Abstract – Shooter localization is considered in a wireless network of microphones. Both the acoustic muzzle blast (MB) from the gunfire and the ballistic shock wave (SW) from the bullet can be detected by the microphones and are considered as measurements. The MB measurements give rise to a standard sensor network problem, similar to time-difference of arrivals in cellular phone networks, and the localization accuracy is good provided that the sensors are well synchronized compared to the MB detection accuracy. The detection times of the SW depend on both shooter position and aiming angle, and we demonstrate that estimation based on these measurements can potentially also reveal the shooting direction beside the position, but again this requires good synchronization. We propose to base the estimation on the time difference of MB and SW at each sensor, which becomes insensitive to synchronization. Cramér-Rao lower bound analysis indicates how a lower bound of the root mean square error depends on the synchronization error for MB and MB-SW differences, respectively. Results from field trials with different type of ammunition show excellent accuracy of the proposed method for both the position and the aiming angle of the shooter.

Keywords: Localization, Sensor Networks, Acoustic, Synchronization

1 Introduction

Several acoustic shooter localization systems are today commercially available, see for instance [1–3]. Typically, one or more microphone arrays are used, each synchronously sampling acoustic phenomena associated with gunfire. An overview is found in [4]. Some of these systems are mobile, and in [5] it is even described how soldiers can carry the microphone arrays on their helmets.

Indeed, less common are shooter localization systems based on singleton microphones geographically distributed in a wireless sensor network. An obvious issue in wireless networks is the sensor synchronization. For localization algorithms that rely on accurate timing like ones based on time difference of arrival (TDOA), it is of major importance that synchronization errors are carefully controlled. Regardless if the synchronization is solved by using GPS or other techniques, see for instance [6–8], the synchronization procedures are associated with costs in battery life or communication resources that usually must be kept at a minimum.

In [9] the synchronization error impact on the localization ability of an urban network is studied by using Monte Carlo simulations. One of the results was that the inaccuracy increased significantly (> 2 m) for synchronization errors exceeding approximately 4 ms. 56 small wireless sensor nodes were modeled.

In this paper we derive fundamental estimation bounds for shooter localization systems based on wireless sensor networks, with the synchronization errors in focus. An accurate method independent of the synchronization errors will be proposed (the MB-SW model), as well as a useful bullet deceleration model. The algorithms are tested on data from a field trial with 10 microphones spread over an area of 100 m and with gunfire at distances up to 400 m. Earlier in [10, 11] we contributed with algorithms and bounds for sensor network based localization using energy measurements.

The outline is as follows. Section 2 sketches the localization principle and describes the acoustical phenomena that are used. Section 3 gives the estimation framework. Section 4 derives the signal models for the muzzle blast (MB), shock wave (SW) and MB-SW, respectively. Section 5 derives expressions for the root mean square error (RMSE) Cramér-Rao lower bound (CRLB) for MB and MB-SW, respectively, and provides numerical results from a realistic scenario. Section 6 presents the results from field trials, and Section 7 the conclusions.
2 Localization Principle

Two acoustical phenomena associated with gunfire will be exploited to determine the shooter’s position: the 
muzzle blast and the shock wave. The principle is to detect and time stamp the phenomena as they reach microphones distributed over an area, and let the shooter’s position be estimated by, in a sense, the most likely point, considering the microphone locations and detection times.

The muzzle blast (MB) is the sound that probably most of us associate with a gun shot; the “bang”. The MB is generated by the pressure depletion in effect of the bullet leaving the gun barrel. The sound of the MB travels at the speed of sound in all directions from the shooter. Provided that a sufficient number of microphones detect the MB, the shooters position can be more or less accurately determined.

The shock wave (SW) is formed by supersonic bullets. The SW has (approximately) the shape of a cone, with the bullet trajectory as axis, and reaches only microphones that happens to be located in the cone. The SW propagates at the speed of sound in direction away from the bullet trajectory, but since it is generated by a supersonic bullet, it always reaches the microphone before the MB, if it reaches the microphone at all. As with the MB, a sufficient number of SW detections may reveal the shooter’s position. The SW detection is however more difficult to utilize, since it depends on the bullet’s speed and ballistic behavior.

Figure 1 shows an acoustic taking of gunfire. The first pulse is the SW, which for distant shooters significantly dominates the MB, not the least if the bullet passes close to the microphone. The figure shows real data, but a rather ideal case. Usually, there are reflections and other acoustic effects that make it difficult to accurately determine the MB and SW times. This issue will however not be treated in this work. We will instead assume that the detection error is stochastic with a certain distribution.

Of course, the MB and SW (when present) can be used in conjunction with each other. One of the ideas described and tested later is to utilize the time difference between the MB and SW detections. This way, the localization is independent of the clock synchronization errors that are always present in wireless sensor networks.

3 Estimation Framework

Assume there are $M$ microphones with known positions $\{p_k\}_{k=1}^M$ in the network detecting the muzzle blast. Without loss of generality, the first $S \leq M$ ones also detect the shock wave. The detected times are denoted $\{y_k^{MB}\}_{1}^M$, and $\{y_k^{SW}\}_{1}^S$, respectively. Each detected time is subject to a detection error $\{e_k^{MB}\}_{1}^M$ and $\{e_k^{SW}\}_{1}^S$, different for all times, and a clock synchronization errors $\{b_k\}_{1}^M$ specific for each microphone.

where $y$ is a vector with the measured detection times, $\varphi$ is a nonlinear function with values in $\mathbb{R}^{1 \times (M+S)}$, and where $\theta$ represents the unknown parameters apart from $x$. The error $e$ is assumed to be stochastic, see Section 4.4. Given the sensor locations in $p \in \mathbb{R}^{M \times 3}$, nonlinear optimization can be performed to estimate $x$, using the nonlinear least squares (NLS) criterion

$$ \hat{x} = \arg \min_{x, \theta} V(x, \theta; p), $$

$$ V(x, \theta; p) = \|y - \varphi^T(x, \theta; p)\|_Q^2. $$

$$ ||v||_Q^2 $$ denotes the Q-norm, that is, $||v||_Q^2 \triangleq v^TQ^{-1}v$. Whenever Q is omitted, $Q = I$ is assumed. The loss function norm $R$ is chosen by consideration of the expected error characteristics. Numerical optimization, for instance the Gauss-Newton method, can here be applied to get the NLS estimate.

In the next section it will become clear that the assumed unknown firing time and the inverse speed of sound enter the model equations linearly. To exploit this fact we identify a sub-linear structure in the signal model and apply the weighted least squares method to the parameters appearing linearly (the separable least squares method). By doing so, the NLS search space is
Table 1: Notation. MB, SW, and MB–SW are different models, and L/N indicates if model parameters or signals enter the model linearly (L) or non-linearly (N).

<table>
<thead>
<tr>
<th>Variable</th>
<th>MB</th>
<th>SW</th>
<th>MB–SW</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td>Number of microphones</td>
</tr>
<tr>
<td>S</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Number of microphones receiving shock wave</td>
</tr>
<tr>
<td>x</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Position of shooter</td>
</tr>
<tr>
<td>p_k</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Position of microphone k</td>
</tr>
<tr>
<td>y_k</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>Measured detection time for microphone at position p_k</td>
</tr>
<tr>
<td>t_0</td>
<td>L</td>
<td>L</td>
<td></td>
<td>Rifle or gun firing time</td>
</tr>
<tr>
<td>c</td>
<td>L</td>
<td>N</td>
<td>N</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>v</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Speed of bullet</td>
</tr>
<tr>
<td>α</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Shooting direction</td>
</tr>
<tr>
<td>b_k</td>
<td>L</td>
<td>L</td>
<td></td>
<td>Clock error for microphone k</td>
</tr>
<tr>
<td>e_k</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>Detection error at microphone k</td>
</tr>
<tr>
<td>r</td>
<td>N</td>
<td>N</td>
<td></td>
<td>Point of origin for shock wave received by microphone k</td>
</tr>
<tr>
<td>β</td>
<td></td>
<td></td>
<td></td>
<td>Mach angle, ( \sin \beta = c/v )</td>
</tr>
</tbody>
</table>

reduced which in turn significantly reduces the computational burden. For that reason, the signal model (1) is rewritten as

\[
y = \varphi_N^T(x, \theta_N; p) + \varphi_L^T(x, \theta_N; p)\theta_L + e. \tag{3}
\]

The NLS problem can then be formulated as

\[
\hat{x} = \arg \min_{x,d_L,\theta_N} V(x, \theta_N, \theta_L; p),
\]

\[
V(x, \theta_N, \theta_L; p) = ||y - \varphi_N^T(x, \theta_N; p) - \varphi_L^T(x, \theta_N; p)\theta_L||_R^2.
\]

Since \( \theta_L \) enters linearly it can be solved for by linear least squares (suppressing the arguments of \( \varphi_L(x, \theta_N; p) \) and \( \varphi_N(x, \theta_N; p) \))

\[
\hat{\theta}_L = \arg \min_{\theta_L} V(x, \theta_N, \theta_L; p)
\]

\[
= (\varphi_L R^{-1} \varphi_N^T)^{-1} \varphi_L R^{-1} (y - \varphi_N^T). \tag{4}
\]

This simplifies the nonlinear minimization to

\[
\hat{x} = \arg \min_{x} V(x, \theta_N, \hat{\theta}_L; p)
\]

\[
= \arg \min_{x, \theta_N} ||y - \varphi_N^T + \varphi_L^T (\varphi_L \varphi_L^T)^{-1} \varphi_L (y - \varphi_N^T)||_R^2. \tag{5}
\]

4 Signal Models

4.1 Muzzle Blast model

According to the clock at microphone k, the muzzle blast (MB) sound is assumed to reach \( p_k \) at the time

\[
y_k = t_0 + b_k + \frac{1}{2} ||p_k - x|| + e_k. \tag{6}
\]

The shooter position \( x \) and microphone location \( p_k \) are in \( \mathbb{R}^n \), where generally \( n = 3 \). However, both computational and numerical issues occasionally motivate a simplified plane model with \( n = 2 \). For all \( M \) microphones, the model is represented on vector form as

\[
y = b + \varphi_L^T (p - 1_M x^T) \theta_L + e, \tag{7}
\]

where

\[
\theta_L = [t_0, 1/c]^T, \tag{8a}
\]

\[
[\varphi_L (p - x^T 1_M)]_{col} k = [1 \ ||p_k - x||]^T, \tag{8b}
\]

and where \( y, b, \) and \( e \) are vectors with elements \( y_k, b_k, \) and \( e_k \), respectively. \( 1_M \) is the vector with \( M \) ones, where \( M \) might be omitted if there is no ambiguity regarding the dimension. Furthermore, \( p \) is \( M \)-by-\( n \), where each row is a microphone position. Note that the inverse of the speed of sound enters linearly.

\( \varphi_L \) is \( 2 \)-by-\( n \). The \( \cdot_L \) notation indicates that \( \cdot \) is part of a linear relation, as described in the previous section. With \( \varphi_N = 0 \) and \( \varphi_L = \varphi_L(x; p) \), the derived least squares criterion is

\[
\hat{x} = \arg \min_{x} -y^T R^{-1} \varphi_L^T (\varphi_L R^{-1} \varphi_L^T)^{-1} \varphi_L R^{-1} y. \tag{9}
\]

This criterion has computationally efficient implementations, that in many applications make the time it takes to do an exhaustive minimization over \( a \), say, 10-meter grid acceptable. The grid-based minimization of course reduces the risk to settle on suboptimal local minimizers, which otherwise could be a risk using greedy search methods. The objective function does, however, behave rather well. Figure 2 visualizes (9) in logarithmic scale for data from a field trial (the norm is \( R = I \)). Apparently, there are only two local minima.

4.2 Shock Wave Model

It is assumed, that the bullet follows a straight line with initial speed \( v_0 \), see Figure 3. Due to air friction, the bullet decelerates, so when the bullet has traveled the distance \( ||d_k - x|| \), for some point \( d_k \) on the trajectory, the speed is reduced to

\[
v = v_0 - r ||d_k - x||, \tag{10}
\]
been reused from the MB model. Below, also \( \phi \) for instance, the curvilinear trajectories proposed by [12], but we propose it for simplicity.

The shock wave from the bullet trajectory propagates at the speed of sound \( c \) with angle \( \beta_k \) to the bullet heading. \( \beta_k \) is the Mach angle defined as

\[
\sin \beta_k = \frac{c}{v} = \frac{c}{v_0 - r||d_k - x||}. \tag{11}
\]

\( d_k \) is now the point where the shock wave piece that reaches microphone \( k \) is generated. The time it takes the bullet to reach \( d_k \) is

\[
\int_0^{||x - d_k||} \frac{d\xi}{v_0 - r \cdot \xi} = \frac{1}{r} \log \frac{v_0}{v_0 - r||d_k - x||}. \tag{12}
\]

This time and the wave propagation time from \( d_k \) to \( p_k \) sum up to the total time from firing to detection,

\[
y_k = t_0 + b_k + \frac{1}{r} \log \frac{v_0}{v_0 - r||d_k - x||} + \frac{1}{r} ||d_k - p_k|| + e_k,
\]

according to the clock at microphone \( k \). Note that the variable names \( y \) and \( e \) for notational simplicity have been reused from the MB model. Below, also \( \varphi, \theta_N, \) and \( \theta_L \) will be reused. When there is ambiguity, a subscript will indicate exactly which entity that is referred to, for instance, \( y^{MB}, \varphi^{SW} \).

It is a little bit tedious to calculate \( d_k \). The law of sines gives

\[
\frac{\sin(90^\circ - \beta_k - \gamma_k)}{||d_k - x||} = \frac{\sin(90^\circ + \beta_k)}{||p_k - x||}, \tag{13}
\]

which together with (11) implicitly defines \( d_k \). We have not found any simple closed form for \( d_k \), so we solve for \( d_k \) numerically. \( \gamma_k \) is trivially induced by the shooting direction \( \alpha \) (and \( x, p_k \)). The vector form is

\[
y = b + \varphi^T_N(x, \theta_N; p) + \varphi^T_L(x, \theta_N; p) \theta_L + e, \tag{14}
\]

where

\[
\varphi_L(x, \theta_N; p) = 1, \tag{15}
\]

\[
\theta_L = t_0, \tag{16}
\]

\[
\theta_N = \left[ \frac{1}{c} \alpha^T \ v_0 \right]^T, \tag{17}
\]

and where the elements of \( \varphi_N(x, \theta_N; p) \in \mathbb{R}^{1 \times S} \) are

\[
\varphi_N(x, \theta_N; p_k) = \frac{1}{r} \log \frac{v_0}{v_0 - r||d_k - x||} + \frac{1}{r} ||d_k - p_k||, \tag{18}
\]

\( d_k \) being the reasonable solution to (11), (13).

### 4.3 MB–SW Model

Motivated by accurate localization despite synchronization errors, we propose the MB–SW model:

\[
y_k = y_k^{MB} - y_k^{SW}
\]

\[
= \varphi_L^{MB,T}(px - x)\theta_L^{MB} - \varphi_N^{MB}(x, \theta_N^{SW}; p_k)
\]

\[
- \varphi_L^{SW}(x, \theta_N; p_k)\theta_N^{SW} + e_k^{MB} - e_k^{SW}, \tag{19}
\]

for \( k = 1, 2 \ldots S \). The key idea is that \( y \) is by cancelation independent of both the firing time \( t_0 \) and the synchronization error \( b \). The drawback, of course, is that there are only \( S \) equations (instead of a total of \( M + S \)) and the detection error increases, \( e_k^{MB} - e_k^{SW} \). However, when the synchronization errors are expected to be significantly larger than the detection errors, and when also \( S \) is sufficiently large (at least as large as the number of parameters), this model is believed to give better localization accuracy. This will be investigated later. The vector form for the MB–SW model is

\[
y = \varphi^T_N(x, \theta_N; p) + e, \tag{20}
\]

where

\[
\varphi_N(x, \theta_N; p) =\]

\[
= \frac{1}{r} ||p_k - x|| - \frac{1}{r} \log \frac{v_0}{v_0 - r||d_k - x||} - \frac{1}{r} ||d_k - p_k||, \tag{21}
\]

\( y = y^{MB} - y^{SW}, e = e^{MB} - e^{SW} \). As before, \( d_k \) is the reasonable solution to (11), (13). The MB–SW least squares criterion is

\[
\hat{e} = \arg\min_{x, \theta_N} ||y - \varphi_N(x, \theta_N; p)||_R^2, \tag{22}
\]
which requires numerical optimization. Numerical experiments indicate that this optimization problem is more prone to local minima, compared to (9) for the MB model, why good starting points for the numerical search are essential. One such starting point could for instance be the MB estimate \( \hat{x}^{\text{MB}} \). Initial shooting direction could be given by assuming, in a sense, the worst possible case; that the shooter aims at some point close to the microphone network.

4.4 Error Model

At an arbitrary moment, the detection errors and clock bias are assumed to be independent stochastic variables with normal distribution,

\[
e^{\text{MB}} \sim \mathcal{N}(0, R^{\text{MB}}),
\]

\[
e^{\text{SW}} \sim \mathcal{N}(0, R^{\text{SW}}),
\]

\[
b \sim \mathcal{N}(0, R^{b}).
\]

For the MB–SW model the error is consequently

\[
e^{\text{MB–SW}} \sim \mathcal{N}(0, R^{\text{MB}} + R^{\text{SW}}).
\]

Note that the (assumed known) error covariance, generally denoted \( R \), dictates the norm used in the weighted least squares criterion. \( R \) also impacts the estimation bounds. This will be discussed now.

5 Cramér-Rao Lower Bound

The accuracy of localization based on the rather general model

\[
y = \varphi(\eta) + e
\]

is, under not too restrictive assumptions, bounded by the Cramér-Rao bound,

\[
\text{Cov} \hat{\eta} \geq I^{-1}(\eta_0).
\]

Here, the location \( x \) is for notational purposes part of the parameter vector \( \eta \). The Cramér-Rao lower bound provides a fundamental estimation limit for unbiased estimators, see [13]. This bound has been analyzed thoroughly in the literature, primarily for AOA, TOA and TDOA, [14–16].

The square matrix \( I \) is the Fisher information matrix, which for \( e \sim \mathcal{N}(0, R) \) takes the form

\[
I(\eta) = \nabla_{\eta} \varphi^T(\eta) R^{-1} \nabla_{\eta} \varphi(\eta).
\]

The bound is evaluated for a specific location, parameter setting, and microphone positioning, collectively \( \eta = \eta_0 \).

5.1 MB Case

For the MB case, the entities in (29) are identified by

\[
\eta = \begin{bmatrix} x^T & \theta_L^T \end{bmatrix}^T,
\]

\[
\varphi(\eta) = \varphi_L(p - 1x^T) \theta_L,
\]

\[
R = R^{\text{MB}} + R^b.
\]

The Jacobian \( \nabla_{\eta} \varphi \) is an \( n + 2 \)-by-\( n + 2 \) matrix, \( n \) being the dimension of \( x \). The bound is

\[
\text{Cov} \hat{x}^{\text{MB}} \geq [I_n \ 0] I^{-1} \begin{bmatrix} I_n \\ 0 \end{bmatrix}.
\]

The LS solution in (4) however gives a shortcut to an \( n \)-by-\( n \) Jacobian,

\[
\nabla_{\eta} \varphi_L \theta_L = \nabla_{\eta} \varphi_L^T(\varphi_L R^{-1} \varphi_L^T)^{-1} \varphi_L R^{-1} y_0
\]

for \( y_0 = \varphi_L^T(p - 1x^T) \theta_L \). At least for \( n = 2 \), this bound can, with some effort, be expressed explicitly.

The equivalent bound is

\[
\text{Cov} \hat{x} \geq [(\nabla_{\eta} \varphi_L^T \theta_L) R^{-1} (\nabla_{\eta} \varphi_L^T \theta_L)]^{-1}.
\]

5.2 MB–SW Case

For the MB–SW case,

\[
\eta = \begin{bmatrix} x^T & \theta_N^T \end{bmatrix}^T,
\]

\[
\varphi(\eta) = \varphi_N(x, \theta_N; p),
\]

\[
R = R^{\text{MB–SW}},
\]

and the estimation bound is analogous to (33) above. The \( 2n + 1 \)-by-\( 2n + 1 \) Jacobian \( \nabla_{\eta} \varphi_N(x, \theta_N; p) \) is probably best evaluated by finite difference methods.

5.3 Numerical Example

We will study a scenario where 14 microphones are deployed in a sensor network to support camp protection, see Figure 4. The microphones are positioned along a road to track vehicles and around the camp site to detect intruders. Of course, the microphones also detect muzzle blasts and shock waves from gunfire, so shooters can be localized and the shooter’s target identified.

A plane model (flat camp site) is assumed, \( x \in \mathbb{R}^2 \), \( \alpha \in \mathbb{R} \). Furthermore, it is assumed that

\[
R^{\text{MB}} + R^b = (\sigma_e^2 + \sigma_b^2) I,
\]

\[
R^{\text{MB–SW}} = \sigma_e^2 I,
\]

and that \( \alpha = 0, \ c = 330 \text{ m/s}, \ v_0 = 700 \text{ m/s}, \ r = 0.63 \). The scenario setup implies that all microphones detect the shock wave, \( S = M \).

**MB model** The localization accuracy using the MB model is bounded below according to

\[
\text{Cov} \hat{x}^{\text{MB}} \geq (\sigma_e^2 + \sigma_b^2) \begin{bmatrix} 64 & -17 \\ -17 & 9 \end{bmatrix} \cdot 10^4.
\]

The root mean square error (RMSE) is consequently bounded according to

\[
\text{RMSE} (\hat{x}^{\text{MB}}) \geq \sqrt{\text{tr} \text{Cov} \hat{x}^{\text{MB}}} \approx 858 \sqrt{\sigma_e^2 + \sigma_b^2} \quad [\text{m}].
\]

Monte Carlo simulations (not described here) indicate that the NLS estimator attains this lower bound for \( \sqrt{\sigma_e^2 + \sigma_b^2} < 0.1 \text{ s} \).
Figure 4: Example scenario. A network with 14 sensors deployed for camp protection. The sensors detect intruders, keep track on vehicle movements, and, of course, locate shooters.

Figure 5: Scene of the shooter localization field trial. There are 10 microphones, 3 shooter positions, and a common target.

Figure 6: Estimated positions \( \hat{x} \) based on the MB model and on the MB–SW model. The diagrams are enlargements of the interesting areas around the shooter positions. The dashed lines identify the shooting directions.

**MB–SW Model**  The localization accuracy using the MB–SW model is bounded according to

\[
\text{Cov} \hat{x}_{\text{MB–SW}}^2 \geq \sigma_e^2 \begin{bmatrix} 28 & 5 \\ 5 & 12 \end{bmatrix} \cdot 10^5, \quad (41)
\]

\[
\text{RMSE} (\hat{x}_{\text{MB–SW}}) \geq 2000\sigma_e \text{ [m]}.
\]

**Discussion**  In this example, the MB–SW model gives at least twice the error of the MB model, provided that there are no synchronization errors. However, in a wireless network we expect the synchronization error to be 10-100 times larger than the detection error, and then the MB–SW error will be substantially smaller than the MB error.

**6 Experimental Data**

A field trial to collect acoustic data on non-military small arms fire is conducted. 10 microphones are placed around a fictitious camp, see Figure 5. The microphones are placed close to the ground and wired to a common recorder with 16-bit sampling at 48 kHz. A total of 42 rounds are fired from three positions and aimed at a common cardboard target. Three rifle and one pistol ammunition are used, see Table 2. Four rounds are fired of each ammunition type at each shooter position, with two exceptions. The pistol is only used at position three. At position three, six instead of four rounds of 308W are fired. All ammunition types are supersonic. However, when firing from position three, not all microphones are subjected to the shock wave.

Light wind, no clouds, around 24 °C are the weather conditions. Little or no acoustic disturbances are present. The terrain is rough. Dense woods surround the test site. There is light bush vegetation within the site. Shooter position 1 is elevated some 20 m, otherwise spots are within ±5 m of a horizontal plane. Positions are determined with less relative error than 1 m, except for shooter position 1, which is determined with 10 m accuracy.
6.1 Detection
The muzzle blast and shock wave reception times are determined by visual inspection of the microphone signals in conjunction with filtering techniques. For shooter position 1 and 2, the shock wave detection accuracy is approximately $\sigma_{SW} \approx 80 \mu s$, and the muzzle blast error $\sigma_{MB}$ is slightly worse. For shooting position 3 the accuracies are generally worse, since the muzzle blast and shock wave components become intermixed in time.

6.2 Numerical Setup
For simplicity, a plane model is assumed. All elevation measurements are ignored and $x \in \mathbb{R}^2$ and $\alpha \in \mathbb{R}$. Localization using the MB model (6) is done by minimizing (9) over a 10 m grid well covering the area of interest, followed by numerical minimization.

Localization using the MB–SW model (19) is done by numerically minimizing (22). The objective function is subject to local optima, why the more robust muzzle blast localization $\hat{x}$ is used as an initial guess. Furthermore, the direction from $\hat{x}$ toward the mean point of the microphones (“the camp”) is used as initial shooting direction $\alpha$. Initial bullet speed is $v = 800 \text{ m/s}$ and initial speed of sound is $c = 330 \text{ m/s}$. $r = 0.63$ is used, which is a value derived from the 308 Winchester ammunition ballistics.

6.3 Results
Figure 6 shows, at three enlarged parts of the scene, the resulting position estimates based on the MB model (blue crosses) and based on the MB–SW (red squares). Apparently, the use of the shock wave significantly improves localization at position 1 and 2, while rather the opposite holds at position 3. Figure 7 visualizes the shooting direction estimates, $\alpha$. Estimate root mean square errors (RMSE) for the three shooter positions are given in Table 3. These statistics indicate that the use of the shock wave from distant shooters cut the error by at least 75%.

Since all microphones are recorded by a common recorder, there are actually no timing errors due to inaccurate clocks. This is of course the best way to conduct a controlled experiment, where any uncertainty renders the dataset less useful. It is simple to add clock errors of any desired magnitude off-line. On the dataset at hand, this is however work under progress. At the moment, there are apparently other sources of error, worth identifying.

There are at least two explanations for the bad performance using the MB–SW model at shooter position 3. One is, that the number of microphones reached by the shock wave is insufficient to make accurate estimates. There are four unknown model parameters, but for the relatively low speed of pistol ammunition, for instance, only one microphone has a valid shock wave detection. Another explanation is, that the increased detection uncertainty (due to SW/MB intermix) impacts the difference model harder. This is illustrated by the numerical example in Section 5.3, where $\text{RMSE}(\hat{x}^{MB}) \geq 858 \sqrt{\sigma_{c}^2 + \sigma_{b}^2}$, but $\text{RMSE}(\hat{x}^{MB-SW}) \geq 2000\sigma_{c}$.

Most likely, there are model inaccuracies both in the ballistic and in the acoustic domain. To that end, there are meteorological uncertainties out of our control. For instance, looking at the MB–SW localizations around shooter position 1 in Figure 6 (red squares), three clusters are identified that correspond to three ammunition types with different ballistic properties. This clustering indicates that the ballistic model is not perfect. Working with three-dimensional data in the plane is of course another model discrepancy, that could have greater impact than we first anticipated. This will be investigated in experiments to come.

Finally, we face numerical uncertainties. There is no guarantee that the numerical minimization programs we have used for the MB–SW model really deliver the global minimum.

Table 2: Ammunition used at the trial. The table accounts for how many rounds that are fired from the different positions. Also, the resulting localization RMSE for the MB–SW model is noted. However, for the Luger Pistol the MB model RMSE is given instead, since only one microphone is located in the SW cone.

<table>
<thead>
<tr>
<th>Type</th>
<th>Caliber</th>
<th>Weight</th>
<th>Velocity</th>
<th>Positions</th>
<th>#Rounds</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>308 Winchester</td>
<td>7.62 mm</td>
<td>9.55 g</td>
<td>847 m/s</td>
<td>1,2,3</td>
<td>4,4,6</td>
<td>11 m</td>
</tr>
<tr>
<td>Heavy Bullet</td>
<td>9.3 mm</td>
<td>15 g</td>
<td>767 m/s</td>
<td>1,2,3</td>
<td>4,4,4</td>
<td>23 m</td>
</tr>
<tr>
<td>Swedish Mauser</td>
<td>6.5 mm</td>
<td>8.42 g</td>
<td>852 m/s</td>
<td>1,2,3</td>
<td>4,4,4</td>
<td>5.6 m</td>
</tr>
<tr>
<td>Luger Pistol</td>
<td>9 mm</td>
<td>6.8 g</td>
<td>400 m/s</td>
<td>3</td>
<td>-,-,-</td>
<td>1.4 m</td>
</tr>
</tbody>
</table>

Table 3: Localization and aim RMSE for the three different shooter positions using the MB and the MB–SW models. The aim error, given for the MB–SW estimates only, is with respect to the aim at $\hat{x}$ against the target, not with respect to the true direction $\alpha$. This way the ability to identify the target is assessed.

<table>
<thead>
<tr>
<th>Sh. pos.</th>
<th>$\hat{x}^{MB}$</th>
<th>$\hat{x}^{D}$</th>
<th>Aim</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105 m</td>
<td>28 m</td>
<td>0.041°</td>
</tr>
<tr>
<td>2</td>
<td>28 m</td>
<td>5.7 m</td>
<td>0.14°</td>
</tr>
<tr>
<td>3</td>
<td>5.2 m</td>
<td>17°</td>
<td></td>
</tr>
</tbody>
</table>
7 Conclusions

We have presented a framework for estimation of shooter location and aiming angle from wireless networks where each node has a single microphone. Both the acoustic muzzle blast (MB) and the ballistic shock wave (SW) contain useful information about the position, but only SW for the aiming angle. A separable nonlinear least squares (SNLS) framework was proposed to limit the parametric search space and to enable the use for global grid-based optimization algorithms (for the MB model), eliminating potential problems with local minima.

For a perfectly synchronized network, both MB and SW measurements should be stacked into one large signal model for which SNLS is applied. However, when the synchronization error in the network becomes comparable to the detection error for MB and SW, the performance quickly deteriorates. For that reason, we propose to use the time difference of MB and SW at each microphone. Then the clock offset is automatically eliminated. The effective number of measurements decreases in this approach, but as the CRLB analysis showed, the root mean square position error decreases significantly in practical networks.

The bullet speed occurs as nuisance parameters in the proposed signal model. Further, the bullet retardation constant was optimized manually. Future work will investigate if the retardation constant should also be estimated, and if these two parameters can be used, together with the MB and SW signal forms, to identify the weapon and ammunition.

References


