Wave digital filter theory is based on a scattering parameter formalism. The one-port network can be described by the incident and reflected waves instead of voltages and currents.

The steady-state voltage waves are defined as

\[ V_\text{A} = \frac{Z_0 + Z}{Z_0 - Z} S \]

where \( Z_0 \) is the characteristic impedance and \( Z \) is the propagation line.

Transmission lines are often referred to as line elements. Transmission lines are sometimes called the characteristic lines, while lossless transmission lines and the steady-state voltage waves are defined as

\[ \frac{V_\text{A} + Z}{V_\text{A} - Z} = S \]

and the impedance is described by \( Z = \frac{1}{T} \). We get

\[ V_\text{A} - A = \frac{1}{L} \]
\[ V_\text{A} + A = V \]

The steady-state voltage waves are defined as

\[ V = \frac{1}{L} \]

where \( L \) is the load resistance, \( B \) is the reflected wave, and \( R \) is a positive real constant, called the characteristic resistance.

A one-port can be described by the reflection function defined as

\[ \frac{V + Z}{V - Z} = S \]
Obviously, a transmission line cannot be described by poles and zeros since the elements in the chain matrix are not rational functions in $s$.

Wave digital filters imitate reference filters built out of resistors and lossless transmission lines by means of incident and reflected voltage waves. Computable digital filter algorithms can be obtained if the reference filter is designed using only such transmission lines. Wave digital filter design involves synthesis of such reference filters. Commensurate-length transmission line filters constitute a special case of distributed element networks that can easily be designed by mapping them to a lumped element structure.

This mapping involves Richards’ variable which is defined as

$$Y = \frac{Z + Y}{Z + Y}$$

where

$$Y = \frac{Z + Y}{Z + Y}$$

and

$$Y = \frac{Z + Y}{Z + Y}$$

The input impedance of a lossless transmission line with characteristic impedance $Z_0$, loaded with an impedance $Z_2$ is

$$Z_{in} = \frac{Z_2}{Z_0} + \frac{Y_2}{Z_0}$$

where

$$Y_2 = \frac{V_2}{I_2}$$

The real frequencies in the $s$- and $Y$-domains are related by

$$\frac{Z}{Z} = \frac{1 + e^{j\omega t}}{1 - e^{j\omega t}}$$

This mapping involves Richards’ variable which is defined as

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The transmission line filters of interest are, with a few exceptions, built using only one-ports.

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Graphs called in analogy:

Transmission Line Filters

Analogies

Interconnection Networks

The corresponding wave-flow graphs to a short-circuit an open-circuit are

Wave-Flow Building Blocks

Transmission Line Filters
Symmetric Two-Port Adaptor

The symbol for the symmetric two-port adaptor that corresponds to a connection of two ports.

The incident and reflected waves for the two-port are

\[ A_1 \quad R_1 \quad B_1 \]

\[ A_2 \quad R_2 \quad B_2 \]

At the interconnection we have, according to Kirchhoff's current and voltage laws

\[ I_1 = I_2 \]

\[ V_1 = V_2 \]

By eliminating voltages and currents we get the following relation between incident and reflected waves for the symmetric two-port adaptor

\[ A_1 \quad V_1 \quad R_1 \quad I_1 \]

\[ B_1 \quad V_1 \quad R_1 \quad I_1 \]

The adaptor coefficient \( a \) is usually written on the side corresponding to port 1. As can be seen, the wave-flow graph is almost symmetric.

Note that \( a = 0 \) for \( R_1 = R_2 \). The adaptor degenerates into a direct connection of the two ports and the incident waves are not reflected at the point of interconnection.

For \( R_2 = 0 \) we get \( a = 1 \) and the incident wave at port 1 is reflected and multiplied by \(-1\) while for \( R_2 = \infty \) we get \( a = 1 \) and the incident wave at port 1 is reflected and multiplied by \(-1\) without a change of sign.

The adaptor coefficient is usually written on the side corresponding to the port, but the flow graph is almost symmetric.

\[ \frac{V_2 + I_2}{V_2 - I_2} = \frac{1 - I_2}{1 + I_2} \]

\[ \frac{V_2}{I_2} \quad \frac{V_2}{I_2} \]

Design of Wave Digital Filters

By eliminating voltages and currents we get the following relation between incident and reflected waves for the symmetric two-port adaptor

\[ z_A = 1_A \]

\[ z_{-A} = 1_{-A} \]

At the interconnection we have, according to Kirchhoff's current and voltage laws

The incident and reflected waves for the two-port are

\[ z_I x + z_A = z_I y + z_{-A} \]

The symbol for the symmetric two-port adaptor that corresponds to a connection of two ports.

\[ z_{-A} = z_I y \quad \text{and} \quad z_{-A} = z_I x + z_{-A} \]

The incident and reflected waves for the two-port are

\[ z_I y + z_{-A} = z_I y + z_{-A} \]

The symbol for the symmetric two-port adaptor that corresponds to a connection of two ports.