Recursive Bayesian Estimation– Bearings-only Applications

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***Abstract*— In this paper Bayesian recursive estimation methods are applied to several bearings-only applications. Both Air-to-Air passive ranging as well as terrain induced constraints for Air-to-Sea applications are discussed. The bearings-only problem is tested on experimental data from a torpedo, i.e., Sea-to-Sea with a passive sonar sensor. The Bayesian estimation problem is solved using both the particle filter method and the marginalized particle filter. As a comparison the range parameterized extended Kalman filter (RPEKF) is used. In a simulation study the particle filter outperforms the RPEKF.**

I. INTRODUCTION

Target tracking using angle measurements in azimuth and elevation is a common technique for many applications using radar, sonar or infra red (IR) sensor information. Typically, radar and sonar sensors are used in an active mode, transmitting energy. In this mode range and possibly range rate are available from the sensor. To avoid the risk of being detected by a hostile target, the sensors are used in a passive mode. Hence, only angle measurements from target induced energy is available. There are also true passive sensors, such as the IR sensor.

The main idea in passive ranging is to use angle information only to estimate the unknown relative range. By using the platform (aircraft, missile, torpedo etc.) and perform certain maneuvers, it is possible to gain observability in the range direction. Often it is difficult to perform maneuvers in an optimal manner. Therefore, the approach is to perform the maneuvering sequence in a deterministic way, exciting the system sufficiently to gain range observability. The issue of optimal maneuvers is not discussed here, but can be found in [26, 27].

Passive ranging applications have been an important research area for several years. The classical method is to use a single *extended Kalman filter* (EKF). A common problem is that a single linearized filter may easily diverge. There exist several approaches to estimate the range using a single tracking filter. As described in [30, 32, 33] a modified spherical/polar coordinate system is preferred instead of using a traditional Cartesian system. In [1] this is studied and it is investigated how to reduce filter divergence problems by selecting the coordinate system. Another approach is to use multiple filters where each filter is parameterized to a sub-interval. By using a bank of filters, cf. [29], the performance is enhanced by introducing the *range parameterized extended Kalman filter* (RPEKF), as described in Section II-B. By using sequential Monte Carlo methods, or particle filters, [10], a single non-linear filter can be used. The unknown range uncertainty is easier addressed and constraints due to terrain or limitations in the system can be incorporated in a natural way.

In Section II the recursive Bayesian estimation problem is formulated together with several methods and algorithms. The EKF and the RPEKF represent the linearized solution assuming Gaussian noise. The *particle filter* (PF) and the marginalized version thereof (MPF) is a sample approximation of the general problem. In Section III a common target tracking model for bearings-only is described. Section IV consists of three bearings-only applications, Air-to-Air, Air-to-Sea and Sea-to-Sea. Both simulations and experimental data are used.

II. BAYESIAN ESTIMATION

Consider the discrete state-space model

$$x_{t+1} = f(x_t, u_t) + G_t w_t, \quad (1a)$$
$$y_t = h(x_t) + e_t, \quad (1b)$$

with state variables $x_t \in \mathbb{R}^n$, input signal u_t and measurements $\mathbb{Y}_t = \{y_i\}_{i=1}^t$, with known probability density functions (pdfs) for the process noise, $p_w(w)$, and measurement noise $p_e(e)$. Here only additive noise is considered, but a more general description is possible. The non-linear prediction density $p(x_{t+1}|\mathbb{Y}_t)$ and filtering density $p(x_t|\mathbb{Y}_t)$ for the Bayesian inference, [17], is given by

$$p(x_{t+1}|\mathbb{Y}_t) = \int_{\mathbb{R}^n} p(x_{t+1}|x_t)p(x_t|\mathbb{Y}_t)\, dx_t, \quad (2a)$$
$$p(x_t|\mathbb{Y}_t) = \frac{p(y_t|x_t)p(x_t|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})}. \quad (2b)$$

These equations are in general not analytically solvable. However, for the important special case of linear-Gaussian dynamics and linear-Gaussian observations the Kalman filter, [19], provides a finite dimensional solution. For a general non-linear or non-Gaussian system, approximate methods must be used. Here we will consider two different approaches of solving the Bayesian equations, EKF and PF. Also a bank of EKFs, for the RPEKF method, are discussed.

A. The Extended Kalman Filter (EKF)

For many non-linear problems the noise assumptions and the nonlinearity are such that a linearized solution assuming Gaussian noise will be a good approximation. This is the idea behind the EKF, [2], where the model is linearized around the

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previous estimate. Here we only briefly present the time- and measurement update for the EKF,

$$\begin{cases} \hat{x}_{t+1|t} = f(\hat{x}_{t|t}, u_t), \\ P_{t+1|t} = F_t P_{t|t} F_t^T + G_t Q_t G_t^T, \end{cases} \tag{3a}$$

$$\begin{cases} \hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(y_t - h(\hat{x}_{t|t-1})), \\ P_{t|t} = P_{t|t-1} - K_t H_t P_{t|t-1}, \\ K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1}, \end{cases} \tag{3b}$$

where we use the linearized matrices

$$F_t = \nabla_x f(x_t)|_{x_t = \hat{x}_{t|t-1}},\ H_t = \nabla_x h(x_t)|_{x_t = \hat{x}_{t|t-1}}. \tag{4}$$

The noise covariances are given as

$$Q_t = \mathrm{Cov}\{w_t\},\ R_t = \mathrm{Cov}\{e_t\}. \tag{5}$$

B. Range Parameterized Extended Kalman Filters

The approach reviewed in this section is a multiple model to estimate the unknown range and velocity. This is a general estimation problem, which for instance can be handled by a Gaussian sum filter, [2]. Here, we focus on the passive ranging problem and use a special method called the *range parameterized extended Kalman filter* (RPEKF) which consists of a bank of extended Kalman filters, each tuned to a certain range. The presentation follows the development in [21]. The RPEKF method described in [4, 23] consists of a bank of extended Kalman filters in Cartesian coordinates, initialized to different range assumptions for the angle-only tracking application. In [29] the filter bank is expressed in modified polar coordinates.

From Fig. 1 we define the range gates for $i = 1, \ldots, N_F$ different range assumptions (filters). For a predefined interval $(r_{\min}, r_{\max})$, the filter sub-intervals are given by

$$r^{(i)} = \frac{r_{\min}}{2}(\rho^i + \rho^{i-1}), \tag{6}$$

$$\rho = \left(\frac{r_{\max}}{r_{\min}}\right)^{1/N_F}. \tag{7}$$

The *coefficient of variation* C_R defines the variance for each sub-interval,

$$C_R = \frac{\sigma^{(i)}}{r^{(i)}} = \frac{2(\rho - 1)}{\sqrt{12}(\rho + 1)}, \tag{8}$$

where $r^{(i)}$ and $\sigma^{(i)}$ are the range and standard deviation for the different filters. Therefore, the variance for each sub-interval is given as $\sigma^{(i)} = r^{(i)} C_R$, where C_R is defined in (8).

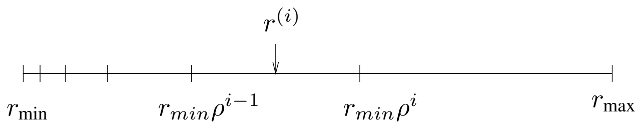

Fig. 1. RPEKF range intervals.

The RPEKF uses the likelihood from each EKF, $p(y_t|i)$, to recursively update its probability according to Bayes' rule

$$w_t^{(i)} = p(i|\mathbb{Y}_t) \propto p(y_t|i) p(i|\mathbb{Y}_{t-1}), \tag{9}$$

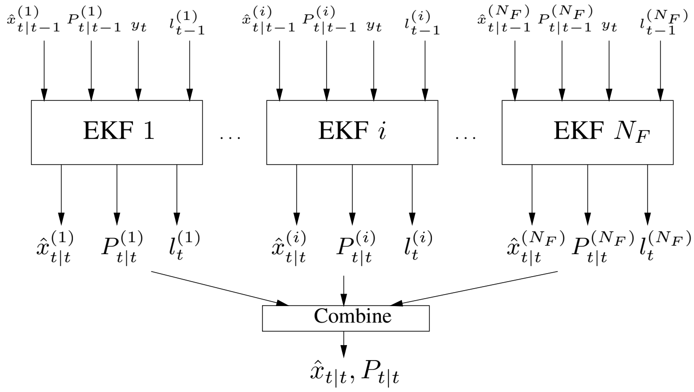

Fig. 2. The structure of the range parameterized EKF.

where y_t is the measurement at time t and $\mathbb{Y}_t = \{y_i\}_{i=1}^t$ the set of all measurements up to current time. The prior distribution is assumed uniform, i.e., $w_t^{(i)} = \frac{1}{N_F}$, $i = 1, \ldots, N_F$. However, if other information is available it could be used to enhance the performance. Under a Gaussian assumption, the likelihood is given from the EKF for the Cartesian case as

$$l_t^{(i)} = p(y_t|i) \propto \frac{1}{\sqrt{|S_t^{(i)}|}} e^{-\frac{1}{2}(y_t - h(\hat{x}^{(i)}_{t|t-1}))^T S_t^{(i)^{-1}} (y_t - h(\hat{x}^{(i)}_{t|t-1}))} \tag{10}$$

$$S_t^{(i)} = H_t^{(i)} P^{(i)}_{t|t-1} (H_t^{(i)})^T + R_t, \tag{11}$$

where R_t is the measurement noise covariance matrix and $H_t^{(i)} = \nabla_x h(x)|_{x = \hat{x}^{(i)}_{t|t-1}}$. The measurement update for each filter is given by the Kalman filter. The combined estimate and covariance can now be expressed as

$$\hat{x}_{t|t} = \sum_{i=1}^{N_F} w_t^{(i)} \hat{x}^{(i)}_{t|t}, \tag{12}$$

$$P_{t|t} = \sum_{i=1}^{N_F} w_t^{(i)} [P^{(i)}_{t|t} + (\hat{x}^{(i)}_{t|t} - \hat{x}_{t|t})(\hat{x}^{(i)}_{t|t} - \hat{x}_{t|t})^T], \tag{13}$$

where $P^{(i)}_{t|t}$ is the covariance and $\hat{x}^{(i)}_{t|t}$ the estimate for the different range filter $i = 1, \ldots, N_F$ and $w_t^{(i)}$ is the normalized likelihood value, $w_t^{(i)} = l_t^{(i)} / \sum_j l_t^{(j)}$. In Fig. 2, the RPEKF idea is summarized. If the filter probability is below a predefined threshold or if some other criterion, such as if the estimated range in a filter is outside the $(r_{\min}, r_{\max})$ interval, the filter is removed from further calculations.

C. The Particle filter (PF)

In this section the presentation of the particle filter theory is according to [6, 10, 13, 20]. The particle filter provides an approximative solution to the discrete time Bayesian estimation problem formulated in (2) by updating an approximative description of the posterior filtering density. Let x_t denote the state of the observed system and $\mathbb{Y}_t = \{y^{(i)}\}_{i=1}^t$ be the set of observed measurements until present time. The particle filter approximates the density $p(x_t|\mathbb{Y}_t)$ by a large set of N samples (particles), $\{x_t^{(i)}\}_{i=1}^N$, where each particle has an assigned relative weight, $\gamma_t^{(i)}$, chosen so that all weights sum to unity. The location and weight of each particle reflect the value of the density in the region of the state space. The particle filter updates the particle location and the corresponding weights recursively with each new observed measurement. For the common special case of additive measurement noise, i.e.,

$$y_t = h(x_t) + e_t, \tag{14}$$

the unnormalized weights are given by

$$\gamma_t^{(i)} = p_e(y_t - h(x_t^{(i)})), \quad i = 1, \dots, N. \tag{15}$$

Using the samples (particles) and the corresponding weights the Bayesian equations can be approximately solved. To avoid divergence a resampling step is introduced. This is referred to as the *Sampling Importance Resampling* (SIR), [13], and is summarized in Algorithm 1.

Algorithm 1 Sampling Importance Resampling

1: Generate N samples $\{x_0^{(i)}\}_{i=1}^N$ from $p(x_0)$.
2: Compute $\gamma_t^{(i)} = p_e(y_t - h(x_t^{(i)}))$ and normalize, i.e., $\bar{\gamma}_t^{(i)} = \gamma_t^{(i)} / \sum_{j=1}^N \gamma_t^{(j)}$, $i = 1, \dots, N$.
3: Generate a new set $\{x_t^{(i\star)}\}_{i=1}^N$ by resampling with replacement N times from $\{x_t^{(i)}\}_{i=1}^N$, with probability $\bar{\gamma}_t^{(j)} = Pr\{x_t^{(i\star)} = x_t^{(j)}\}$.
4: $x_{t+1}^{(i)} = f(x_t^{(i\star)}, u_t, w_t^{(i)})$, $i = 1, \dots, N$ using different noise realizations, $w_t^{(i)}$.
5: Increase t and iterate to step 2.

As the estimate for each time we choose the minimum mean square estimate, i.e.,

$$\hat{x}_t = \mathbb{E}\{x_t\} = \int_{\mathbb{R}^n} x_t p(x_t|\mathbb{Y}_t) dx_t \approx \sum_{i=1}^N \gamma_t^{(i)} x_t^{(i)}. \tag{16}$$

The particle filter approximates the posterior pdf, $p(x_t|\mathbb{Y}_t)$, by a finite number of particles. However, asymptotically the approximated pdf converges to the true one, [10].

D. The Marginalized Particle filter (MPF)

If the state-space model contains a linear-Gaussian substructure this can be used to obtain better estimates using the marginalized particle filter (MPF), [3, 8, 9, 11, 12, 15, 28, 31]. For many target tracking applications it is common to use linear-Gaussian dynamics. All non-linearities are then in the measurement relation. Particularly, if not all state variables are present in the measurement relation marginalization can be applied. This is the case for all the applications described in this paper.

Denote the state vector $x_t = \begin{pmatrix} x_t^l & x_t^n \end{pmatrix}^T$, with linear states $x_t^l \in \mathbb{R}^l$ and nonlinear states $x_t^n \in \mathbb{R}^n$. Furthermore, $\mathbb{X}_t^n = \{x_i^n\}_{i=0}^t$ and $\mathbb{Y}_t = \{y_i\}_{i=0}^t$. Using Bayes' theorem we obtain

$$p(\mathbb{X}_t^n, x_t^l|\mathbb{Y}_t) = p(x_t^l|\mathbb{X}_t^n, \mathbb{Y}_t) p(\mathbb{X}_t^n|\mathbb{Y}_t), \tag{17}$$

where $p(\mathbb{X}_t^n|\mathbb{Y}_t)$ is given by a PF and $x_t^l|\mathbb{X}_t^n$ is linear-Gaussian, i.e., $p(x_t^l|\mathbb{X}_t^n, \mathbb{Y}_t)$ is given by the KF. Let us partition the state vector x_t in two parts, $x_t^p \in \mathbb{R}^p$ and $x_t^k \in \mathbb{R}^k$, which are estimated using the PF and the KF respectively, so

$$x_{t+1}^p = A_t^p x_t^p + A_t^k x_t^k + w_t^p, \qquad w_t^p \in \mathcal{N}(0, Q_t^p), \tag{18a}$$
$$x_{t+1}^k = F_t^p x_t^p + F_t^k x_t^k + w_t^k, \qquad w_t^k \in \mathcal{N}(0, Q_t^k), \tag{18b}$$
$$y_t = h_t(x_t^n) + e_t, \qquad e_t \in \mathcal{N}(0, R_t), \tag{18c}$$

where the noise is assumed to be independent. This is no restriction, since the case of dependent noise can be reduced to the case of independent noise using a Gram-Schmidt procedure [18]. This is important since in many target tracking applications, the noise is dependent. In Algorithm 2 the MPF is summarized for the model given in (18).

Algorithm 2 Marginalized Particle Filter (MPF)

1: Initialization: For $i = 1, \dots, N$, initialize the particles, $x_{0|-1}^{p,(i)} \sim p_{x_0^p}(x_0^p)$ and set $\{x_{0|-1}^{k,(i)}, P_{0|-1}^{(i)}\} = \{\bar{x}_0^k, \bar{P}_0\}$. Set $t = 0$.
2: For $i = 1, \dots, N$, evaluate the importance weights $\gamma_t^{(i)} = p(y_t|\mathbb{X}_t^{p,(i)}, \mathbb{Y}_{t-1})$ according to the likelihood

$$p(y_t|\mathbb{X}_t^p, \mathbb{Y}_{t-1}) = \mathcal{N}(h_t(x_t^p), R_t) \tag{19}$$

and normalize $\tilde{\gamma}_t^{(i)} = \frac{\gamma_t^{(i)}}{\sum_{j=1}^N \gamma_t^{(j)}}$.
3: PF measurement update: Resample N particles with replacement according to,

$$\Pr(x_{t|t}^{p,(i)} = x_{t|t-1}^{p,(j)}) = \tilde{q}_t^{(j)}.$$

4: PF time update and Kalman filter update
  1) Kalman filter measurement update,

$$\hat{x}_{t|t}^{k,(i)} = \hat{x}_{t|t-1}^{k,(i)}, \; P_{t|t} = P_{t|t-1}. \tag{20}$$

  2) PF time update: For $i = 1, \dots, N$,

$$x_{t+1|t}^{p,(i)} \sim p(x_{t+1|t}^p|\mathbb{X}_t^{p,(i)}, \mathbb{Y}_t), \tag{21}$$

  where

$$p(x_{t+1}^{p,(i)}|\mathbb{X}_t^{p,(i)}, \mathbb{Y}_t) = \mathcal{N}(A_t x_t^{p,(i)} + A_t^k \hat{x}_{t|t}^{k,(i)}, \; A_t^k P_{t|t} (A_t^k)^T + Q_t^p). \tag{22}$$

  3) Kalman filter time update,

$$\begin{aligned} \hat{x}_{t+1|t}^{k,(i)} &= F_t^k \hat{x}_{t|t}^{k,(i)} + F_t^p x_t^{p,(i)} + \\ &\quad L_t(x_{t+1|t}^{p,(i)} - A_t^p x_t^{p,(i)} - A_t^k \hat{x}_{t|t}^{k,(i)}), \\ P_{t+1|t} &= F_t^k P_{t|t} (F_t^k)^T + Q_t^k - L_t M_t L_t^T, \\ M_t &= A_t^k P_{t|t} (A_t^k)^T + Q_t^p, \\ L_t &= F_t^k P_{t|t} (A_t^k)^T M_t^{-1}, \end{aligned}$$

5: Set $t := t + 1$ and iterate from step 2.

III. Target Tracking Model

There are several possible tracking models for passive ranging problems. The choice of coordinate system, [1, 30, 32, 33], can be one issue. Also modeling of non-kinematic properties, such as intensity [7], could in principle improve the result, but in practice it may be troublesome. In this section a simple tracking model is described. Initialization and target maneuverability are also addressed.

A. Models

Here, we will focus on a discrete-time linear dynamics so all nonlinearities occur in the measurement relation. The described model will then be used in all the applications described in Section IV. For an overview of models within this structure or other common tracking models we refer to [15, 24, 25]. We assume position and velocity coordinates as state variables, and we denote the relative Cartesian state vector $x(t)$, for the difference between the tracked object (target) x^{tg} and the aircraft (tracking platform) x_t^{o}

$$x_{t+1} = x_t^{\text{tg}} - x_t^{\text{o}}. \tag{24}$$

Both target and aircraft are described by linear state equations

$$x_{t+1}^{\text{tg}} = F_t x_t^{\text{tg}} + G_t u_t^{\text{tg}}, \tag{25}$$

$$x_{t+1}^{\text{o}} = F_t x_t^{\text{o}} + G_t u_t^{\text{o}}. \tag{26}$$

Hence, in relative coordinates the state equation is

$$x_{t+1} = F_t \underbrace{(x_t^{\text{tg}} - x_t^{\text{o}})}_{x_t} + G_t \underbrace{u_t^{\text{tg}}}_{w_t} - G_t \underbrace{u_t^{\text{o}}}_{u_t}. \tag{27}$$

Since the target input signal is unknown, it is considered as process noise in the model, $w_t = u_t^{\text{tg}}$. If we assume that the state vector consists of Cartesian position and velocity components

$$x_t = \begin{pmatrix} X_t & Y_t & Z_t & \dot{X}_t & \dot{Y}_t & \dot{Z}_t \end{pmatrix}^T, \tag{28}$$

then the system matrices are given by

$$F = \begin{pmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, G = \begin{pmatrix} T^2/2 & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & 0 & T^2/2 \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{pmatrix}. \tag{29}$$

The observation relation consists of azimuth and elevation angle measurements or possibly only azimuth measurements and we assume an additive noise term

$$y_t = h(x_t) = \begin{pmatrix} \varphi \\ \theta \end{pmatrix} + e_t = \begin{pmatrix} \arctan(Y_t/X_t) \\ \arctan(-Z_t/\sqrt{X_t^2 + Y_t^2}) \end{pmatrix} + e_t. \tag{30}$$

In our approach, a Gaussian distribution is applied, $e_t \in \mathcal{N}(0, R)$.

In the above model the process noise represents the maneuverability of the unknown target. For long range applications, when the target is assumed not to have detected the tracker a common model used is to assume a straight flying path, i.e., a small process noise. For maneuvering targets a higher value of the noise can be used, but this will reduce performance. Often it may be necessary to introduce multiple turn models, which could be handled by using for instance the *Interacting Multiple Model* (IMM), [5], instead of single extended Kalman filters in the filter bank. Another possibility is to use a change detector and adjust the estimate or system when a maneuver is detected, [16]. In [7], some aspects of the angle-only problem are described in more detail.

In the model only the position states, X_t, Y_t and Z_t, are present in the measurement relation. Using the notation from Section II-D this means that $x_t^n = \begin{pmatrix} X_t & Y_t & Z_t \end{pmatrix}^T$ and $x_t^l = \begin{pmatrix} \dot{X}_t & \dot{Y}_t & \dot{Z}_t \end{pmatrix}$. Hence, the particle filter dimension is reduced from $\mathbb{R}^6$ to $\mathbb{R}^3$. This means that a lot of computational time is saved, since the Kalman filter handles the linear-Gaussian subspace more efficiently. Even though the MPF introduces extra calculations this is a more efficient method for many system, since the number of particles needed can be reduced. In [22] this is analyzed more thoroughly, where the computation complexity of the MPF is discussed in a radar tracking application using a similar dynamical model as the one described in this section. Also note that the marginalization improves the performance, i.e., reduce the estimation variance.

B. Initialization

There are many possible ways to initialize a passive ranging estimator. Classical initialization methods are based on measurement initialization for Kalman filters, see for instance [5, 7]. In the implemented angle-only application the filter initialization is performed in Cartesian coordinates, projecting the assumed range hypothesis to the line-of-sight (LOS) using the measured angles. The velocity consists of the known velocity for the tracking platform. The unknown target velocity is accounted for in the initial uncertainty covariance. The initial value of the relative state vector, assuming no knowledge of the target velocity, is for each filter

$$x_0^{(i)} = \begin{pmatrix} r^{(i)} \cos\varphi_m \cos\theta_m \\ r^{(i)} \sin\varphi_m \cos\theta_m \\ -r^{(i)} \sin\theta_m \\ 0 - \dot{X}^{\text{o}} \\ 0 - \dot{Y}^{\text{o}} \\ 0 - \dot{Z}^{\text{o}} \end{pmatrix}, \tag{31}$$

where φ_m and θ_m are the measured angle values. The initial state covariance matrix in the Line-of-Sight (LOS) system is in Cartesian coordinates given by

$$P_{0_{\text{LOS}}}^{(i)} = \left(\begin{array}{ccc|c} (\sigma^{(i)})^2 & 0 & 0 & \\ 0 & (r^{(i)}\Delta\varphi)^2 & 0 & O_{3\text{x}3} \\ 0 & 0 & (r^{(i)}\Delta\theta)^2 & \\ \hline & O_{3\text{x}3} & & (\Delta v)^2 I_{3\text{x}3} \end{array}\right), \tag{32}$$

where Δv is the maximal uncertainty in the target velocity, $\Delta\varphi$ and $\Delta\theta$ are the angle measurement noise standard deviation. The initial covariance matrix is calculated using the rotation matrix R_{rot} as

$$P_0^{(i)} = R_{\text{rot}} P_{0_{\text{LOS}}}^{(i)} R_{\text{rot}}^T, \; R_{\text{rot}} = \left(\begin{array}{c|c} (R_{I,B})^T & O_{3\text{x}3} \\ \hline O_{3\text{x}3} & (R_{I,B})^T \end{array}\right), \tag{33}$$

where

$$R_{I,B} = \begin{pmatrix} \cos\varphi_m & \sin\varphi_m & 0 \\ -\sin\varphi_m & \cos\varphi_m & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_m & 0 & -\sin\theta_m \\ 0 & 1 & 0 \\ \sin\theta_m & 0 & \cos\theta_m \end{pmatrix}, \tag{34}$$

evaluated at the initial measurement angles. If a single doppler radar measurement is present the initial range and range rate uncertainty can be set to the measurement error for the radar, thus increasing performance substantially.

IV. Passive Ranging Applications

A. Air-to-Air Passive Ranging

In Fig. 3 an air-to-air passive ranging is illustrated for an IR sensor. This section is based on a part of the passive ranging application from [21], where the RPEKF in Cartesian coordinates is compared to the PF method in a Monte Carlo simulation study.

Fig. 3. Air-to-air passive ranging.

We assume that the target is non-maneuvering and that angle observations are available with a sample period of $T = 1$ s. The initialization for PF is similar to the EKF based approach, but instead of assuming Gaussian distribution around different working points, we use a uniform distribution in the range direction. In the azimuth and elevation the particles are drawn randomly from a Gaussian density around the initial measurement strobe. The particle filter implementation is straightforward and in accordance with Section II-C.

The relative target height is 4000 m above the tracking platform. The measurement noise was assumed Gaussian with angle standard deviation $\sigma_\varphi = \sigma_\theta = 1$ mrad for the IR sensor. The standard deviation for the initial velocity used to calculate P_0 was 200 m/s. The realizations in the Monte Carlo simulation use independent measurement noise and the initial position for each realization is perturbed by a uniform distribution around its nominal value. The RPEKFs have range interval $r_{\min} = 1$ and $r_{\max} = 64$ km, with N_F=6 filters, yielding $C_R = 0.1925$ as in [4, 29]. For the particle filter method, $N = 60000$ particles are used. In the evaluation $N_{MC} = 40$ Monte Carlo simulations were performed, over $L = 70$ s using the same scenario, but with different measurement noise realizations. The performance is evaluated using the root mean square error for each time, given in Fig. 4, according to [14]

$$\text{RMSE(t)} = \sqrt{\frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} ||x_t^{\text{true}} - x_{t,j}||_2^2}, \tag{35}$$

where $x_{t,j}$ denotes the estimate at time t, for Monte Carlo simulation j. For the angle-only tracking problem, only position coordinates are used in the RMSE calculation. As seen the PF outperforms the RPEKF in terms of RMSE performance. To reduce the RMSE to almost the same level as for the particle filter, more than $N_F \gtrsim 60$ filters had to be used. The MPF RMSE plot will be included in the final version of the paper.

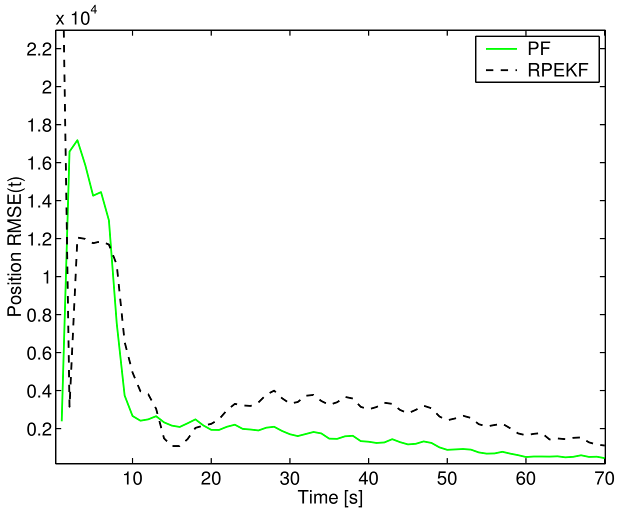

Fig. 4. Position RMSE(t) for PF and RPEKF using $N_{MC} = 40$ Monte Carlo simulations.

B. Air-to-Sea Passive Ranging

In this section we consider an air-to-sea passive ranging application from [20] as illustrated in Fig. 5. By maneuvering the aircraft, estimates of range and range rate become available as IR sensor information is processed. We use the passive particle filter tracking system described in Section IV-A for the position estimation. The main objective in this section is

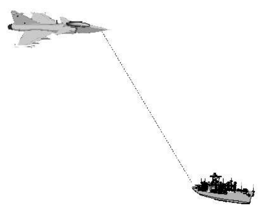

Fig. 5. Passive ranging for the air-to-sea application.

to merge terrain type information from a terrain database to the kinematic part in order to discard regions uninteresting to the tracking application. Terrain induced tracking constraints improve tracking performance and reduce the computational complexity. Here we only distinguish between land and sea, but for other applications, more detailed terrain type information could be used.

The tracking filter is implemented according to the particle filter method presented in Algorithm 1. In the particle filter each particle, $x_t^{(i)}$, represents a possible target location.

Hence, using the terrain database, particles are discarded if the position is within a restricted area (land), whereas particles belonging to accepted regions (sea) are accepted and assigned with a weight according to a likelihood function.

1) Terrain constraints via measurement update: A natural approach to introduce constraints is by using the importance weights calculated in the measurement update. Here we can interpret the database as an extra sensor in a larger sensor fusion context.

$$\begin{aligned} w_t^{(i)} &= p_{e_t}(e_t) = p_{e_t}(y_t - h(x_t^{(i)})) \\ &= \begin{cases} p_{e_t}(y_t - h(x_t^{(i)})), & x_t^{(i)} \in \text{Sea area} \\ 0, & \text{otherwise} \end{cases}. \end{aligned} \tag{36}$$

2) Terrain constraints via time update: By introducing the constraints in the state equation, each particle is accepted if the predicted position, after the additive process noise has been added, belongs to a sea region in the database.

$$\begin{aligned} p_{v_t}(v_t) &= p_{v_t}(x_{t+1}^{(i)} - f(x_t^{(i)})) \\ &= \begin{cases} p_{v_t}(x_{t+1}^{(i)} - f(x_t^{(i)})), & x_t^{(i)} \in \text{Sea area} \\ 0, & \text{otherwise} \end{cases}. \end{aligned} \tag{37}$$

It is easy to impose other constraints. For example, hard constraints on state-variables such as velocity and acceleration. The fact that sea-targets are close to the surface can be easily handled. All these constraints are difficult or extremely troublesome to handle with classical Kalman filter techniques.

In a simulation study we consider the range estimation problem using an IR sensor, which measures azimuth and elevation angles relative to the target. The particle filter is initialized around the first measurement strobe, where we used an IR sensor with a total angle error of 1 mrad. The range values are drawn from a uniform distribution, over the range of interest, and we used $N = 5000$ particles. We consider a track-while-scan (TWS) application, where the sample period is $T = 1$ s. The aircraft's velocity was about 250 m/s and a constant height sinusoidal maneuver was performed to gain observability. The state equation and measurement relation are similar to those described in Section IV-A. The target model used in the simulations assumes a small constant velocity. The terrain database has a resolution of 50 m.

In Fig. 6 the scenario is presented together with the marginal position densities in each direction, $p(X)$ and $p(Y)$, for time $t = 1$ s, when constraints are used. The marginal distributions are zero over land areas. As a result, the number of particles needed is reduced when imposing the terrain constraints. The terrain constraints were incorporated in the calculation of the importance weights as described in (36). By performing maneuvers, only particles that satisfy the motion and measurement models acquire high enough probability. In Fig. 7 the scenario is presented for time $t = 15$ s. As seen, the particle cloud is located in the vicinity of the true target.

C. Sea-to-Sea Passive Ranging

Modern torpedo systems are equipped with an acoustic seeker, which is similar to the electro-magnetic radar. This sound based sensor is referred to as *sonar*. In the active mode both range and bearing to a target are available. To avoid being detected by a hostile target and to reduce the risk of hostile counter-measurements, it is often important to minimize the active mode usage. In the passive mode, the acoustic sensor just listens for target related sounds. Hence in principle only the direction can be measured. Most sonar based systems do not measure the elevation angle, therefore only bearing information is available. When the torpedo is tracking a sea target using the passive sensor mode the range estimation must be performed by maneuvering the torpedo to gain observability. In this section we focus on a torpedo system and apply the passive tracking techniques from Section IV-A, using both the RPEKF and particle filter methods. The bearing information is from experimental sonar data acquired from a torpedo system provided by Saab Bofors Underwater Systems.

The scenario in the position plane is given in Fig. 8 where torpedo way-points are marked with time indices. The true position of the target (ship) is not known since no true position information is available. However, it is known that the ship follows a rather straight path, with nearly constant velocity as

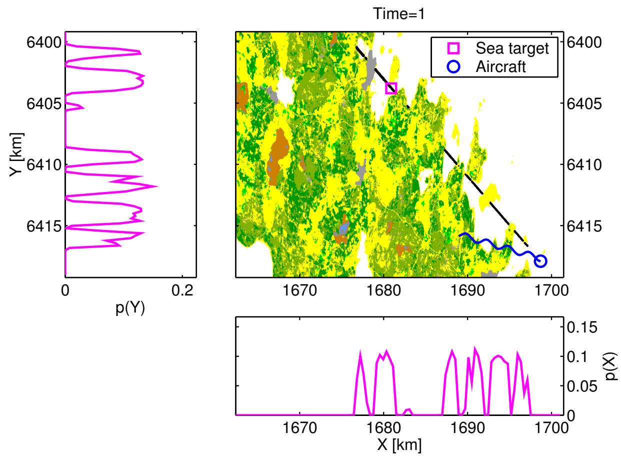

Fig. 6. The sea target position in RT90 coordinates and marginalized position pdf using the particle filter with constraints at $t = 1$ s. The particle cloud and the future trajectory of the aircraft are also shown.

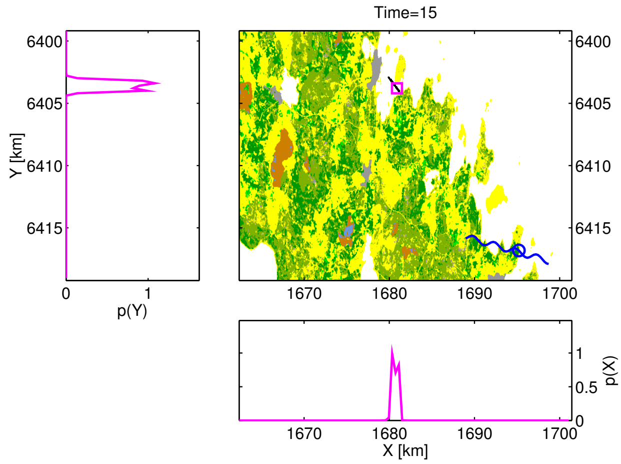

Fig. 7. Particle filter with constraints at $t = 15$ s.

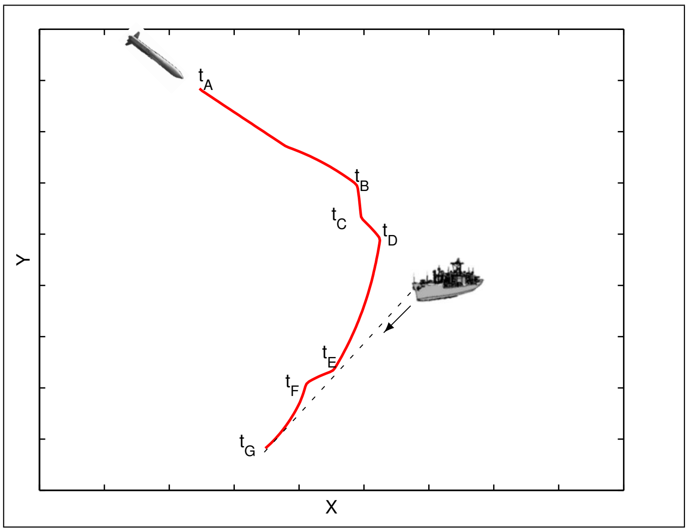

Fig. 8. Torpedo trajectory from experimental data with time indications at way-points and and an indication of the ship's true trajectory.

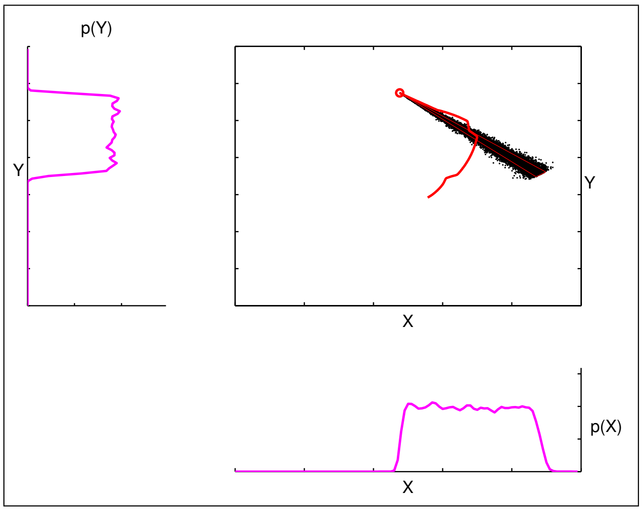

Fig. 9. Torpedo trajectory, particle cloud and marginalized densities for target position at $t = t_A$.

indicated in the figure. Since the torpedo approaches the target from behind in the final phase ($t > t_F$) and impact occurs at $t = t_G$, the probable approximate true target position is indicated with a dashed line in Fig. 8.

Between the initial time t_A and first major maneuver at t_B the torpedo follows a relatively straight trajectory. During this time the range estimation is not particularly good since the small maneuver and the relative geometry to the target does not reveal much range information. At t_B the first major maneuver is performed to gain observability, followed by maneuvers at t_C and t_D. During the long interval $t_D - t_E$ a straight path is followed, where the main objective is to decrease the distance to the ship. Finally, at t_E and t_F maneuvers are performed to gain observability and to approach the target from behind. The bearing measurements from the sonar are given with a sample period of T s, and the total number of measurements is close to 300. The position scale and the actual values of the maneuver times are not presented in any plot, since it is confidential information. We apply both the RPEKF and the particle filter to the experimental torpedo data. The filters are initialized in accordance with Section IV-A using the initial bearing measurement. We use $N = 15000$ particles for the PF. In Fig. 9, the output from the particle filter is shown for $t = t_A$. Both the target position and the marginalized target position probability densities are given. Initially, no maneuver is made so the range can not be estimated. The full torpedo position trajectory is shown, where the current torpedo position is indicated with a small circle. Here we did not utilize any external data about the range uncertainty. In practice the range uncertainty region can be reduced by using data from other external sources.

Since we estimate the relative velocity, the target speed and heading can be calculated as illustrated in Fig. 10 for PF and RPEKF method. The minimum mean square estimate of target position from the particle filter is presented in Fig. 11 together with the estimation from the RPEKF method using $N_F = 6$ filters. The range uncertainty and how fast it converges depends on how the initial uncertainty region was chosen and on the number of filters and particles.

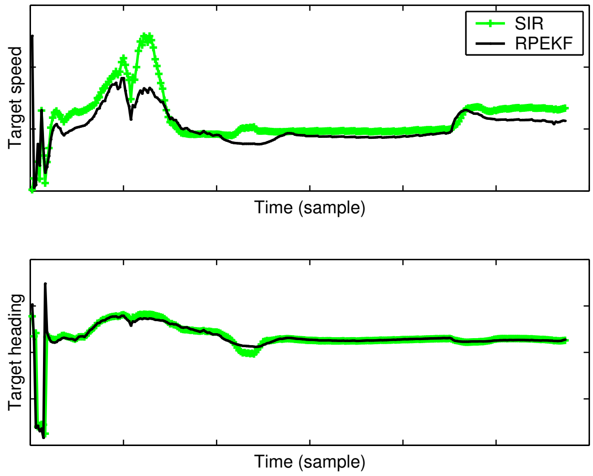

Fig. 10. Target speed and heading estimate.

V. Conclusions

In this paper Bayesian estimation methods for several bearings-only applications are discussed. The particle filter can handle any noise distribution. Hence, the initialization is easy and optimal for the bearings-only problem, since it is natural to initialize the filter with a uniform range distribution over all possible target distances. If the angle measuring sensor is non-Gaussian this can also be incorporated. If constraints are present on the system state or from external sources, such as terrain information, the particle filter can easily incorporate these, hence improving performance. This is impossible or very troublesome with the EKF approach. The particle filter also improves the estimation performance since no linearizations are necessary. If a linear-Gaussian substructure is present, the MPF can be used. This will then further improve performance and reduce the computational complexity. As

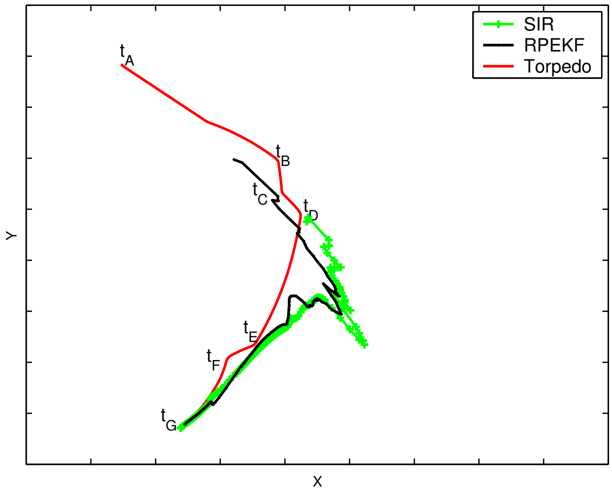

Fig. 11. Torpedo trajectory, minimum mean square estimate of target position using PF and RPEKF methods.

a comparison to the simulation based particle filter, a bank of EKFs are used, in the RPEKF method. In an Air-to-Air passive ranging simulation study the PF outperformed the RPEKF method using the RMSE as performance measure. These methods were also successfully applied to experimental data from a torpedo system using a sonar sensor.

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