, Sweden	[IS19, IS39, IS23]
GT Silicon, India	[IS33]
REW Insurance Consulting Services, USA	[IS14]
Movelo AB, Sweden	[IS14, IS15, IS52, IS53, IS54, IS55, IS56, IS57]
OHB Sweden AB, Sweden	[IS34]
SafeLine AB, Sweden	[IS8]

program. Moreover, the PI has received 2.6 MSEK from CENIIT for the related, but non-overlapping, research project "Complex Acoustic Surveillance and Tracking", in which sensor-fusion methods for target tracking and localization using acoustic fields are studied.

10 Independent line of research

The proposed project will be carried out at the Div. of Automatic Control, LiU, which is internationally recognized for its excellence in sensor fusion, system identification, and control theory. The department have several related, but non-overlapping, research projects within the area of sensor fusion, funded by the Swedish Research Council, Vinnova, ELLIIT, etc. The proposed project is unique in its focus on sensor-fusion and machine learning methods for tensor-field observation from arrays of vector sensors. Still, positive cross fertilization effects with related projects are foreseen.

The PI was recruited as part of a division strategy to strengthen the experimental activities and broaden the research in the area of sensor fusion. The proposed research is an important part of this strategy and an opportunity for the PI to employ a new PhD student and thereby fortify his own research direction within the division. The division offers several PhD courses on topics related to the proposed research; courses on target tracking, sensor fusion, and adaptive control are given on a regular basis by, among others, the co-PI Gustaf Hendeby. Thus, the division constitutes a stimulating and inspiring environment for training PhD students to become independent researchers producing high-quality research.

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